QATs

Quality Assessment Tasks

UNIT 3 ■ OUTCOMES 1 & 3

VCE Mathematical Methods (CAS)

SCHOOL-ASSESSED COURSEWORK

Introduction

Outcome 1
Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Outcome 3
Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Task
Test – multiple-choice, short-answer and extended response items.

The task has been designed to allow achievement up to and including the highest level in the Performance Descriptors and meets a broad range of **key knowledge** and **key skills** related to each outcome.

This is one of two tests. The two tests contribute 20 of the total marks (60) allocated for SAC in Unit 3.

This task will be marked out of 40 and then will be converted to a proportion of the contribution of this task to SAC in this unit.

The marks for each question are indicated in brackets.

The test is of 90 minutes duration. The formula sheet for end-of-year examinations may be used in the test, and you may bring up to four A4 (two sides of two pages) of summary notes.

Access to an approved CAS calculator will be allowed.

CAS TECHNOLOGY Indicates where use of the technology is specifically required in order to answer the question.

Answer in spaces provided or as indicated.

Your teacher will advise you of any variation to the above conditions.

NAME:

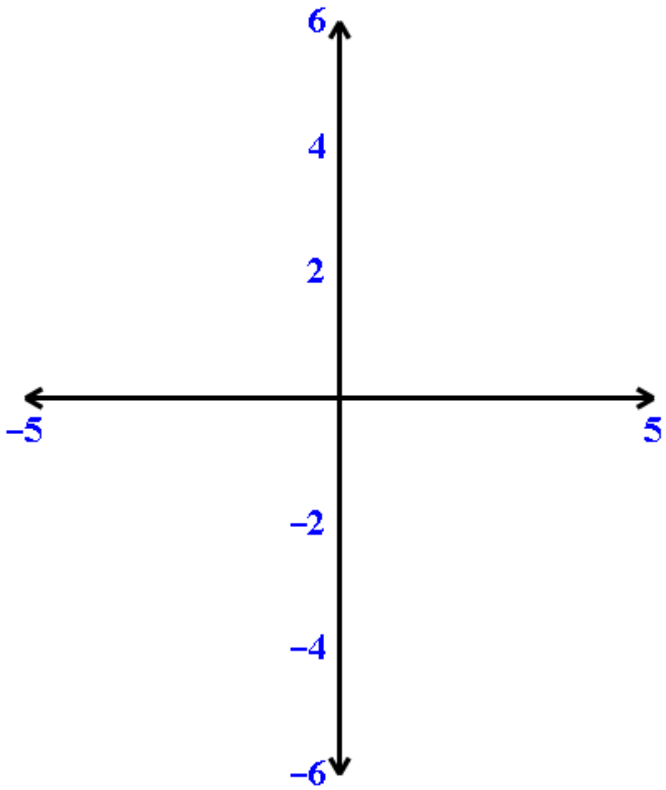
Task

MARKS: /20 /10 /10

Part A: Short Response

Question 1

Plot the function $f(x) = 2\sqrt{4-x} - 2 \quad -5 \le x \le 5$ on the axes provided. Include values of intercepts and endpoints. (3 marks)



Question 2

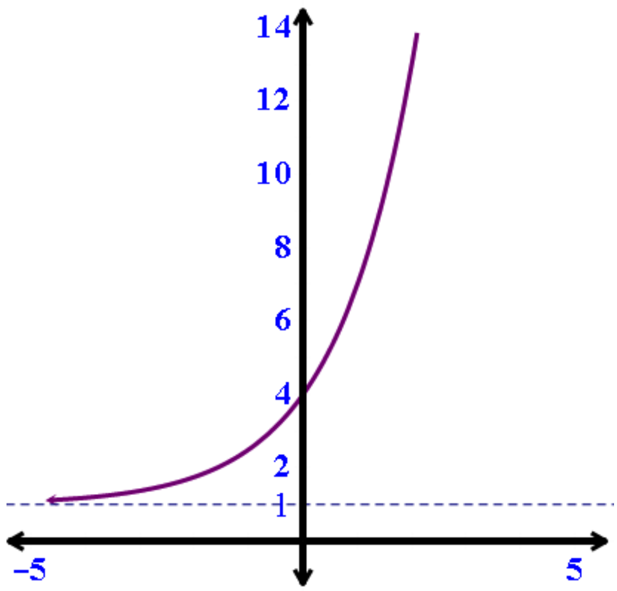
For the equation: $(x+2)(x-1)(x+a) = x^3 + bx^2 + 2x - 8$, find the values of a and b.

(2 marks)

Task

Question 3

An exponential function of the form $f(x) = A.2^x + b$ has a graph as shown.



a. Write down the value of *b*.

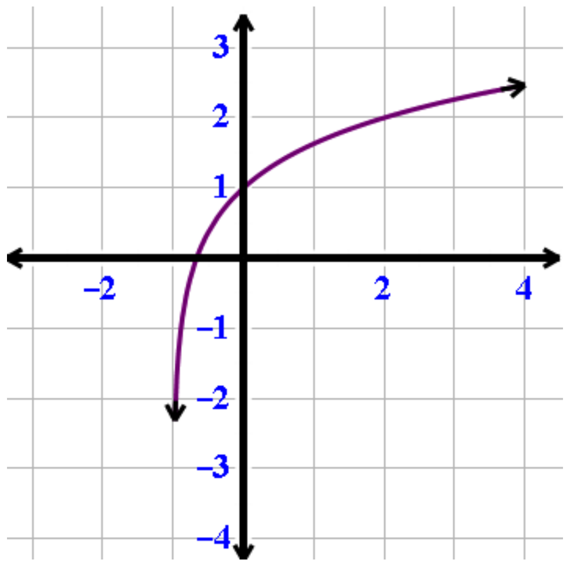
b. Hence or otherwise find the value of *A*.

c. Now calculate the value of *f(1)*.

(1 + 2 + 1= 4 marks)

Question 4

Consider the function $f(x) = \log_3(x+1)+1$ the graph of which is shown.



a) Determine the inverse function of *f(x)*.

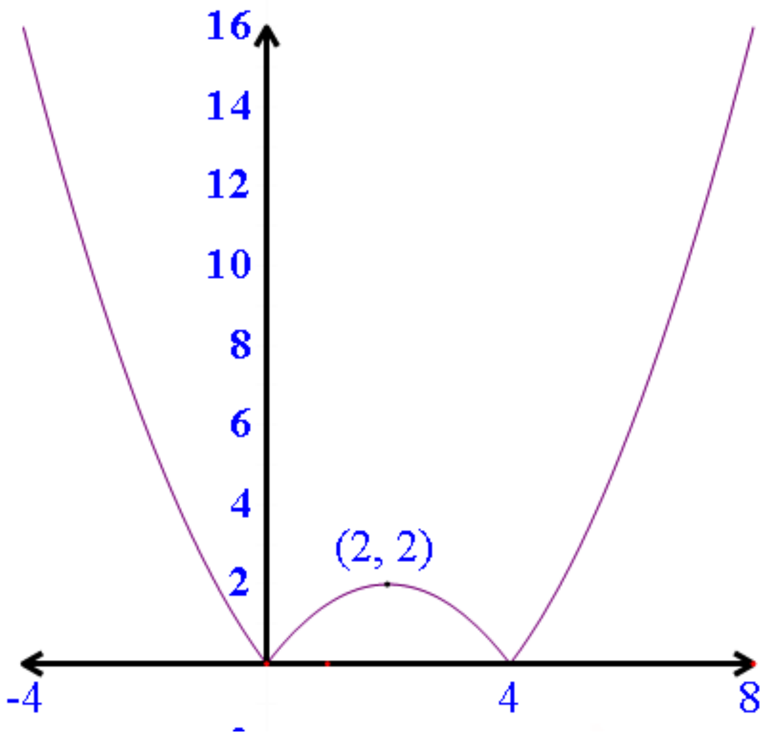
b) Sketch the graph of the inverse function on the axes provided. **It is not necessary to find the values of the points of intersection**.

(2 + 2 marks= 4 marks)

Task

Question 5

The graph of a function of the form $f(x) = \left|a(x-h)^2 + k\right|$ is shown.



a) Write down the values of *h* and *k*.

$h =$

$k =$

b) Hence determine the value of *a*.

(2 + 1= 3 marks)

Task

NOTE TO STUDENT: ONLY DO QUESTIONS 6 AND 7 (LOGARITHMS AND EXPONENTIALS) IF DIRECTED BY YOUR TEACHER.

Question 6

Consider the functions $f(x) = e^x$ and $g(x) = \ln(3x)$.

Fully define the function f(g(x), including its maximal domain.

(2 marks)

Question 7

Solve $\log_a(8) + \log_a(2) = 2\log_5(25)$ for a. Show all working.

(2 marks)

Task

NOTE TO STUDENTS: ONLY DO QUESTION 8 (CIRCULAR FUNCTIONS) IF DIRECTED BY YOUR TEACHER.

Question 8

a) Describe the transformations necessary to change sin(x) into $\sin\left(2\left(x-\frac{\pi}{3}\right)\right)+1$

b) What is the period of the transformed function?

(3 + 1= 4 marks)

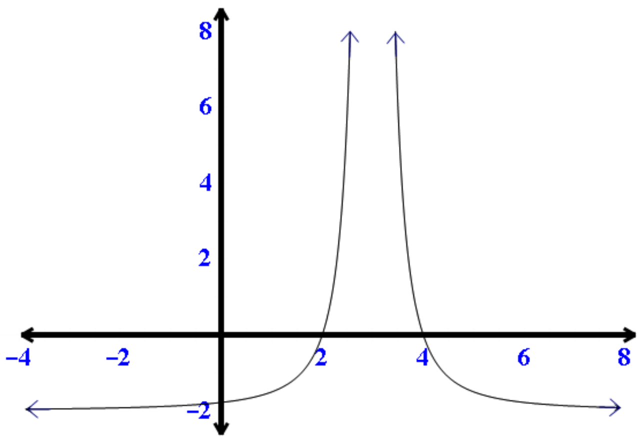
Task

Part B: Multiple Choice and Extended Response

One mark for each correct multiple choice. Circle your correct response.

Question 9

The equation of the function with the graph shown could be:



A. $f(x) = (x-3)^2 - 2$

B. $f(x) = \dfrac{1}{(x-3)} - 2$

C. $f(x) = \dfrac{1}{(x+3)^2} - 2$

D. $f(x) = \dfrac{1}{(x-3)^2} - 2$

E. $f(x) = \dfrac{1}{(x+3)} - 2$

Task

Question 10

A cubic of the form $f(x)=ax^3+bx^2+cx+d$ passes through the points (-1, 0), (0, -1), (1, -6) and (2, 9). A matrix equation that can be used to find the values of *a, b, c* and *d* is:

A. $$\begin{bmatrix}-1 & 1 & -1 & 1\\ 0 & 0 & 0 & 1\\ 1 & 1 & 1 & 1\\ 8 & 4 & 2 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}=\begin{bmatrix}0\\ 1\\ 6\\ 9\end{bmatrix}$$

B. $$\begin{bmatrix}-1 & 1 & -1 & 1\\ 0 & 0 & 0 & 1\\ 1 & 1 & 1 & 1\\ 8 & 4 & 2 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}=\begin{bmatrix}0\\ -1\\ -6\\ 9\end{bmatrix}$$

C. $$\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 0 & 0 & 1\\ 1 & 1 & 1 & 1\\ 8 & 4 & 2 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}=\begin{bmatrix}0\\ -1\\ -6\\ 9\end{bmatrix}$$

D. $$\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}=\begin{bmatrix}-1 & 1 & -1 & 1\\ 0 & 0 & 0 & 1\\ 1 & 1 & 1 & 1\\ 8 & 4 & 2 & 1\end{bmatrix}\begin{bmatrix}0\\ -1\\ -6\\ 9\end{bmatrix}$$

E. $$\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}=\begin{bmatrix}-1 & 1 & -1 & 1\\ 0 & 0 & 0 & 1\\ 1 & 1 & 1 & 1\\ 8 & 4 & 2 & 1\end{bmatrix}\begin{bmatrix}0\\ -1\\ -6\\ 9\end{bmatrix}$$

Task

CAS TECHNOLOGY

Question 11

A quartic with equation $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$ has x-intercepts at:

A. (-3, 0), (-2, 0), (-1, 0), (2, 0)

B. (-2, 0), (1, 0), (2, 0), (3, 0)

C. (-3, 0), (-2, 0), (1, 0)

D. (-3, 0), (1, 0), (2, 0)

E. (-3, 0), (-2, 0), (1, 0), (2, 0)

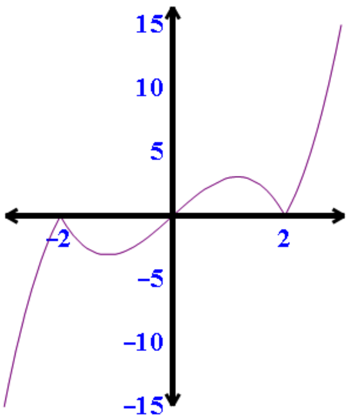
Task

CAS TECHNOLOGY

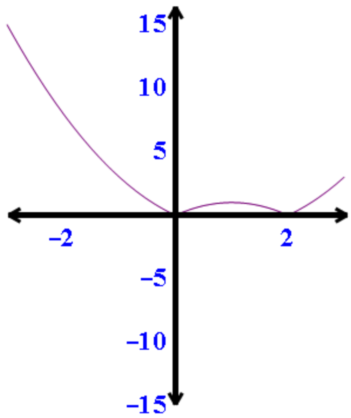
Question 12

The graph of the function $f(x) = |x^2 - 4|\ x$ is:

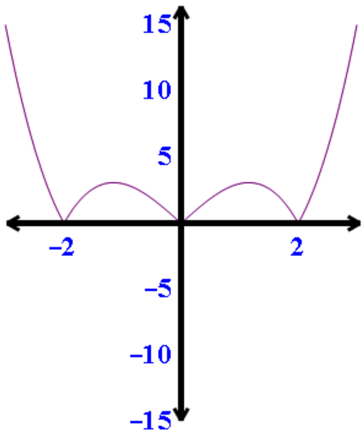
A.



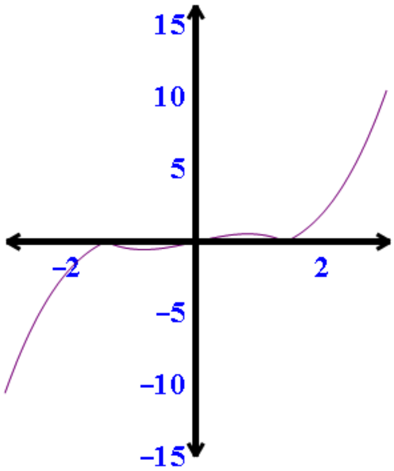
D



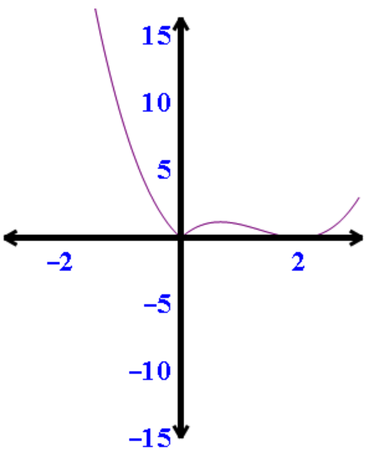
B.



E



C.



Task

CAS TECHNOLOGY

Question 13

The exact values of the coordinate(s) of intersection between the functions $f(x) = 2x^2e^x$ and $g(x) = e^x$ are:

A. $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ *and* $\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$

B. $\left(-\sqrt{2}, e^{-\sqrt{2}}\right)$ only

C. $\left(\frac{-\sqrt{2}}{2}, e^{\frac{-\sqrt{2}}{2}}\right)$ only

D. $\left(-\sqrt{2}, e^{-\sqrt{2}}\right)$ *and* $\left(\sqrt{2}, e^{\sqrt{2}}\right)$

E. $\left(\frac{-\sqrt{2}}{2}, e^{\frac{-\sqrt{2}}{2}}\right)$ *and* $\left(\frac{\sqrt{2}}{2}, e^{\frac{\sqrt{2}}{2}}\right)$

Question 14

For the product of the functions $f(x) = 2^x$ and $g(x) = x^2$ which of the following statements is NOT true?

A. $f(x).g(x)$ has a minimum turning point at $x = 0$

B. $f(x).g(x)$ has an asymptote at $x = 0$

C. $f(x).g(x)$ has two turning points, one of which is a maximum and the other a minimum

D. $f(1).g(1) = 2$

E. The domain of $f(x).g(x)$ = domain of $f(x)$

Task

Question 15

If $\log_6(b) + \log_6(c) = 1$ then b and c could be:

A. 1 and 0

B. 2 and 4

C. 6 and $\frac{1}{3}$

D. 12 and $\frac{1}{2}$

E. 6 and 0

NOTE TO STUDENTS: ONLY ANSWER QUESTIONS 16 TO 18 (LOGARITHMS AND EXPONENTIALS) IF DIRECTED BY YOUR TEACHER.

CAS TECHNOLOGY

Question 16

The sum of the solutions of the equation $\ln(2x^2 - 8x + 10) = 2$ is closest to:

A. 4

B. 0.3585

C. 3.6415

D. 3.9998

E. 0.6931

CAS TECHNOLOGY

Question 17

The expression 2^{4x-3} is the same as:

A. 8×2^{4x}

B. 16^{x-3}

C. 8^{x-3}

D. $\frac{16^x}{8}$

E. 2^x

Task

Question 18

The expression $\ln(16)+4\ln(3)-\ln(6)$ is equivalent to:

A. $ln(22)$

B. 4

C. $3ln(6)$

D. $ln(18)$

E. $ln(215)$

NOTE TO STUDENTS: ONLY ANSWER QUESTIONS 19 TO 21 (CIRCULAR FUNCTIONS) IF DIRECTED BY YOUR TEACHER.

Question 19

The general solution to $2\cos(3x)+1=2$ is:

A. $\frac{2n\pi}{3} \pm \frac{\pi}{3}$

B. $\frac{2\pi}{3} \pm \frac{\pi}{9}$

C. $\frac{n\pi}{3} \pm \frac{\pi}{9}$

D. $\frac{2n\pi}{3} \pm \frac{\pi}{9}$

E. $\frac{n\pi}{3} \pm \frac{\pi}{3}$

Task

Question 20

$\sin\left(\frac{5\pi}{12}\right)$ is the same as:

A. $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right)$

B. $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

C. $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

D. $2\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)$

E. $2\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

Question 21

For the function $f(x) = \tan(4x),\ 0 \le x \le \frac{\pi}{3}$ the equation of asymptote is:

A. $x = \frac{\pi}{8}$

B. $x = \frac{(2k+1)\pi}{8}$

C. $x = \frac{(2k+1)\pi}{4}$

D. $x = \frac{3\pi}{8}$

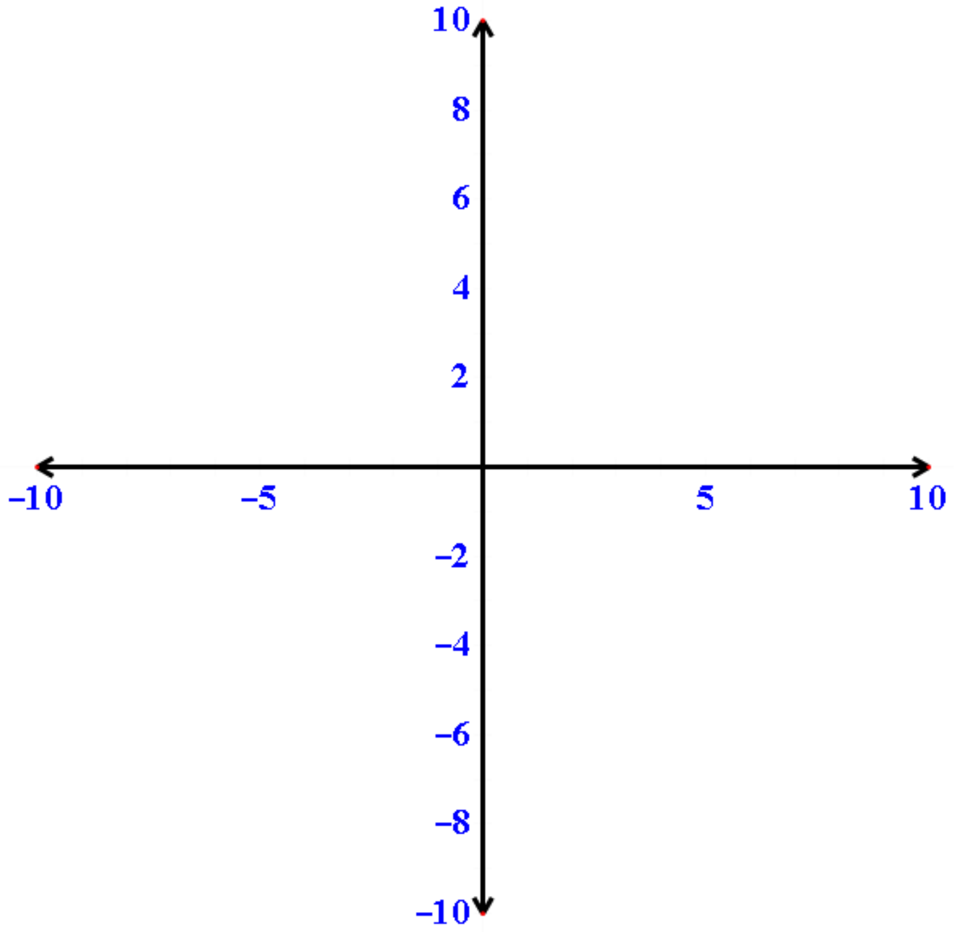
E. There is no asymptote within this domain

Task

CAS TECHNOLOGY

Question 22 (Extended response)

a) Graph the linear equation $f(x) = x - 1$ on the axes provided. (1 mark)



b) On the same set of axes, sketch the function $g(x) = \frac{1}{4}(x+3)^2 - 9$ (2 marks)

c) Use your calculator to determine the exact values of the coordinates of intersection of $f(x)$ and $g(x)$.

(3 marks)

Task

In "Turning Point Form" , $h(x)=\frac{1}{4}(x+3)^2+k$, k translates the parabola parallel to the y-axis. There is one value of k for this family of curves that makes $f(x) = x - 1$ a tangent to $h(x)$.

d) Let $f(x) = g(x)$ and show algebraically that

$$0=\frac{1}{4}x^2+\frac{1}{2}x+\frac{13}{4}+k$$

(2 marks)

e) Hence, write down the values of the coefficients a, b, and c of the new equation in standard form.

$a =$ $b =$ $c =$

(1 mark)

f) Now find the value of k that makes $f(x)$ tangent to $h(x)$.

(1 mark)

Teacher Advice

This is the Test 1 – Functions, as suggested to be undertaken in week 9 in the **sample teaching sequence** on page 142 of the VCAA Study Design.

Part A of the test has been designed to *be done as a technology free task if teachers so wish*.

This test covers assessment in:
- Functions and graphs
- Transformations
- Polynomial functions
- Exponential and logarithmic functions

Circular Functions may also be assessed in this test. There are alternative questions for Part A (Question 6 and 7 *or* Question 8) and the Multiple Choice of Part B (Questions 16- 18 *or* 19-21).

This test contributes 10 of the 60 SAC marks in Unit 3.

The coursework scores for this test are:
Outcome 1 7.5 marks 75%
Outcome 3 2.5 marks 25%
TOTAL 10 marks
This weighting can be used in the conversion of their raw mark out of 40.
For example, a score of 28 results in:

OUTCOME 1

$$\frac{28}{40}\times 10\times 0.75 = 5.25$$
$$= 5\,(\text{rounded})$$

OUTCOME 3

$$\frac{28}{40}\times 10\times 0.25 = 1.75$$
$$= 2\,(\text{rounded})$$

The above can be established in an Excel file.

This QAT has been designed to meet the highest level in the Performance Descriptors provided by VCAA for each outcome in Unit 3 in the VCAA Mathematics Study Design February 2010.

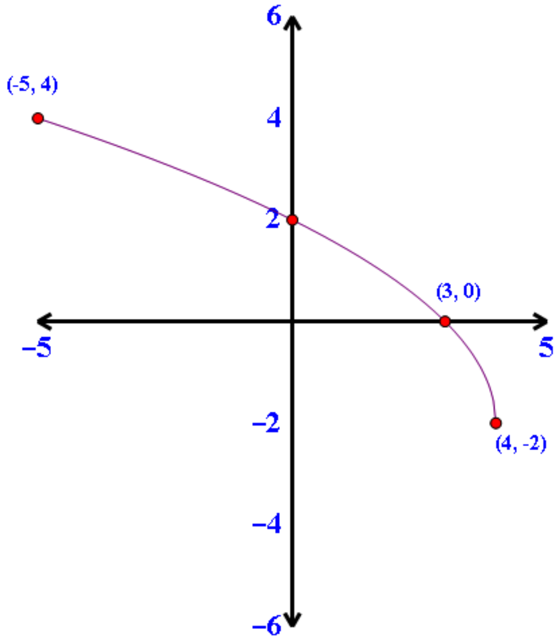
Solution Pathway

Question 1

Shape and position: 1 mark

Endpoints: 1 mark

Axes intercepts: 1 mark



Question 2

$$(x+2)(x-1)\left(x+a\right)=x^3+bx^2+2x-8$$

Expanding the constants of the left had side gives -2a = -8

Therefore a = 4 (1 mark)

$$\therefore\ (x+2)(x-1)\left(x+4\right)=x^3+bx^2+2x-8$$

$$1\times-2\times3=-6=1+b-2-8$$

Let x = -1 gives $-6=b-11$ (1 mark)

$$\therefore\ b=5$$

Question 3

a) The value of the asymptote is 1. Therefore b = 1.

b) When x = 0, f(0) = 4 = A.2^0 + 1

$\therefore$ 3 = A

c) $f(x) = 3.2^x + 1$ so $f(1) = 3x2 + 1 = 7$

Solution Pathway

Question 4

a) Swap x and y:

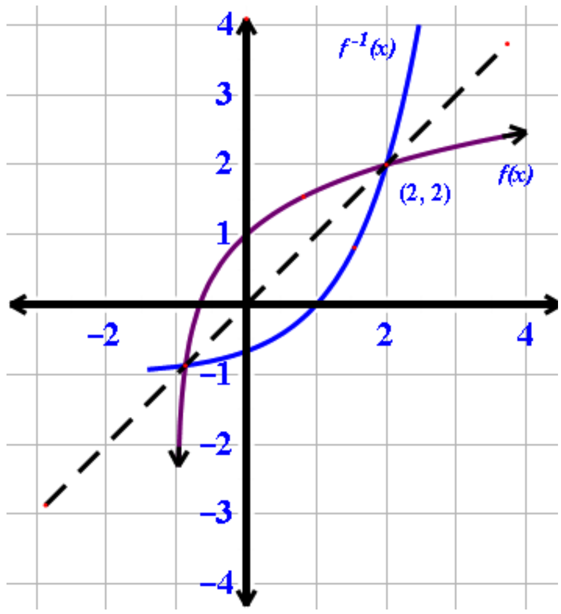
$$x = \log_3(y+1)+1$$
$$x-1 = \log_3(y+1)$$
$$3^{x-1} = y+1$$
$$\therefore f^{-1}(x) = 3^{x-1}-1$$

1 mark

1 mark

b) The (2, 2) intersection should be exact and the other one should be close for both marks to be awarded.

Allocate 1 mark only if shape is correct but intersections are too far from correct positions.



Question 5

a) The graph is the modulus of a parabola with a turning point of (2, 2), hence h = k = 2

b) By substitution of x = 0 the value of a = 0.5 or ½

Question 6

$f(g(x)) = 3x \quad 0 < x < \infty$

1 mark for the equation and 1 mark for the correct domain.

Solution Pathway

Question 7

$\log_a(8)+\log_a(2)=2\log_5(25)$

$RHS=2\times 2=4$

$LHS=\log_a(8\times 2)$

$\quad=\log_a(16)$

$\therefore \log_a(16)=4 \qquad (1\,mark)$

$16=a^4$

$\therefore a=2 \qquad (1\,mark)$

Solution Pathway

Question 8

Alternative question if testing Circular Functions

a) Dilation by a factor of 2 parallel to the y-axis.

Translation by $\frac{\pi}{3}$ units to the right and 1 unit up.

b) Period = $\frac{2\pi}{2} = \pi$

Question 9

Answer is **D.** The function is a truncus with asymptotes at x = 3 and y = -2.

Question 10

$$f(x) = \frac{1}{(x-3)^2} - 2$$

Answer is **B.** $\begin{bmatrix} -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -6 \\ 9 \end{bmatrix}$

Question 11

Answer is **E.** (-3, 0), (-2, 0), (1, 0), (2, 0). Use the calculator to factorise the original equation.

Question 12

Answer is **A**. Enter the equation into the calculator graph pane and compare to the ones given.

Question 13

Answer is **E.** $\left(\frac{-\sqrt{2}}{2}, e^{\frac{-\sqrt{2}}{2}}\right)$ *and* $\left(\frac{\sqrt{2}}{2}, e^{\frac{\sqrt{2}}{2}}\right)$. Use $solve(y = 2x^2e^x \text{ and } y = e^x, x, y)$.

Question 14

Answer **B** is not true. The equation of the asymptote is $y = 0$, not $x = 0$ as stated in the question.

Question 15

Answer is **D**. The values of b and c must multiply together to give 6. D is the only possibility given that satisfies this condition.

Solution Pathway

Question 16

Answer is **A.** There are two solutions to the equation that can be found by using $\boldsymbol{solve}(\ln(2\boldsymbol{x}^2 - 8\boldsymbol{x} + 10) = 2, \boldsymbol{x})$ and gives 0.3585 and 3.6415. The sum of these solutions is 4.

Question 17

Answer is **D**. This can be checked using the equivalence capability of the calculator. Algebraically the same result can be found as follows:

$$\begin{aligned} 2^{4x-3} &= 2^{4x} \times 2^{-3} \\ &= \left(2^4\right)^x \times 2^{-3} \\ &= 16^x \times \frac{1}{8} \\ &= \frac{16^x}{8} \end{aligned}$$

Question 18

Answer is **C.**

$$\begin{aligned} \ln(16) + 4\ln(3) - \ln(6) &= \ln(16) + \ln(3^4) - \ln(6) \\ &= \ln\left(\frac{16 \times 81}{6}\right) \\ &= \ln\left(\frac{1296}{6}\right) \\ &= \ln(216) \\ &= 3\ln(6) \end{aligned}$$

Question 19

Answer is **D**. $\frac{2\boldsymbol{n}\pi}{3} \pm \frac{\pi}{9}$

$$\begin{aligned} 2\cos(3\boldsymbol{x}) + 1 &= 2 \\ 2\cos(3\boldsymbol{x}) &= 1 \\ \cos(3\boldsymbol{x}) &= \frac{1}{2} \end{aligned}$$

Use the general solution formula $\boldsymbol{if}\ \cos(3\boldsymbol{x}) = \boldsymbol{a}\ \boldsymbol{then}\ 3\boldsymbol{x} = 2\boldsymbol{n}\pi \pm \cos^{-1}(\boldsymbol{a})$

Solution Pathway

$$3x = 2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 2n\pi \pm \frac{\pi}{3}$$

$$\therefore x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$$

Question 20

Answer is **B**.

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

Hence use sin(u+v) = sin(u)cos(v) + cos(u)sin(v)

$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

Question 21

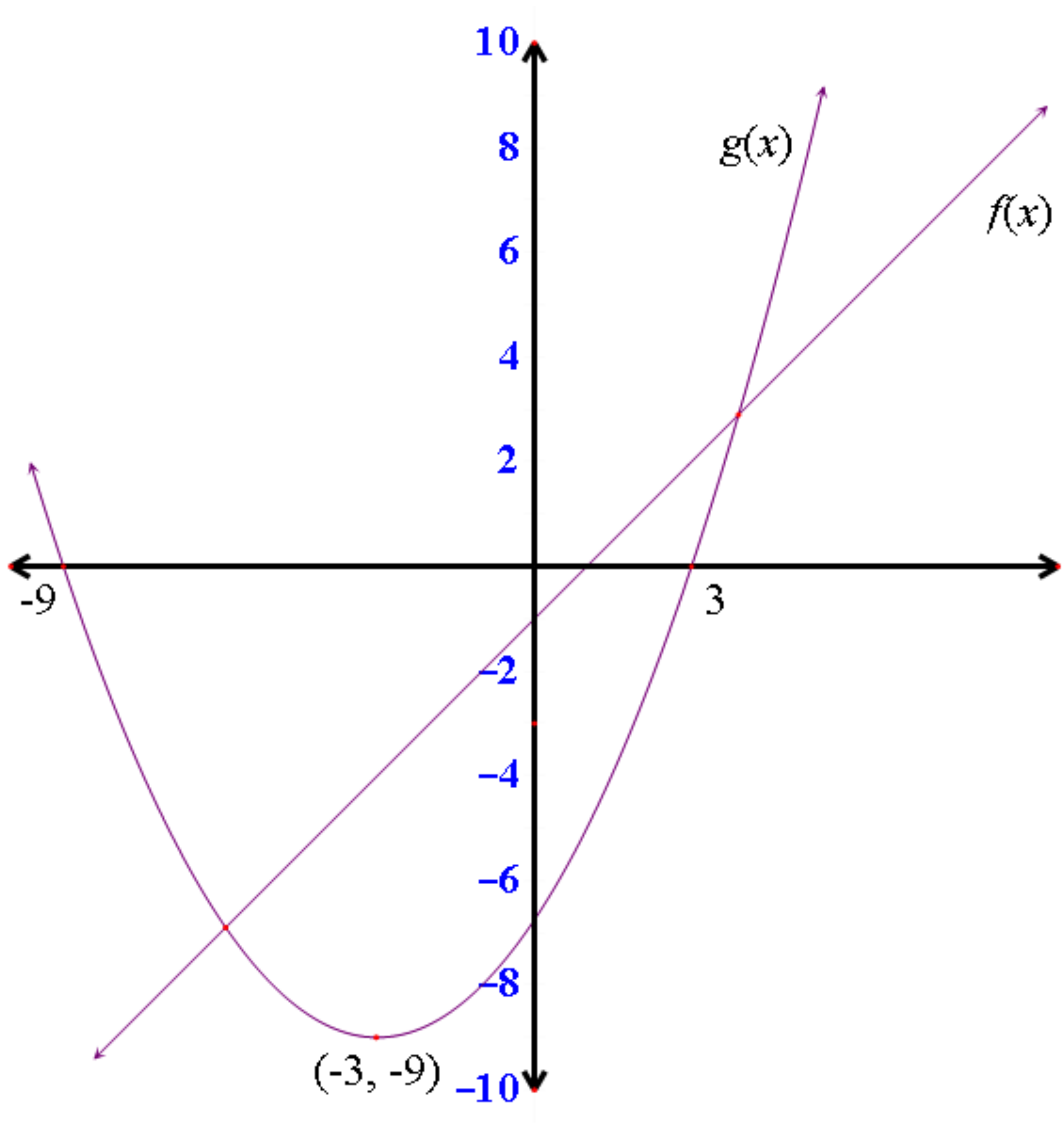
Answer **A**. $x = \frac{\pi}{8}$.

The formula for the asymptotes of a tangent graph is $x = \frac{(2k+1)\pi}{2n}$ where n = 4 and k = 0 for this example.

Solution Pathway

Question 22

a) and b)



c) Using solve(y = x - 1 and y = $\frac{1}{4}(x+3)^2 - 9$,x,y) gives $x = -1 \pm \sqrt{6}$ and $y = -2 \pm 2\sqrt{6}$.

Solution Pathway

d) $$x - 1 = \frac{1}{4}(x+3)^2 + k$$

$$x - 1 = \frac{1}{4}(x^2 + 6x + 9) + k \quad \text{(1mark)}$$

$$x - 1 = \frac{1}{4}x^2 + \frac{6}{4}x + \frac{9}{4} + k \quad \text{(1 mark)}$$

$$0 = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{13}{4} + k$$

e) $a = \frac{1}{4}$ $b = \frac{1}{2}$ $c = \frac{13}{4} + k$ (1 mark)

f) Use a, b and c to find when the determinant equals zero, and the calculator to carry out the computation:

$$solve\left(0 = \left(\frac{1}{2}\right)^2 - 4.\left(\frac{1}{4}\right).\left(\frac{13}{4} + k\right), k\right)$$ gives k = -3 (1 mark)