Neap:

Trial Examination 2011

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Write **your name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2011 VCE Mathematical Methods (CAS) Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

Given $f(x) = 4 - 2x - x^2$ and $g(x) = |x - 7|$, the value of $f(g(5))$ is

A. -4
B. 4
C. 8
D. 12
E. 124

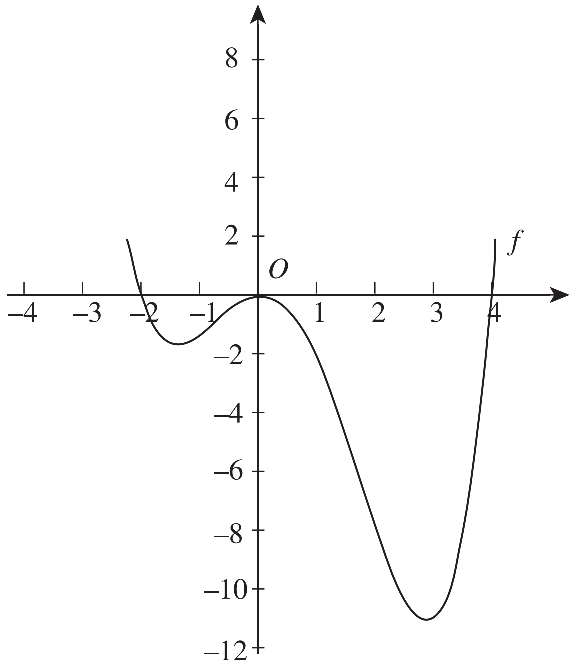
Question 2

Finding $\lim\limits_{h \to 0} \dfrac{\log_e(e + h) - 1}{h}$ is equivalent to finding

A. $f'(e)$, where $f(x) = \log_e(x)$.
B. $f'(e)$, where $f(x) = \dfrac{\log_e(x)}{x}$.
C. $f'(1)$, where $f(x) = \log_e(x)$.
D. $f'(1)$, where $f(x) = \log_e(x + e)$.
E. $f'(0)$, where $f(x) = \log_e(x)$.

Question 3

The graph of the quartic function $y = f(x)$ is shown below.



The range of f equals $[-11, \infty]$ and its absolute minimum occurs at $x = 3$.

Given that f undergoes a transformation to a new function $g(x) = f(1 - 3x) + 2$, then the resultant coordinate on g, corresponding to the absolute minimum on f, is

A. $\left(-\frac{8}{9}, -13\right)$

B. $\left(-\frac{2}{3}, -9\right)$

C. $\left(-\frac{10}{9}, -9\right)$

D. $\left(-\frac{2}{3}, -13\right)$

E. $\left(-\frac{8}{9}, -9\right)$

Question 4

Which of the following statements **does not** describe the graph of $f(x) = 1 - 5\cos(2x - \pi)$?

A. The amplitude is 5.

B. The period is π .

C. The range of the function equals [–4, 6].

D. The graph of $f(x) = 1 - 5\cos(2x - \pi)$ is the same as the graph of $g(x) = 5\cos(2x - \pi)$ with a vertical translation of 1 unit up, followed by a reflection in the x-axis.

E. The graph of $f(x) = 1 - 5\cos(2x - \pi)$ is the same as the graph of $h(x) = -5\cos(2x)$ with a horizontal translation of $\frac{\pi}{2}$ units right and a vertical translation of 1 unit up.

Question 5

The transformation $T : R^2 \rightarrow R^2$ is defined by:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

The equation of the image of the curve $y = x^3$ under T is

A. $2y = 2(x - 1)^3 - 2$

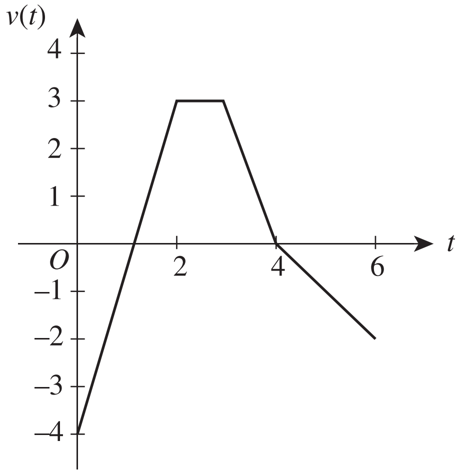
B. $y = \frac{1}{2}(x - 1)^3 - 2$

C. $y = 2(x + 3)^2 - 2$

D. $y = 2(x - 1)^3 - 2$

E. $2y = (x + 1)^3 + 2$

Question 6



The graph shown above represents the velocity, $v(t)$, in cm/s, of a body moving in a straight line over a 6 second interval. Note that the body has constant velocity 3 cm/s for $2 \le t \le 3$.

Which of the following statements regarding the position of the body, $x(t)$, at time *t* seconds is correct?

A. $x(2) < x(0) < x(4) < x(6)$

B. $x(2) < x(0) < x(6) < x(4)$

C. $x(6) < x(0) < x(2) < x(4)$

D. $x(0) < x(6) < x(2) < x(4)$

E. $x(0) < x(2) < x(6) < x(4)$

Question 7

After a workplace initiated its fitness program, employee sick days were recorded for some of the following weeks as shown in the table below.

Weeks after fitness program	Number of weekly sick days taken
0	60
1	57
2	55
4	51
8	43
12	37
16	31

The Human Resources Department thought the decreases might be exponential.

Which function best describes the weekly sick days (*N*) in terms of the week number, *n* after the program started?

A. $N = 60(e^{-0.06n})$

B. $N = 60(1.06)^{-n}$

C. $N = 60 - 2.4n + 0.06n^2$

D. $N = 60(0.96)^n$

E. $N = 60 - 1.8n$

Question 8

Consider that k is a positive integer and the polynomial $p(x) = 5x^{2k+1} - 10x^{2k} + 3x^{2k-1} + 5$.

When $p(x)$ is divided by $x + 1$, the remainder equals

A. 0
B. 2
C. 4
D. –8
E. –13

Question 9

The functions f, g and h are defined for all real numbers on their maximal domain according to the following rules:

$$f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x + 1$$

$$g(x) = x^3 + x^2 + x + 1$$

$$h(x) = x^{-\frac{2}{3}}$$

Which of these functions has an inverse?

A. f and g only
B. f and h only
C. g only
D. All three have an inverse.
E. Neither f nor g nor h has an inverse.

Question 10

The graphs of $y = \cos(3x)$ and $y = \sin(3x)$ intersect at the smallest two positive x values of $x = \frac{\pi}{12}$ and $\frac{5\pi}{12}$.

If each graph is dilated by a scale factor of $\frac{1}{2}$ from the x-axis and by a scale factor of 3 from the y-axis, the new coordinates for these points of intersection will be

A. $\left(\frac{\pi}{36}, \frac{\sqrt{2}}{4}\right)$ and $\left(\frac{5\pi}{36}, -\frac{\sqrt{2}}{4}\right)$

B. $\left(\frac{\pi}{6}, \sqrt{2}\right)$ and $\left(\frac{5\pi}{6}, -\sqrt{2}\right)$

C. $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{4}\right)$ and $\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{4}\right)$

D. $\left(\frac{\pi}{36}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{5\pi}{36}, -\frac{\sqrt{2}}{2}\right)$

E. $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

Question 11

The function $f(x) = \begin{cases} \frac{1}{3}x^2 & \text{if } x \in [p, 3] \\ 0 & \text{otherwise} \end{cases}$ is a probability density function.

The value of p is

A. 0

B. 1

C. $\sqrt[3]{26}$

D. $\sqrt[3]{18}$

E. $\sqrt[3]{20}$

Question 12

A netball player is practising his goal shooting. He has a probability of 0.7 of scoring a goal with each attempt and successive attempts are independent. He has 30 attempts.

The probability that the number of goals scored is within 1 standard deviation of his expected number number of goals is closest to

A. 0.4490

B. 0.5708

C. 0.6812

D. 0.8389

E. 0.9298

Question 13

The continuous random variable X has a normal distribution with mean 50 and standard deviation of 5. The continuous random variable Z has the standard normal distribution.

The probability that X is between 47.5 and 57.5 is equal to

A. $\Pr(-0.5 < Z < 2)$

B. $\Pr(-1.5 < Z < 0.5)$

C. $\Pr(-2.5 < Z < 7.5)$

D. $\Pr(-0.5 < Z < 2.5)$

E. $\Pr(-1.5 < Z < 1)$

Question 14

A teacher needs to select three students to represent the class. In the class there are four boys and eight girls. The probability that there is at least one student of each gender is

A. $\frac{8}{11}$

B. $\frac{9}{11}$

C. $\frac{8}{9}$

D. $\frac{2}{9}$

E. $\frac{26}{55}$

Question 15

The discrete random variable X has the following distribution:

x	1	2	4	6	8	10
$\Pr(X = x)$	p	p	p	$2p$	$3p$	$2p$

The median of X is

A. 4

B. 5

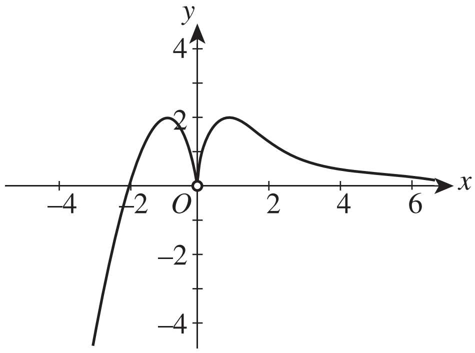
C. 6

D. 7

E. 8

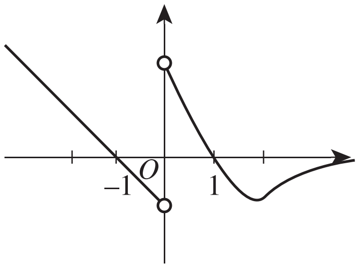
Question 16

The graph of the function $y = f(x)$ is shown below:

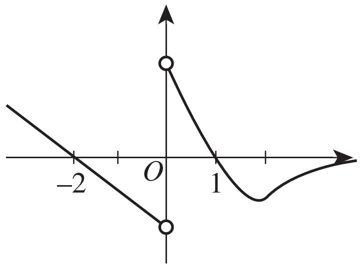


Which of the following could represent the graph of $y = f'(x)$?

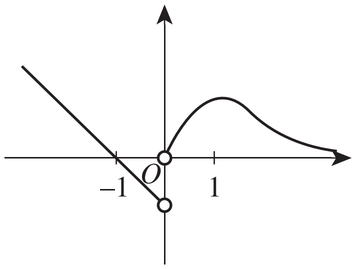
A.



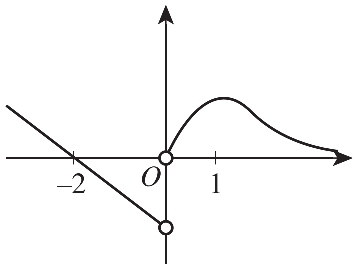
B.



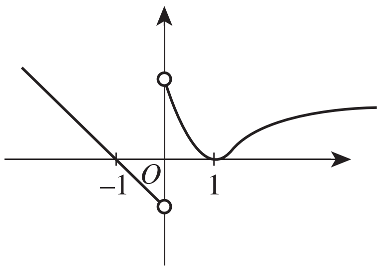
C.



D.



E.



Question 17

For the function $f(x) = |9 - x^2|$, the derivative at $x = 3$ is

A. 6

B. –6

C. ±6

D. 0

E. does not exist

Question 18

For the function $f(x) = x^{\frac{3}{5}} - 2$, the equation of the normal at $x = 0$ is

A. undefined.

B. $y = 0$

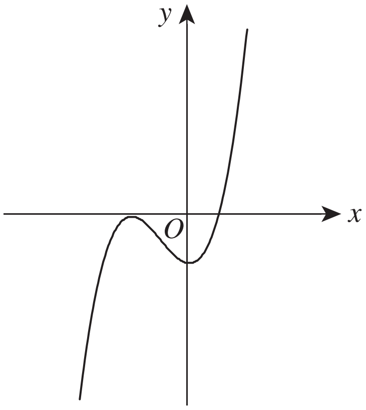
C. $y = -2$

D. $x = 0$

E. $x = -2$

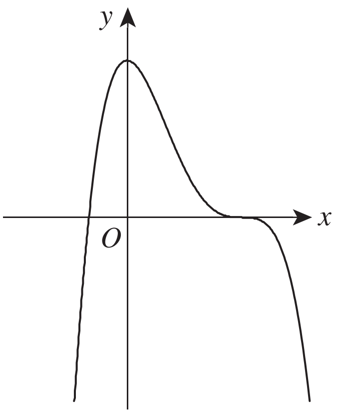
Question 19

The graph of the gradient function $y = f'(x)$ is shown below:

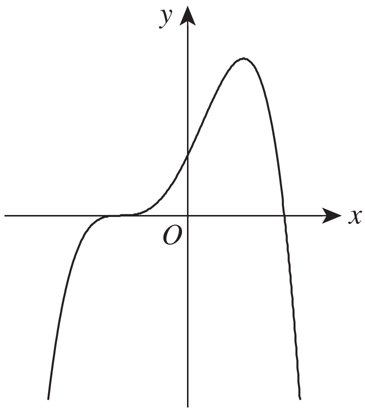


Which of the following could represent the graph of the function $f(x)$?

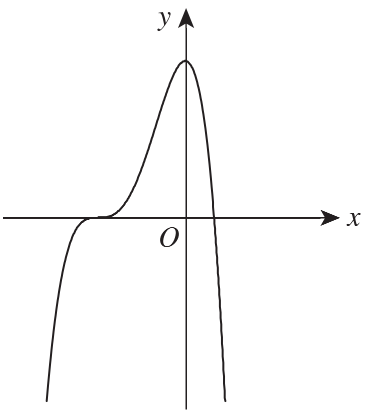
A.



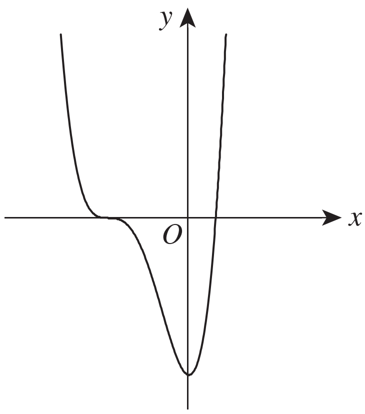
B.



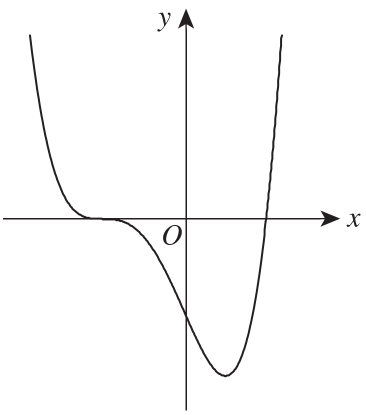
C.



D.



E.



Question 20

Let f be a differentiable function for all real x, where $f(x) \le 0$ for all $x \in [0, a]$.

If $\int_0^a f(x)dx = -2$, then $\int_0^{\frac{a}{2}} 2f(2x)dx$ is equal to

A. $-\frac{1}{2}$

B. -1

C. -2

D. -4

E. -8

Question 21

Events A and B are independent events of a sample space with

$\Pr(A) = p$ and $\Pr(B) = q$, where $0 < p < 1$ and $0 < q < 1$.

$\Pr(A'|B')$ is equal to

A. $\frac{1-p-q}{1-q}$

B. $\frac{pq}{1-q}$

C. $\frac{1-pq}{1-q}$

D. $1-p$

E. $1-q$

Question 22

The area bounded by the curve $y = \log_e\left(\frac{1}{x}\right)$, the x-axis and the line $x = a$, where $a > 1$, is equal to

A. $1 + \log_e(a)$

B. $1 - \log_e(a)$

C. $\log_e(a) - 1$

D. $a - a\log_e(a) - 1$

E. $1 - a + a\log_e(a)$

END OF SECTION 1

Neap:

Trial Examination 2011

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

MATHEMATICAL METHODS (CAS) FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: r^2h

volume of a cone: $\frac{1}{3}\ r^2h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin(A)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Matrices

transition matrices: $S_n = T^n \times S_o$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{Var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\ dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\ dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$

END OF FORMULA SHEET

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

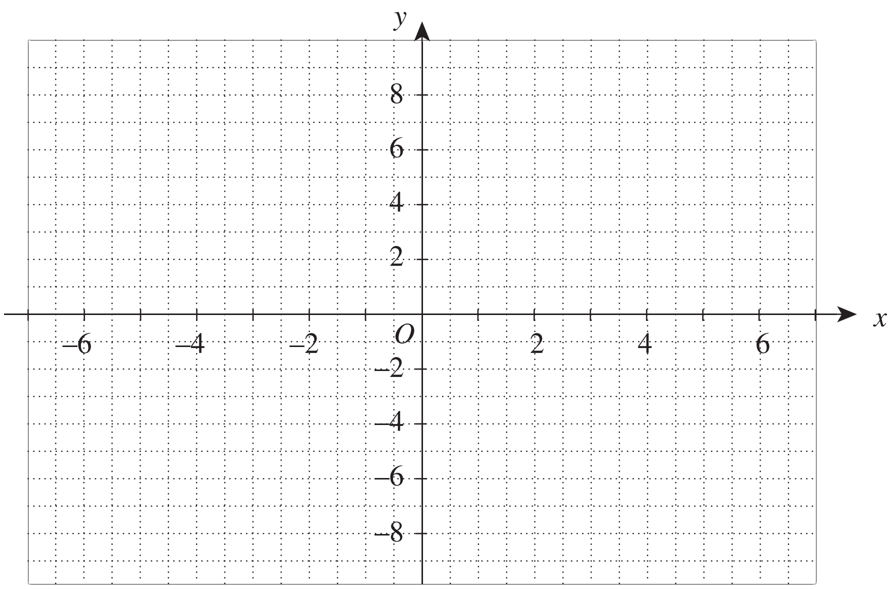
Question 1

A function, f, is given by the rule $f(x) = \log_e((x-1)^2 - m)$, where m is a constant.

a. State the maximal domain over which f is defined for the case where $m > 0$.

2 marks

b. For $m = 4$, sketch the graph of f on the axes provided, indicating coordinates of any axes intercepts and the equations of any asymptotes.



3 marks

Consider the function $g : (-\infty, -2) \to R,\ g(x) = \log_e((x-1)^2 - m)$, where m is a constant.

c. Find the maximum value of m.

2 marks

d. Show that $g^{-1}(x) = 1 - \sqrt{e^x + m}$ and state its domain and range.

3 marks

e. Consider the family of functions of the form $f(x) = \log_e((x-1)^2 - m)$.

i. Write down an expression for $f'(x)$ and hence state the conditions under which the family of graphs given by $y = f(x)$ has a local minimum turning point.

Let $m = -1$ and consider the following sequence of transformations applied to f

- a reflection in the x-axis, followed by a dilation away from the y-axis of scale factor $\frac{1}{2}$, followed by a dilation of scale factor 2 from the x-axis.

ii. If g is the function obtained from the sequence of transformations described above, find $g(x)$.

iii. The graph of $y = g(x)$ from **part e. ii.** is now translated A units in the positive y-direction so that the area enclosed by the graph, the y-axis, x-axis and the line $x = 4$ is 23.5 square units.

Find A correct to the nearest integer.

2 + 2 + 2 = 6 marks
Total 16 marks

Question 2

The Military Intelligence Agency (MIA) uses clay targets for the shooting practice of its trainees. There are two grades of clay targets. Clay targets which have a mass of between 130 g and 170 g are graded as A-class, all others are B-class.

There are two manufacturers, Clarget and Shotrite, which produce the clay targets. Clarget find their production process produces clay targets whose mass is normally distributed with a mean of 150 g and a standard deviation of 25 g.

a. Find the percentage, correct to 2 decimal places, of Clarget's clay target production that would be graded as A-class.

1 mark

b. Clarget would like at least 80% of their production to be A-class targets.

Find the largest value of the standard deviation, to the nearest gram, required to achieve this whilst keeping the mean clay target mass at 150 g.

3 marks

Shotrite use a process that produces clay targets one at a time. The mass of each clay target is independent of the mass of any other clay target. Shotrite know that, on average, 60% of the clay targets they produce are A-class.

c. If the first clay target produced on a particular day is a B-class clay target, find the probability that the next 5 clay targets produced by Shotrite are A-class.

1 mark

d. If Shotrite's first clay target produced on a particular day is an A-class target, find the probability that exactly 3 of the next 5 clay targets produced are A-class.

1 mark

Major Jane Blonde is an engineer at the MIA. She has designed a process in the MIA workshop which produces clay targets. The clay targets are produced one at a time. Major Jane Blonde has been asked to assess the quality of the production process. She finds:

- If an A-class clay target is produced, then the probability that the next clay target is A-class is p.
- If a B-class clay target is produced, then the probability that the next clay target is B-class is $p - 0.4$.

On a particular day it is known that $p = 0.8$.

e. If the first clay target produced on this particular day is a B-class clay target, find the probability that one of the next two clay targets is A-class.

2 marks

f. If the first clay target produced on this particular day is a B-class clay target, find the probability that the fourth clay target produced is A-class.

2 marks

g. Find the long-term proportion of clay targets that can be expected to be assessed as A-class for this value of p.

1 mark

Major Jane Blonde wishes to alter the production process so that, if the first clay target produced is A-class, then the probability that the fourth target produced is also A-class is 0.8.

h. Find the value of p, correct to 4 decimal places, which will enable this to occur.

3 marks

Total 14 marks

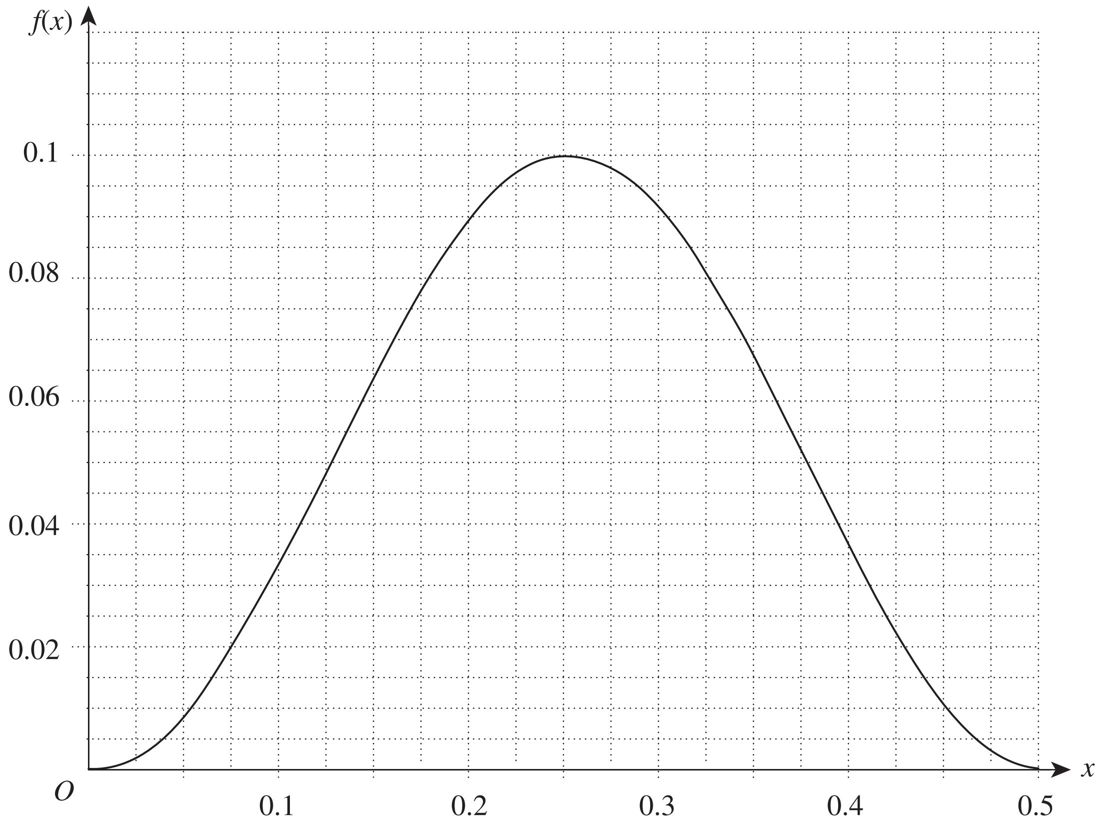
Question 3

Speed humps are widely used to slow traffic. The top of the cross-section of a basic speed hump, called the profile of the speed hump, can be modelled by a sine curve. To be effective, they have to give vehicles a severe jolt when driven across too fast. A basic speed hump function has the form $f(x) = b\left[\frac{1}{2} + \frac{1}{2}\sin\left(\frac{\pi(4x-d)}{2d}\right)\right]$, where $f(x)$ is the height in metres and x is the distance from one edge of the hump. This models the height, in metres, of what is called a sine curve hump of width d metres.

a. What is the maximum height in metres of the sine curve hump?

1 mark

b. A typical sine curve hump is shown in the graph below.



i. Determine the values of b and d.

ii. Show that this sine curve hump can be expressed as a cosine function with rule $y = \frac{1}{20}(1 - \cos(4\pi x))$.

Some speed humps are designed so that the maximum angle of the sine curve hump with those parameters does not exceed 15°.

iii. Determine whether this speed hump has this design.

A dilation factor of k from the x-axis can be applied to the hump equation so that the transformed equation will have a maximum angle of exactly 15°.

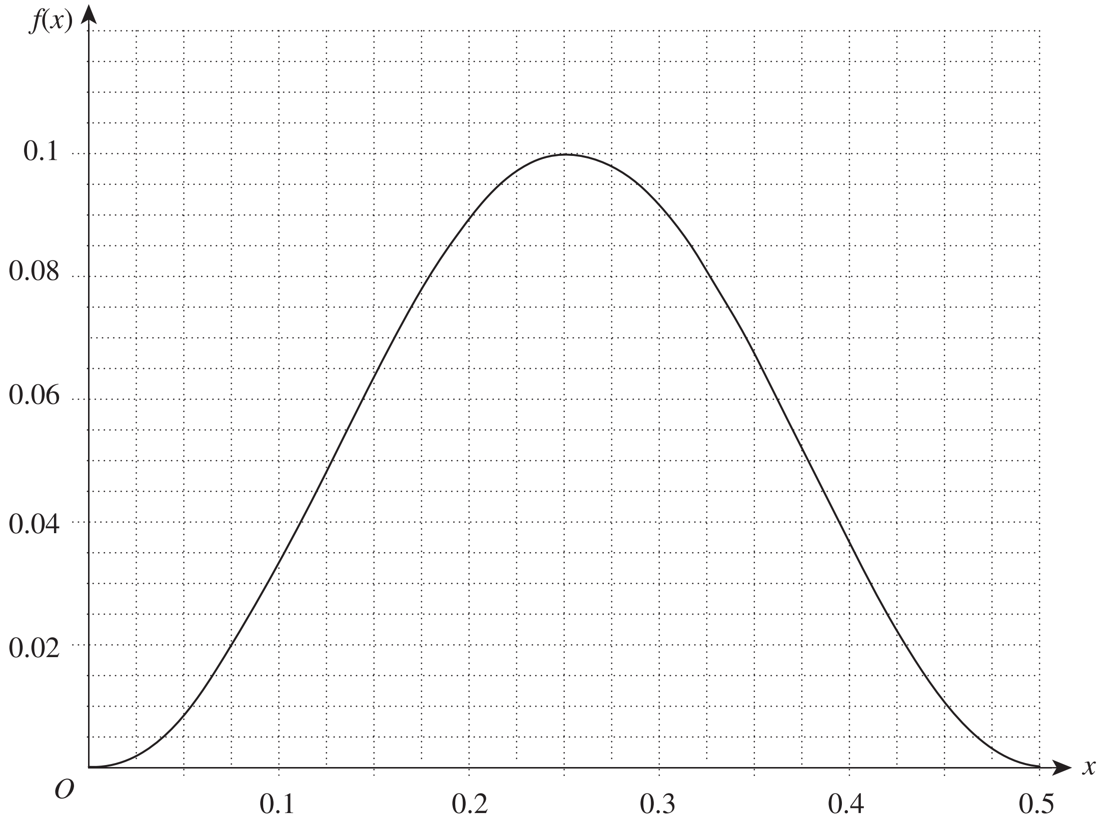
iv. Show that $k = \frac{5(2 - \sqrt{3})}{\pi}$.

2 + 2 + 3 + 2 = 9 marks

c. The hump can be made more sharp or gentle by changing the function to

$g(x) = b\left[\frac{1}{2} + \frac{1}{2}\sin\left(\frac{\pi(4x - d)}{2d}\right)\right]^p$.

i. On the axes below, sketch $y = g(x)$ for the cases where $p = \frac{1}{2}$ and $p = \frac{3}{2}$ with the parameters b and d unchanged. The graph shown on the axes is for $p = 1$.



ii. Give a reason as to why the profile for $p = \frac{1}{2}$ may be less suitable than that for $p = \frac{3}{2}$.

2 + 1 = 3 marks

A particular street is to have 5 speed humps constructed out of concrete using the profile for either $p = \frac{3}{2}$ or $p = 1$.

d. Assume each speed hump is placed in the centre of the road and extends for a length of 3 metres.

i. Write down an integral expression which gives the extra volume of concrete required for a single hump with $p = 1$ compared with $p = \frac{3}{2}$ in cubic metres.

ii. Hence find, correct to three decimal places, in cubic metres, the extra volume of concrete required to complete the 5 speed humps for the street.

2 + 1 = 3 marks
Total 16 marks

Question 4

Consider the function $f: R \to R, f(x) = (2-x)^2(3-x)^3$.

a. Find the x-coordinates of each of the stationary points of f and state the nature of each of these stationary points.

4 marks

Let $g: R \to R, g(x) = (a-x)^2(b-x)^3$, where a and b are real constants.

b. For what values of a and b will g have only one stationary point? State the nature of this stationary point.

2 marks

Now suppose that $b > a$.

c. Write down, in terms of a and b, the possible values of x for which $(x, g(x))$ is a stationary point of g.

2 marks

d. Let $h : R \to R,\ h(x) = (a - x)^m (b - x)^n$, where a and b are real constants, with $b > a$ and m and n are positive integers.

i. Write down in terms of a, b, m and n, the possible values of x for which $(x, h(x))$ is a stationary point of h.

ii. For what values of m and n will h have a stationary point which is equidistant from $x = a$ and $x = b$?

3 + 1 = 4 marks
Total 12 marks

END OF QUESTION AND ANSWER BOOKLET