Neap:

Trial Examination 2011

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 12 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Write your **name** and **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2011 VCE Mathematical Methods (CAS) Units 3 & 4 Written Examination 1.

Instructions

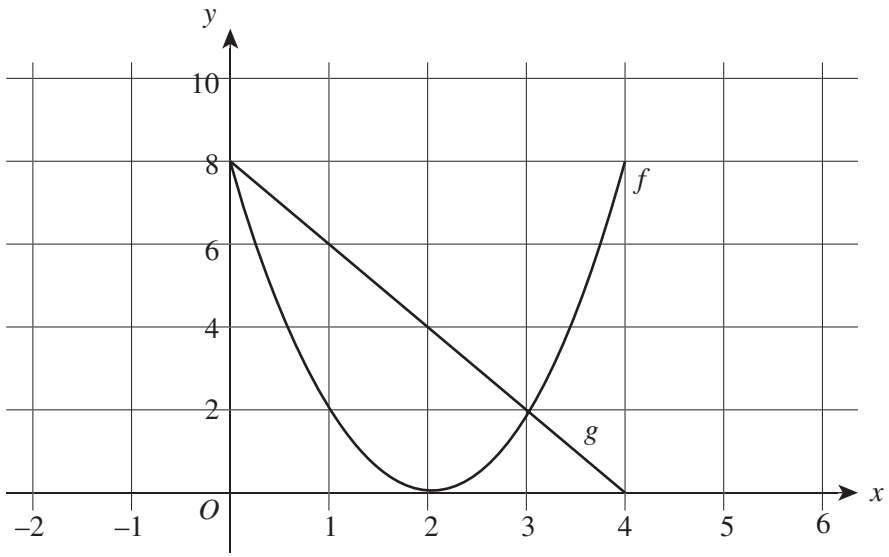
Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1



Consider the functions f and g, whose graphs are shown above.

a. Find $(f-g)(1)$.

_____

_____

_____

_____

_____

1 mark

b. Find $f(g^{-1}(4))$.

_____

_____

_____

_____

_____

1 mark

Question 2

a. Determine the relationship between p and q such that the simultaneous linear equations shown below will have a unique solution.

$$2x + py = 4$$
$$5x + qy = 6$$

1 mark

b. Determine the values of m and n such that the simultaneous linear equations shown below will have an infinite set of solutions.

$$y = mx + n$$
$$3x - 7y = 2$$

2 marks

Question 3

Consider the function with rule $f(x) = x^2 - 2x + 3$ on the domain $[0, \infty)$.

a. Find the equation of the tangent to the graph of f at $x = 2$.

_____

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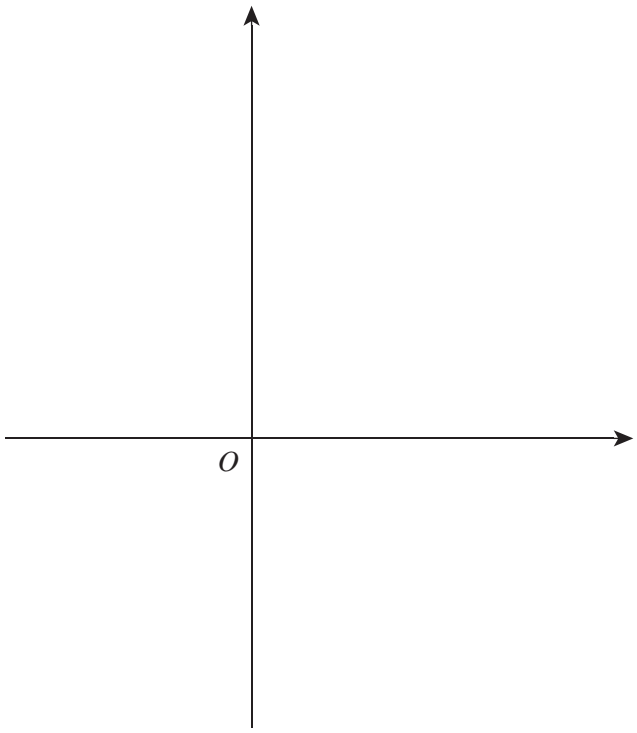
1 mark

b. On the axes provided sketch the graph of f and its tangent at $x = 2$.



1 mark

c. Suppose the tangent line drawn is used to approximate values of $f(x)$. A satisfactory approximation occurs if the resulting error is at most 0.4.

i. Find the error when $x = 2.5$.

_____

_____

_____

_____

ii. Show that the greatest value of x which can be used for this approximation is $\frac{10 + \sqrt{10}}{5}$.

_____

_____

_____

_____

_____

_____

_____

_____

2 + 2 = 4 marks

Question 4

Let X be a normally distributed variable with mean 8 and variance 16, and let Z be the random variable with standard normal distribution.

a. Find $\Pr(X > 8)$.

_____

_____

_____

1 mark

b. Find k such that $\Pr(X < 0) = \Pr(Z > k)$.

_____

_____

_____

_____

_____

_____

2 marks

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Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

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MATHEMATICAL METHODS (CAS) FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Matrices

transition matrices: $S_n = T^n \times S_o$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

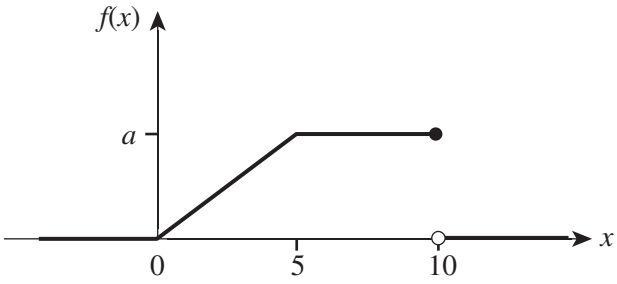
variance: $\mathrm{Var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\ dx$	$\mu = \int_{-\infty}^{\infty} xf(x)\ dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$

END OF FORMULA SHEET

Question 5

The continuous random variable X has a distribution with probability density function shown by the graph below.



a. Find the value of a.

2 marks

b. Determine the value of the median, m, for the continuous random variable, X.

3 marks

Question 6

The discrete random variable X has the probability distribution:

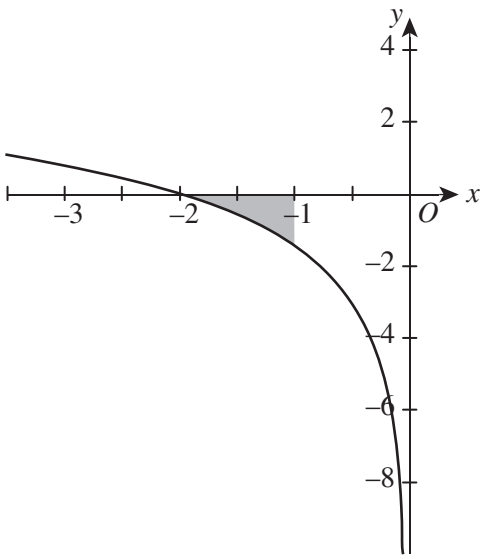
x	-1	0	1	2	3
$\Pr(X = x)$	p^2	q	$\frac{1 + 3p}{4}$	$\frac{1}{8}$	$\frac{p}{2}$

Given that the mean of X is 1, find the value of p.

4 marks

Question 7

The graph of $f: R^{-} \rightarrow R, f(x) = \log_e\left(\frac{x^2}{4}\right)$ is shown below.



a. Find the derivative of $x\log_e\left(\frac{x^2}{4}\right)$.

2 marks

b. Use your answer to **part a.** to find the area of the shaded region in the form $a - \log_e(b)$, where a and b are integers.

2 marks

Question 8

The normal to the curve with equation $y = \sqrt{a - x}$ at $x = b$, where a and b are real constants and $a > 0$ and $b < a$, passes through the origin.

Find the value of b.

4 marks

Question 9

A particle moves in a straight line along the x-axis so that its position, $x(t)$, at time t seconds, $t \geq 0$, is given by $x = \sin(\pi t^2)$.

a. Find expressions for the velocity and acceleration of the particle at time t.

_____

_____

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2 marks

b. Find the times for which the particle is stationary.

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3 marks

Question 10

Let g be a differentiable function defined for all positive values of x such that the following three conditions hold:

I. $g(1) = 0$

II. The tangent to the graph of g at $x = 1$ is inclined at 45° to the positive x-axis.

III. $\frac{d}{dx}(g(2x)) = g'(x)$

a. Determine the value for $g'(2)$.

_____

_____

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_____

_____

2 marks

b. Prove that $g(2x) = g(x) + g(2)$.

_____

_____

_____

_____

_____

_____

2 marks

END OF QUESTION AND ANSWER BOOKLET