



Trial Examination 2011

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

$$(f-g)(1) = f(1) - g(1)$$

$$\begin{aligned} \text{a.} \quad &= 2 - 6 \\ &= -4 \end{aligned}$$

A1

$$\text{b.} \quad \text{As } g(2) = 4, \text{ we have } g^{-1}(4) = 2. \text{ Thus } f(g^{-1}(4)) = f(2) = 0.$$

A1

Question 2

$$\text{a.} \quad \text{The simultaneous equations can be represented in matrix form as } \begin{bmatrix} 2 & p \\ 5 & q \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}. \text{ These equations}$$

$$\text{will have a unique solution if } \det \begin{bmatrix} 2 & p \\ 5 & q \end{bmatrix} \neq 0.$$

$$\therefore \text{The relationship required is } 2q - 5p \neq 0 \text{ or } p \neq \frac{2}{5}q.$$

A1

b. Rearranging the simultaneous equations and representing them in matrix form:

$$\begin{aligned} -mx + y &= n \dots(1) \\ 3x - 7y &= 2 \dots(2) \end{aligned} \quad \begin{bmatrix} -m & 1 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} n \\ 2 \end{bmatrix}$$

$$\text{It is not a unique solution if } \det \begin{bmatrix} -m & 1 \\ 3 & -7 \end{bmatrix} = 0,$$

$$\therefore (7m - 3) = 0$$

$$m = \frac{3}{7}$$

A1

And if equation (1) is identical to equation (2), an infinite set of solutions will occur.

$$\text{equation (1) } y = \frac{3}{7}x + n$$

$$\text{equation (2) } y = \frac{3}{7}x - \frac{2}{7} \quad \therefore n = -\frac{2}{7}$$

A1

Question 3

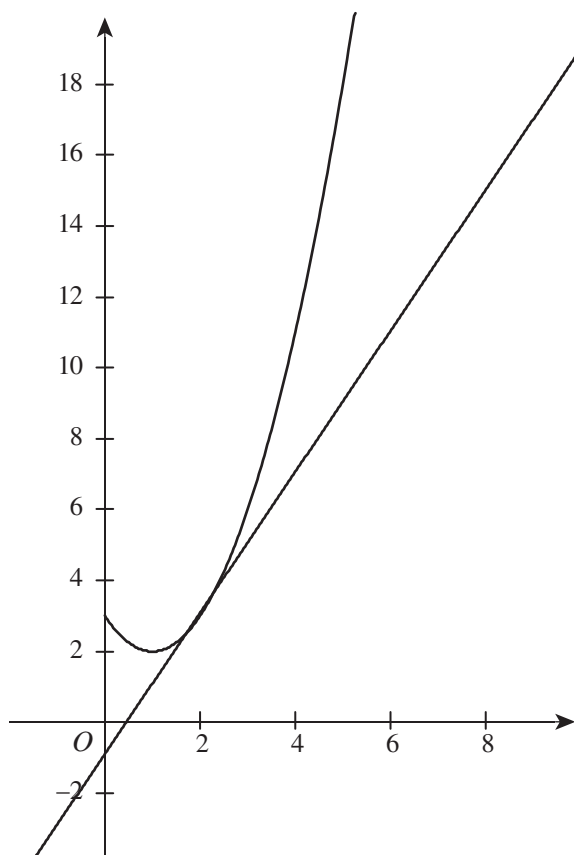
$$\text{a.} \quad f(x) = x^2 - 2x + 3 \Rightarrow f'(x) = 2x - 2$$

$$\text{Thus } f'(2) = 4 - 2 = 2 \text{ and } f(2) = 4 - 4 + 3 = 3.$$

$$\text{Equation of the tangent at } x = 2 \text{ is given by } y - 3 = 2(x - 2) \Rightarrow y = 2x - 1.$$

A1

b.



Parabola over domain with tangent shown

A1

c. i. The size of the error is given by the vertical distance between the graphs.

$$\text{Error} = x^2 - 2x + 3 - (2x - 1) = (x - 2)^2$$

M1

$$x = 2.5 \text{ gives error} = \frac{1}{4}.$$

A1

ii. Now $(x - 2)^2 \leq 0.4$, giving $-\sqrt{0.4} \leq (x - 2) \leq \sqrt{0.4}$.

M1

$$\begin{aligned} \text{Thus the largest value of } x \text{ will be } \sqrt{0.4} + 2 &= \sqrt{\frac{4}{10}} + 2 \\ &= \frac{2}{\sqrt{10}} + 2 \\ &= \frac{\sqrt{10}}{5} + 2 \\ &= \frac{10 + \sqrt{10}}{5} \end{aligned}$$

A1

Question 4

$$\begin{aligned} \text{a. } \Pr(X > 8) &= \Pr\left(Z < \frac{8-8}{4}\right) \\ &= \Pr(Z > 0) \\ &= 0.5 \end{aligned} \quad \text{A1}$$

$$\begin{aligned} \text{b. } \Pr(X < 0) &= \Pr\left(Z < \frac{0-8}{4}\right) && \text{M1} \\ &= \Pr(Z < -2) \\ &= \Pr(Z > 2) \\ \therefore k &= 2 && \text{A1} \end{aligned}$$

Question 5

a. Area below the curve must equal 1.

$$\therefore \text{Area} = \frac{1}{2}(5)a + 5a = 1 \quad \text{M1}$$

$$\frac{15}{2}a = 1$$

$$\therefore a = \frac{2}{15} \quad \text{A1}$$

b. m is such that $\Pr(X < m) = \frac{1}{2}$.

$$\text{Here, } \Pr(X < 5) = \frac{5}{2}a = \frac{5}{2} \times \frac{2}{15} = \frac{1}{3} \quad \text{M1}$$

\therefore require:

$$\Pr(5 < X < m) = \frac{1}{6} \quad \text{M1}$$

$$\Rightarrow \frac{2}{15}(m-5) = \frac{1}{6}$$

$$m-5 = \frac{5}{4}$$

$$m = 6\frac{1}{4} \text{ or } 6.25 \quad \text{A1}$$

Question 6

$$E(X) = 1$$

$$\sum xp(x) = 1$$

$$(-1)p^2 + 0 + (1)\left(\frac{1+3p}{4}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{p}{2}\right) = 1$$

M1

$$-p^2 + \frac{1+3p}{4} + \frac{1}{4} + \frac{3p}{2} = 1$$

$$-4p^2 + 1 + 3p + 1 + 6p = 4$$

$$-4p^2 + 9p - 2 = 0$$

A1

$$4p^2 - 9p + 2 = 0$$

$$(4p-1)(p-2) = 0$$

$$p = \frac{1}{4} \text{ or } p = 2$$

A1

$$\text{Given } \Pr(X = -1) = p^2, \therefore 0 < p < 1, \therefore p = \frac{1}{4}$$

A1

Question 7

$$\text{a. } \frac{d}{dx}\left(x \log_e\left(\frac{x^2}{4}\right)\right) = 1 \cdot \log_e\left(\frac{x^2}{4}\right) + x \cdot \frac{\frac{x}{2}}{\frac{x^2}{4}}$$

M1

$$= \log_e\left(\frac{x^2}{4}\right) + 2$$

A1

$$\text{b. } \text{Area} = \left| \int_{-2}^{-1} \log_e\left(\frac{x^2}{4}\right) dx \right|$$

$$\text{Using } \int \left(\log_e\left(\frac{x^2}{4}\right) + 2\right) dx = x \log_e\left(\frac{x^2}{4}\right) + c$$

M1

$$\int \log_e\left(\frac{x^2}{4}\right) dx = x \log_e\left(\frac{x^2}{4}\right) - \int 2 dx + c$$

$$= x \log_e\left(\frac{x^2}{4}\right) - 2x + c$$

A1

$$\therefore \text{Area} = \left| \left[x \log_e\left(\frac{x^2}{4}\right) - 2x \right]_{-2}^{-1} \right|$$

$$= \left| \left(-\log_e\left(\frac{1}{4}\right) + 2 \right) - \left(-2\log_e(1) + 4 \right) \right|$$

$$= \left| -\log_e\left(\frac{1}{4}\right) - 2 \right|$$

$$= 2 + \log_e\left(\frac{1}{4}\right)$$

$$= 2 - \log_e(4)$$

A1

Question 8

At $x = b$, $y = \sqrt{a - b}$

$$\frac{dy}{dx} = \frac{1}{2}(-1)(a - x)^{-\frac{1}{2}} \quad \text{M1}$$

$$= \frac{-1}{2\sqrt{a - x}}$$

\therefore gradient of normal $= 2\sqrt{a - x}$

Gradient of normal at $x = b$ is $2\sqrt{a - b}$. A1

Gradient of line through $(0, 0)$ and $(b, \sqrt{a - b})$ is $\frac{\sqrt{a - b}}{b}$. A1

$$\sqrt{a - b} = \frac{\sqrt{a - b}}{b}$$

$$2 = \frac{1}{b} \quad \text{as } a \neq b$$

$$b = \frac{1}{2} \quad \text{A1}$$

Question 9

a. $x = \sin(\pi t^2)$ so we have $v = \frac{dx}{dt} = 2\pi t \cos(\pi t^2)$ A1

$$\begin{aligned} a &= \frac{dv}{dt} = 2\pi(t \cdot (-2\pi t) \sin(\pi t^2) + 1 \cdot \cos(\pi t^2)) \\ &= -4\pi^2 t^2 \sin(\pi t^2) + 2\pi \cos(\pi t^2) \end{aligned} \quad \text{A1}$$

b. $v = 0$ which occurs at: M1

$$t = 0 \text{ and } \cos(\pi t^2) = 0 \Rightarrow \pi t^2 = \frac{\pi}{2} + k\pi, k \in Z$$

$$t^2 = \frac{1}{2} + k \Rightarrow t = \sqrt{\frac{1}{2} + k} \quad \text{M1}$$

Velocity is zero at $t = 0$ and $t = \sqrt{\frac{1}{2} + k}, k \in Z^+ \cup \{0\}$. A1

Question 10

a. Condition II states that $g'(1) = \tan(45^\circ) = 1$.

Using condition III, we have $\frac{d}{dx}(g(2x)) = g'(x) \Rightarrow 2g'(2x) = g'(x)$ by the Chain Rule. M1

So $g'(1) = 2g'(2)$, giving $1 = 2g'(2) \Rightarrow g'(2) = \frac{1}{2}$. A1

b. Using condition III, we have $\frac{d}{dx}(g(2x)) = g'(x)$. Integrating both sides with respect to x :

$$g'(2x) = g'(x) \Rightarrow g(2x) = g(x) + c \quad \text{M1}$$

Let $x = 1$, $g(2) = g(1) + c$.

Using condition I, $g(1) = 0$ and $c = g(2)$, giving $g(2x) = g(x) + g(2)$. A1