Mathematical Association of Victoria
Trial Exam 2011

MATHEMATICAL METHODS (CAS)

STUDENT NAME ____

Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

Note
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 19 pages with a detachable sheet of miscellaneous formulas at the back.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- Write your **student name** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

If $f:\left(-\infty,\frac{1}{2}\right] \to R$, where $f(x)=\sqrt{(1-2x)}$ and $g: R\setminus\left\{\frac{1}{2}\right\} \to R$, where $g(x)=\frac{1}{1-2x}$ then the domain of *fg* is

A. R^+

B. $R\setminus\left\{\frac{1}{2}\right\}$

C. $\left(-\infty,\frac{1}{2}\right)$

D. $\left(-\infty,\frac{1}{2}\right]$

E. R

Question 2

Let $f: R \to R$, where $f(x)=e^x+k$, where k is a real constant. If *f* and f^{-1} have two points of intersection then

A. $k<-1$
B. $k<0$
C. $k>1$
D. $k\le 0$
E. $k\le -1$

Question 3

The minimum value of *f* where $f(x)=|2x|+|2x-a|+|2x+a|$, where a is a real constant is

A. 0
B. $2a$
C. 2
D. $a+2$
E. $a-2$

TURN OVER

Question 4

If $f(x)=ax^2+bx+c$, where *a*, *b* and *c* are real constants, then

A. $f(x+y)=f(x)+f(y)-c$

B. $f(x+y)=f(x)+f(y)+c$

C. $f(x+y)=f(x)+f(y)-c+2axy$

D. $f(x+y)=f(x)+f(y)-c-2axy$

E. $f(x+y)=f(x)+f(y)$

Question 5

The number of solutions for the equation $|\sin(x)+3\cos(x)|=3$, for $-\pi\le x\le\pi$ is

A. 0
B. 2
C. 3
D. 4
E. 5

Question 6

The general solution to the equation $2\sin(2x)=1$ is given by

A. $x=15(12n+5)$ or $15(12n+1)$, $n\in Z$

B. $x=2n\pi+\dfrac{\pi}{2}$, $n\in Z$

C. $x=45(4n+1)$, $n\in Z$

D. $x=n\pi+\dfrac{5\pi}{12}$ or $n\pi+\dfrac{\pi}{12}$, $n\in Z$

E. $x=n\pi+\dfrac{\pi}{4}$, $n\in Z$

Question 7

The equations of all the asymptotes of the graph of *f* with equation

$f(x)=2\tan\left(3x+\dfrac{\pi}{2}\right)+1,\ -\dfrac{\pi}{3}\le x\le\dfrac{\pi}{3}$ are

A. $x=-\dfrac{\pi}{3}$ and $x=0$ and $x=\dfrac{\pi}{3}$

B. $x=-\dfrac{\pi}{3}$ and $x=\dfrac{\pi}{3}$

C. $x=-\dfrac{\pi}{6}$ and $x=\dfrac{\pi}{6}$

D. $x=-\dfrac{\pi}{2}$ and $x=\dfrac{\pi}{2}$

E. $x=-\dfrac{\pi}{2}+1$ and $x=\dfrac{\pi}{2}+1$

Question 8

If $f(x) = ax^3 + bx^2 + cx + d$, $f'(-1) = 3$ and there is a stationary point of inflection at (2, 4) then a, b, c and d are respectively

A. $-\frac{1}{27}, \frac{2}{9}, -\frac{4}{9}$ and $\frac{116}{27}$

B. $\frac{1}{9}, -\frac{2}{3}, \frac{4}{3}$ and $\frac{28}{9}$

C. $\frac{1}{27}, -\frac{2}{9}, \frac{4}{9}$ and $\frac{100}{27}$

D. $1, 6, 12$ and 12

E. $-1, -6, -12$ and -4

Question 9

If $h(x) = x^2 + x$ and $g(x) = |x|$ then

A. $h'(g(x))g'(x) = \begin{cases} 2x+1, & x > 0 \\ 2x-1 & x < 0 \end{cases}$

B. $h'(g(x))g'(x) = \begin{cases} 2x+1, & x > 0 \\ -2x-1 & x < 0 \end{cases}$

C. $h'(g(x))g'(x) = \begin{cases} 2x+1, & x \geq 0 \\ 2x-1 & x < 0 \end{cases}$

D. $h'(g(x))g'(x) = \begin{cases} 2x+1, & x \geq 0 \\ -2x-1 & x < 0 \end{cases}$

E. $h'(g(x))g'(x) = \begin{cases} 2x-1, & x > 0 \\ 2x+1 & x < 0 \end{cases}$

Question 10

The curve of *f* with equation $f(x) = e^{|x|+2}$

A. has a sharp point at $x = e^2$.

B. has a stationary point at $x = 0$.

C. has a positive gradient for $x \geq 0$.

D. has a negative gradient for $x \leq 0$.

E. cannot be differentiated at $x = 0$.

Question 11

The gradient of the normal to the curve of *f* with equation $y = f(x) = 2\log_e(1-x) + 3$ at the *y*-intercept is

A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 1

E. 2

TURN OVER

Question 12

The average rate of change of the curve of f with equation $f(x) = x^{\frac{1}{2}}\sin(2x)$ from $x = 0$ to $x = \pi$ is

A. 0.21
B. 0
C. -0.21
D. -2.10
E. 3.54

Question 13

The area bounded by the curve with equation $f(x) = \sqrt{(2-x)} + 3$ and the lines with equations $x = -2$ and $x = 2$ and the x-axis is approximated using left endpoint rectangles of width 1 unit. This area equals

A. $\frac{52}{3}$
B. $15 + \sqrt{3} + \sqrt{2}$ and is an overestimate of the actual area
C. $13 + \sqrt{3} + \sqrt{2}$ and is an overestimate of the actual area
D. $15 + \sqrt{3} + \sqrt{2}$ and is an underestimate of the actual area
E. $13 + \sqrt{3} + \sqrt{2}$ and is an underestimate of the actual area

Question 14

The area bounded by the curve with equation $f(x) = \log_e(x + a)$, where $a > 1$, and the axes can be found by evaluating

A. $\int_0^a f(x)dx$

B. $\int_{-a}^0 f(x)dx$

C. $\int_{1-a}^0 f(x)dx$

D. $\int_{a+1}^0 f(x)dx$

E. $\int_0^{1-a} f(x)dx$

Question 15

If $\dfrac{d(x\sin(x))}{dx}=\sin(x)+x\cos(x)$, then an antiderivative of $x\cos(x)$ is given by

A. $\dfrac{d(x\sin(x))}{dx}-\sin(x)$

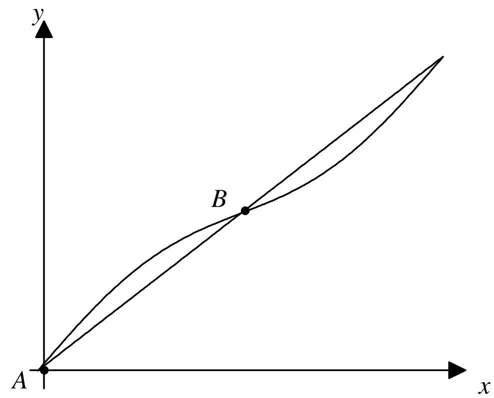
B. $\int(x\sin(x))dx-\sin(x)$

C. $\int(x\sin(x))dx+\cos(x)$

D. $x\sin(x)-\int(\sin(x))dx$

E. $x\sin(x)-\int(\cos(x))dx$

Question 16



Parts of the graphs of f and g with rules $f(x)=\sin(x)+2x$ and $g(x)=2x$ are shown. The area between the curves from A to B is

A. 0.086
B. 2
C. $2\pi^2+2$
D. $1-\cos(3)$
E. -2

Question 17

The continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}\dfrac{\pi}{2}\sin(\pi x), & 0\le x\le 1\\ 0 & , \text{ elsewhere}\end{cases}.$$

If $\Pr(0.1\le X\le a)=0.5$, then the value of a, correct to four decimal places is

A. 0.5156
B. -0.5156
C. 0.5156 and -0.5156
D. 0.3510
E. 0.2527

TURN OVER

Question 18
In a quiz game of 20 questions, the probability of getting an answer correct is 0.6. It is known that a particular player has, inclusively, between 15 and 18 correct answers. The probability the player has exactly 15 correct is closest to
A. 0.6119
B. 0.0745
C. 0.5968
D. 0.4596
E. 0.0005

Question 19
A die is weighted to provide the following probability distribution.

Number	1	2	3	4	5	6
Probability	0.2	0.1	0.1	0.3	0.1	0.2

Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 5\}$. Which one of the following statements is *not* correct?
A. $\Pr(A \cup B) = 0.5$
B. $\Pr(A | B) = \frac{2}{3}$
C. $\Pr(A' \cap B) = 0.1$
D. $\Pr(A' \cup B') = 0.8$
E. Events A and B are independent.

Question 20
For a particular psychological experiment on the behaviour of mice, it has been found that 80% of the mice that went right on a previous experiment will go right on this trial and 60% of those mice that went left on the previous experiment will go right on this trial. Suppose 50% of the mice went right on the first trial, then the prediction for the next trial would be given by
A. $\begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$
B. $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}\begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}$
C. $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}\begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}$
D. $\begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$
E. $\begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}\begin{bmatrix} 0.8 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}$

Question 21
A fair die is tossed. If 2, 3 or 5 occurs, the player wins that number of dollars. If 1, 4 or 6 occurs, the player losses that number of dollars. The expected gain or loss to a player is

A. lose $\$\frac{1}{6}$
B. win $\$\frac{1}{6}$
C. neither win nor lose
D. win $\$1$
E. lose $\$1$

Question 22
Scores on an examination are normally distributed with mean of 76 and standard deviation of 15. The top 15 percent of the students receive As and the bottom 10% receive Fs. The minimum score to receive A and the minimum score to pass (not receive F) are
A. score A: 84, score a pass: 70
B. score A: 90, score a pass: 68
C. score A: 92, score a pass: 57
D. score A: 94, score a pass: 56
E. score A: 91, score a pass: 61

END OF SECTION 1

TURN OVER

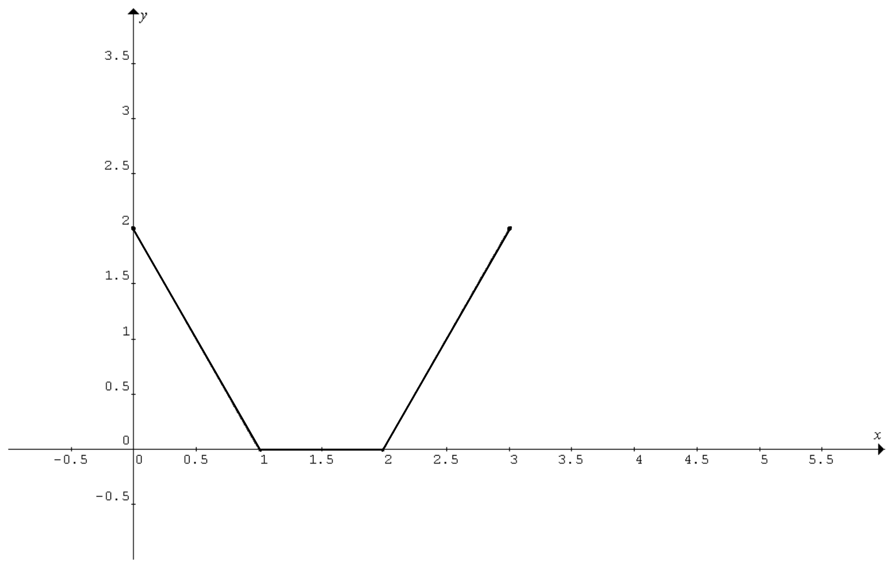
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. The graph of $h:[0,3]\rightarrow R$, where $h(x)=|x-1|+|x-2|-1$ is shown below.



i. Complete the following so that $h(x)$ is written as a hybrid function.

$$h(x)=\begin{cases} \_\_\_\_\_\_\_\_\_\_, & 0\le x\le 1 \\ \_\_\_\_\_\_\_\_\_\_, & 1<x\le 2 \\ 2x-4, & \_\_\_\_\_\_\_\_ \end{cases}$$

ii. Find the average value of h.

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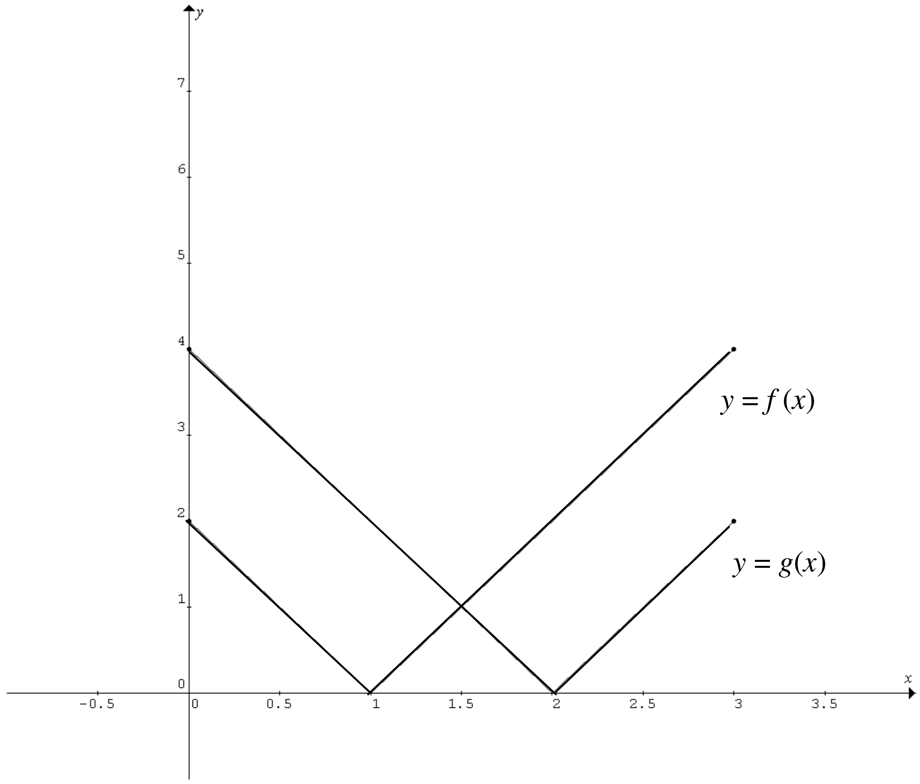
3 + 2 = 5 marks

Question 1- continued

b.



The graphs of *f* and *g* are shown above.

i. Sketch the graph of $f + g$ on the set of axes above.

ii. The average value of $f + g$ is $\frac{10}{3}$. Sketch the line with equation $y = \frac{10}{3}$ on the above graph for $0 \le x \le 3$.

iii. Find the area bounded by the line $y = \frac{10}{3}$ and the graph of $f + g$.

2 + 3 + 1 = 6 marks

Question 1- continued
TURN OVER

c. Consider the family of curves given by $p(x) = |\log_e(x-1)| + |\log_e(x-2)| + c$ where c is a real constant.

i. If $x = a$ is an asymptote of p, what is the value of a.

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ii. Give the exact coordinates of the sharp point of p in terms of c.

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1 + 1 = 2 marks

Total 13 marks

Question 2

a. Show that the derivative of $y = \dfrac{\sin(x)}{\cos(x)}$ is $\dfrac{dy}{dx} = \dfrac{1}{\cos^2(x)}$.

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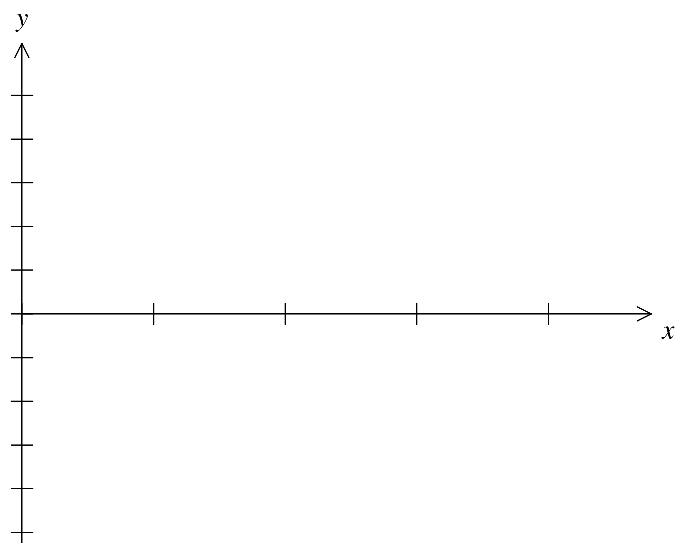
1 mark

b. On the axes provided, sketch the graph of $f(x) = \dfrac{dy}{dx} = \dfrac{1}{\cos^2(x)}$, $0 \le x \le \pi$.



2 marks

c. Find **i.** y **ii.** $\dfrac{dy}{dx}$, when $x = \dfrac{\pi}{4}$.

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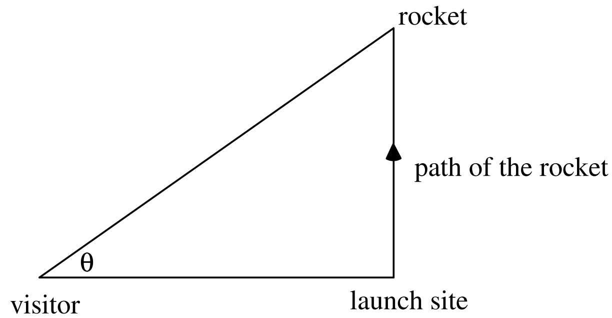
2 marks

Question 2 - continued
TURN OVER

A visitor to a rocket range watches a rocket fired vertically from the ground. The visitor is 3 km from the launch site of the rocket. She observes the rocket at an angle of elevation of θ radians as shown in the diagram below.



d. State the domain of θ.

1 mark

e. Let h km be the height of the rocket above the ground. Show that the height of the rocket is given by $h = 3\tan(\theta)$.

1 mark

f. Find $\frac{dh}{d\theta}$. What does this derivative signify?

2 marks

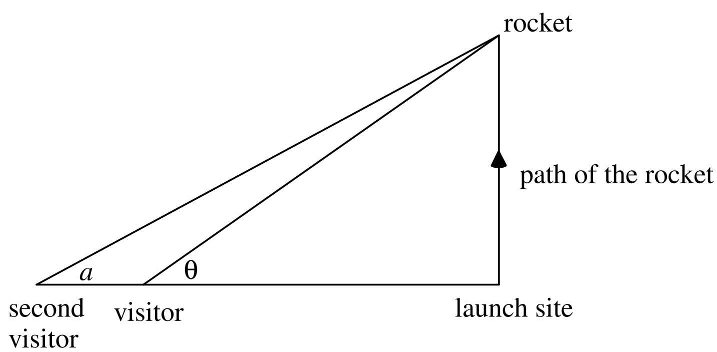
Question 2 - continued

The visitor estimates that the angle, θ, is changing at the rate of 20 degrees per second when the angle of elevation is 45 degrees.

g. Write a mathematical equation for this rate in radians per second and hence find the velocity of the rocket at this time in kilometres per second.

3 marks

A second visitor stands in a direct line behind the first visitor and 4 kilometres from the launch site. (See diagram.)



h. **i.** Calculate the angle α radians, correct to 4 decimal places, when $\theta = \frac{\pi}{4}$.

1 mark

Question 2 - continued
TURN OVER

ii. Hence, find the rate at which α is changing, correct to two decimal places, when $\theta = \frac{\pi}{4}$.

3 marks

Total 16 marks

Question 3
A chocolate factory makes teddy bear statues. The weight of the statues is normally distributed with a mean of 1000 grams and standard deviation of 4 grams.

a. Find the probability, correct to four decimal places, a statue weighs between 992 grams and 1010 grams.

1 mark

b. Statues whose weight does not lie between 992 grams and 1010 grams are rejected as being overweight or underweight. If 1200 statues are manufactured over a particular period, find the number of statues, correct to the nearest whole number, likely to be rejected during this period of production.

1 mark

c. To reduce the number of overweight statues to 10% **of rejects**, the machine can be modified to change the standard deviation while retaining the mean at 1000 grams. Find, to the nearest whole number, what the new standard deviation should be.

2 marks

d. Five statues are selected at random. What is the probability, correct to three decimal places, that at least one of the statues is outside the desired weight of 992 grams to 1010 grams? Assume the standard deviation is 4.

2 marks

Question 3 - continued
TURN OVER

Each month the chocolate statues are sold to a department store. There is an 85% chance the department store will buy statues this month if it bought them last month. If the department store did not buy the statues last month there is a 30% chance they will buy them this month.

Suppose the department store bought statues last month.

e. **i**. Find the probability, correct to four decimal places, that the department store will buy statues for the next three months.

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ii. Find the probability, correct to four decimal places, that it buys statues for two of the next three months.

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iii. Find the steady state probability that the department store buys the statues each month.

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1 + 2 + 1 = 4 marks

Question 3 - continued

At a different department store the probability it buys statues this month if it bought them last month is p. If it did not buy statues last month the probability it will buy statues this month is $p-0.1$. This department store bought statues in January and the probability it will buy statues in March is 0.7.

f. Show that $p=\frac{8}{11}$.

2 marks

g. What is the expected number of months, from February to March that the department store will buy statues? Give the answer correct to one decimal place.

3 marks

Total 15 marks

TURN OVER

Question 4

Let $y = f(x) = a(x-b)^5 + c$, where a, b and c are positive real constants.

a. In terms of a, b and c find the **coordinates** of the

i. x-intercept

ii. y-intercept.

iii. Give the coordinates of the stationary point of inflection in terms of a and b if the value of the x-intercept is the same as the y-intercept.

iv. If the coordinates of the stationary point of inflection are (2, 3) for what values of a will the x-intercept be negative?

1 + 1 + 2 + 2 = 6 marks

Question 4 - continued
TURN OVER

Consider the function g with equation $g(x) = 2x^5 - 2x^4 - 4x^3 + 4x^2 + 2x$. g can be written in the form $g(x) = A(x-B)^3(x-C)^2 + D$.

b. **i.** How many stationary points does g have?

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ii. Find the values of A, B, C and D.

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iii. Give the coordinates of the local minimum.

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iv. For what values of x is g a strictly increasing function?

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v. Let $g_1 : \left[-1, -\frac{1}{5}\right] \to R$, where $g_1(x) = 2x^5 - 2x^4 - 4x^3 + 4x^2 + 2x$.

Find the coordinates, correct to three decimal places, of the point where $g_1 = g_1^{-1}$.

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1 + 2 + 1 + 2 + 2 = 8 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOK