Year 2011

VCE

Mathematical Methods CAS

Trial Examination 2

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Victorian Certificate of Education
2011

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS CAS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 34 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple- choice questions, and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

The solution of the equation $|x-2k|<k$ where $k>0$ is equal to

A. $(k,3k)$

B. $(-\infty,3k)$

C. $(-k,3k)$

D. $(k,2k)$

E. $[2k,3k)$

Question 2

The average value of the function $f(x)=2x+2\sin\left(\frac{\pi x}{2}\right)$ for $0\le x\le 2$ is equal to

A. $2+\frac{4}{\pi}$

B. $4+\frac{8}{\pi}$

C. 2

D. 4

E. $\frac{720}{\pi^2}+2$

Question 3

The graph of $y = \dfrac{ax+b}{x+c}$ where *a*, *b* and *c* are non-zero real constants, has two asymptotes with the equations

A. $x = -c$ and $y = b$

B. $x = -c$ and $y = a$

C. $x = c$ and $y = 1$

D. $x = c$ and $y = b$

E. $x = -c$ and $y = -\dfrac{b}{a}$

Question 4

Let $f(x) = x^2$, $g(x) = \dfrac{1}{x}$ and $h(x) = \cos(x)$. Then $\dfrac{d}{dx}(\tan(x))$ is equal to

A. $h(f(g(x)))$

B. $h(g(f(x)))$

C. $f(h(g(x)))$

D. $g(f(h(x)))$

E. $g(h(f(x)))$

Question 5

Consider the function $f(x) = \sqrt{x-a} + \sqrt{b-x}$ defined on its maximal domain.
If $b > a > 0$ then which of the following is **true?**

A. The domain is $[a,b]$ and the range is $[0,\infty)$.

B. The domain is $[a,b]$ and the range is $\left[\sqrt{b-a}, \sqrt{2(b-a)}\right]$.

C. The domain is (a,b) and the range is *R*.

D. The domain is (a,b) and the range is $[0,\infty)$.

E. The domain is *R* and the range is $[0,\infty)$.

Question 6

Consider the simultaneous linear equations

$px + 3y + z = 5$
$2x - y + 2pz = 3$ Which of the following is **true?**
$x + 4y + pz = 6$

A. There is no unique solution when $p^2 = 1$ and infinitely many solutions when $p = -1$.

B. There is no unique solution when $p^2 = 1$ and no solution when $p = 1$.

C. There is no unique solution when $p^2 \neq 1$ and no solution when $p \neq -1$.

D. There is a unique solution when $p^2 \neq 1$ and infinitely many solutions when $p \neq -1$.

E. There are infinitely many solutions when $p = 1$ and no solution when $p = -1$.

Question 7

The graph of the function $y = f(x)$ passes through the point $(2, -3)$. The graph of the function $y = 1 - f(x+1)$ would pass through the point

A. $(1, 4)$

B. $(3, 4)$

C. $(1, -2)$

D. $(3, -2)$

E. $(3, 2)$

Question 8

If a is a positive real number, then consider the function $f : R \to R$ where $f(x) = |x - a|$, which of the following is **false**?

A. The maximal domain is R and the range is $[0, \infty)$.

B. The function is not differentiable at $x = a$.

C. The function is not continuous at $x = a$.

D. An equivalent function is $f : R \to R$ where $f(x) = \begin{cases} x - a & \text{for } x \geq a \\ a - x & \text{for } x < a \end{cases}$

E. An equivalent function is $f : R \to R$ where $f(x) = |a - x|$

Question 9

For the simultaneous linear equations

$z = 3$

$x + y = 5$ an equivalent matrix equation is

$y - x = -1$

A. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$

C. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$

Question 10

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \dfrac{2}{\pi\sqrt{9 - x^2}} & \text{for } 0 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

Then $\text{var}(X)$ is closest to

A. 4.5

B. 2.59

C. 1.91

D. 0.92

E. 0.85

Question 11

A certain curve has a gradient given by $4e^{-\frac{x}{2}}$, the particular curve which passes through the origin, is given by

A. $y=-2e^{-\frac{x}{2}}$

B. $y=-8e^{-\frac{x}{2}}$

C. $y=2\left(1-e^{-\frac{x}{2}}\right)$

D. $y=8\left(1-e^{-\frac{x}{2}}\right)$

E. $y=-8\left(1-e^{-\frac{x}{2}}\right)$

Question 12

Using a linear approximation, with $f(x)=e^{-x}$, then $\dfrac{1}{e^{0.99}}$ is equal to

A. $f(1)+0.01f'(1)$

B. $f(1)-0.01f'(1)$

C. $f(-1)+0.01f'(-1)$

D. $f(-1)-0.01f'(-1)$

E. 0.3716

Question 13

The gradient of the normal to the curve $y=8\cos\left(\dfrac{x}{2}\right)$ at the point where $x=\dfrac{2\pi}{3}$ is equal to

A. $-\dfrac{\sqrt{3}}{24}$

B. $-\dfrac{\sqrt{3}}{6}$

C. $\dfrac{\sqrt{3}}{6}$

D. 2

E. $-2\sqrt{3}$

Question 14

If $f(x)=\dfrac{x^2}{g(x)}$ and $g(3)=2$ and $g'(3)=1$ then $f'(3)$ is equal to

A. $\dfrac{3}{4}$

B. 3

C. 6

D. 9

E. $-\dfrac{3}{4}$

Question 15

If $\int_0^2 f(x)\,dx=2$ then $\int_2^0 \left(2x-f(x)\right)dx$ is equal to

A. –2

B. –4

C. –6

D. 2

E. 4

Question 16

If Z has the standard normal distribution with $\Pr(Z>b)=B$ and $\Pr(Z>a)=A$,
where $0<a<3$, $0<b<3$, $0<A<1$ and $0<B<1$.
Then $\Pr(-a<Z<-b)$ is equal to

A. $A-B$

B. $B-A$

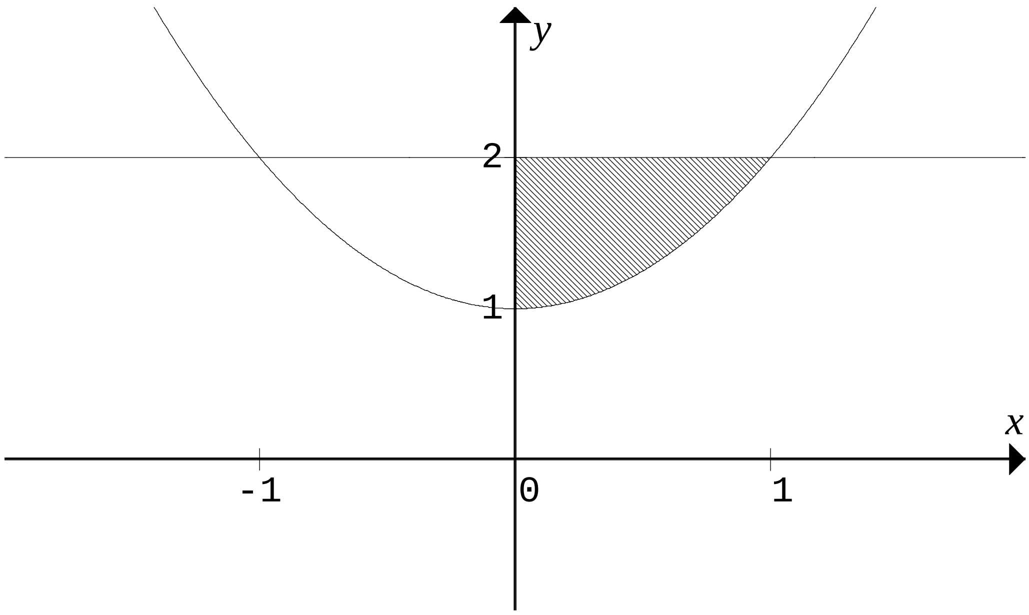
C. $A+B-1$

D. $1-(A+B)$

E. $A+B-0.5$

Question 17

The graphs of $y = x^2 + 1$ and $y = 2$ are shown below.



The shaded area is equal to

A. $\int_0^1 \left(x^2 + 1\right) dx$

B. $\int_0^1 \left(x^2 - 1\right) dx$

C. $\int_0^1 \left(2 - x^2\right) dx$

D. $\int_1^2 \sqrt{x-1}\, dx$

E. $\int_1^2 \sqrt{1-x}\, dx$

Question 18

Consider the function $f: D \to R$ with the rule $f(x) = x^3 - 5ax^2 + 7a^2x - 3a^3$ where a is a positive real number. The function will have an inverse function, provided that the domain D is equal to

A. $(a, 3a)$

B. $(-\infty, 2a)$

C. $\left(-\infty, \frac{7a}{3}\right)$

D. $(2a, \infty)$

E. $\left(\frac{7a}{3}, \infty\right)$

Question 19

Events A and B are independent events of a sample space with $\Pr(A) = a$ and $\Pr(B) = b$ where $0 < a < 1$ and $0 < b < 1$. Then $\Pr(A' \cup B')$ is equal to

A. $2 - (a + b + ab)$

B. $1 - a - b$

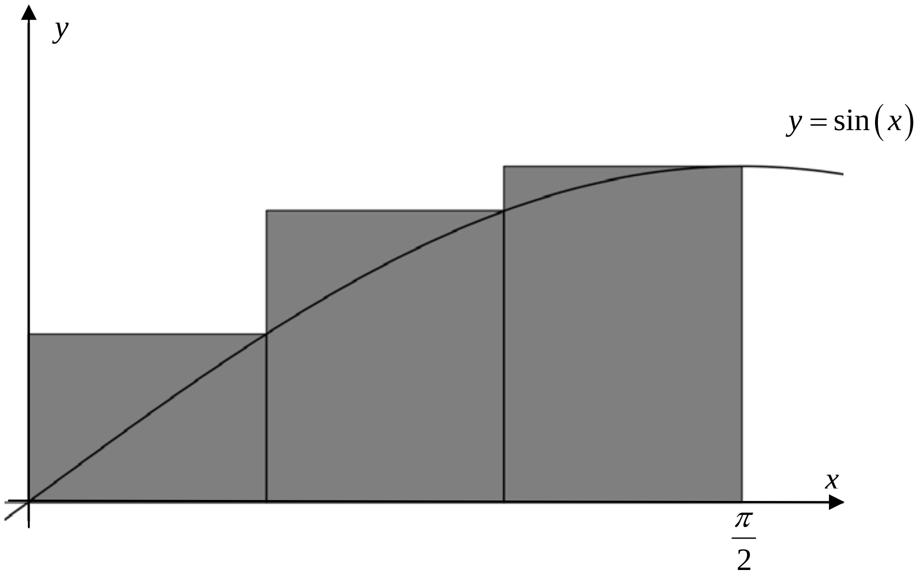
C. $1 - (a + b) + ab$

D. $1 - ab$

E. $a + b - ab$

Question 20

The area of the three shaded rectangles, each of equal width, can be used as an approximation for the area between the curve $y = \sin(x)$, the x-axis $x = 0$ and the line $x = \frac{\pi}{2}$.



The value of this approximation is closest to

A. 0.022

B. $\frac{\pi}{4}$

C. $\frac{\pi(\sqrt{3}+3)}{12}$

D. $\frac{\pi(\sqrt{3}+2)}{12}$

E. 1

Question 21

A discrete random variable has a binomial distribution. The expression

$1-\left(0.7^{10}+10\times 0.7^{9}\times 0.3+45\times 0.7^{8}\times 0.3^{2}\right)$ represents the probability of

A. at least two successes in ten trials each with a probability of success equal to 0.3

B. at least two successes in ten trials each with a probability of success equal to 0.7

C. more than two successes in ten trials each with a probability of success equal to 0.3

D. more than two successes in ten trials each with a probability of success equal to 0.7

E. at least three successes in ten trials each with a probability of success equal to 0.7

Question 22

The general solution of the equation $a\sin(2x)+b\cos(2x)=0$ is $x=\dfrac{n\pi}{2}-\dfrac{\pi}{6}$ where $n\in Z$, then

A. $a=\sqrt{3}$ and $b=1$

B. $a=\sqrt{3}$ and $b=-1$

C. $a=-\sqrt{3}$ and $b=2$

D. $a=1$ and $b=-\sqrt{3}$

E. $a=1$ and $b=\sqrt{3}$

END OF SECTION 1

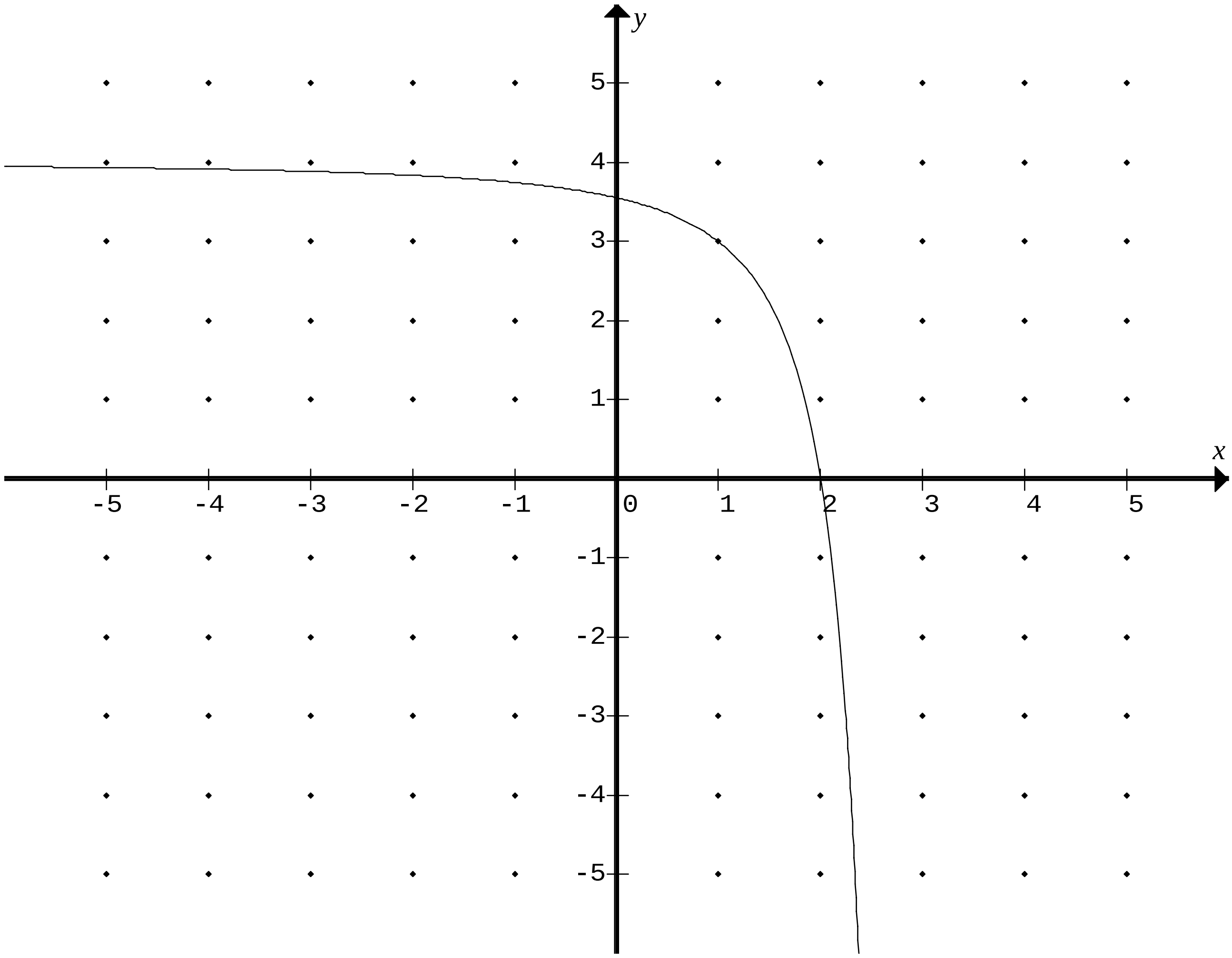
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Part of the graph of $m:(-\infty,3)\to R$, $m(x)=4-\dfrac{4}{(x-3)^2}$ is shown below.



i. State in words, the transformations required, so that the graph of $y = 4 - \dfrac{4}{(x-3)^2}$ is produced from the graph of $y = \dfrac{1}{x^2}$.

2 marks

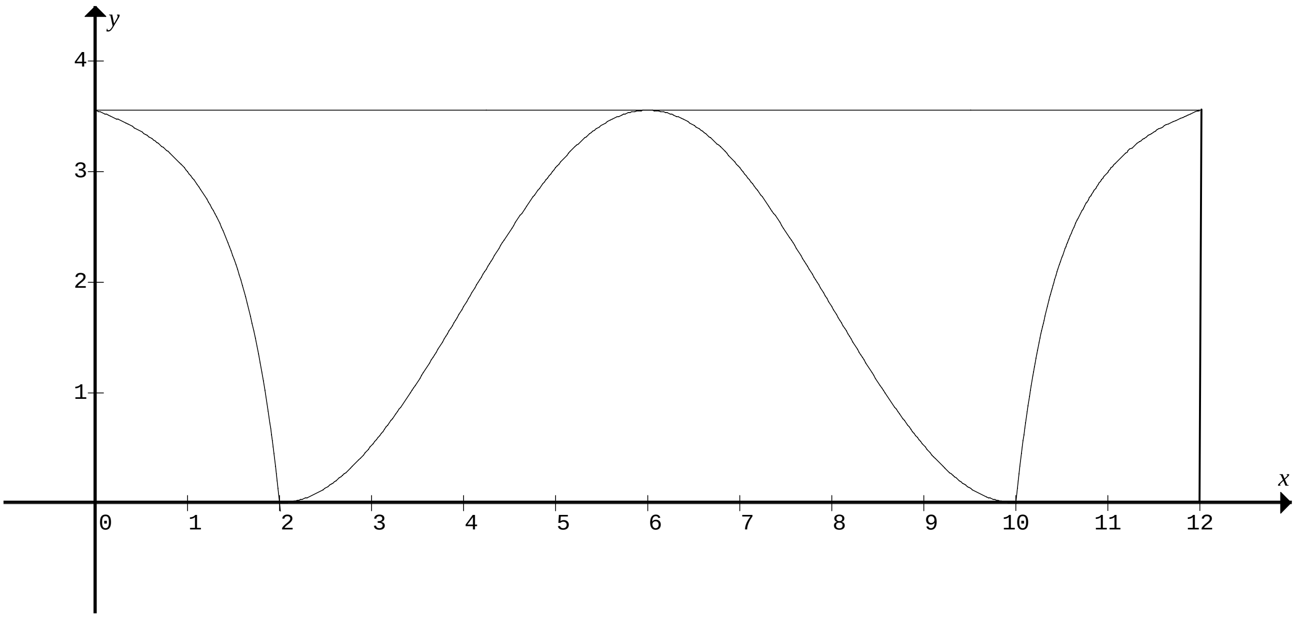
ii. Find m^{-1}**,** the inverse function of m.

3 marks

iii. On the set of axes on the previous page, sketch the graph of m^{-1}.
Label the axes intercepts with their exact values and state the equations of any asymptotes, for the graph of m^{-1}.

3 marks

b. The diagram below is part of the design of the cross-section of a bridge.
All measurements are in metres.



The bridge is composed of three graphs

$f:[0,2]\rightarrow R\ ,\ f(x)=4-\dfrac{4}{(x-3)^2}$

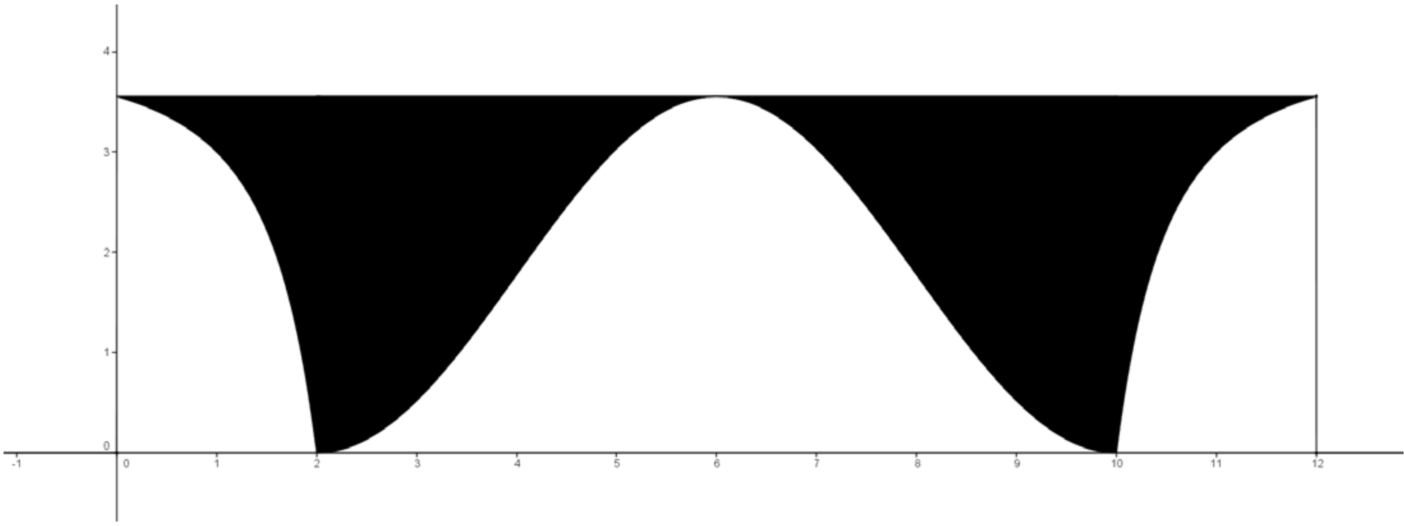
$g:[2,10]\rightarrow R\ ,\ g(x)=b\sin^2\left(n(x-c)\right)$

$h:[10,12]\rightarrow R\ ,\ h(x)=p+\dfrac{r}{(x-s)^2}$

Where the graph of $h(x)$ is the reflection of the graph of $f(x)$ in the line $x=6$.

i. Find the values of b , n , c , p , r and s.

3 marks



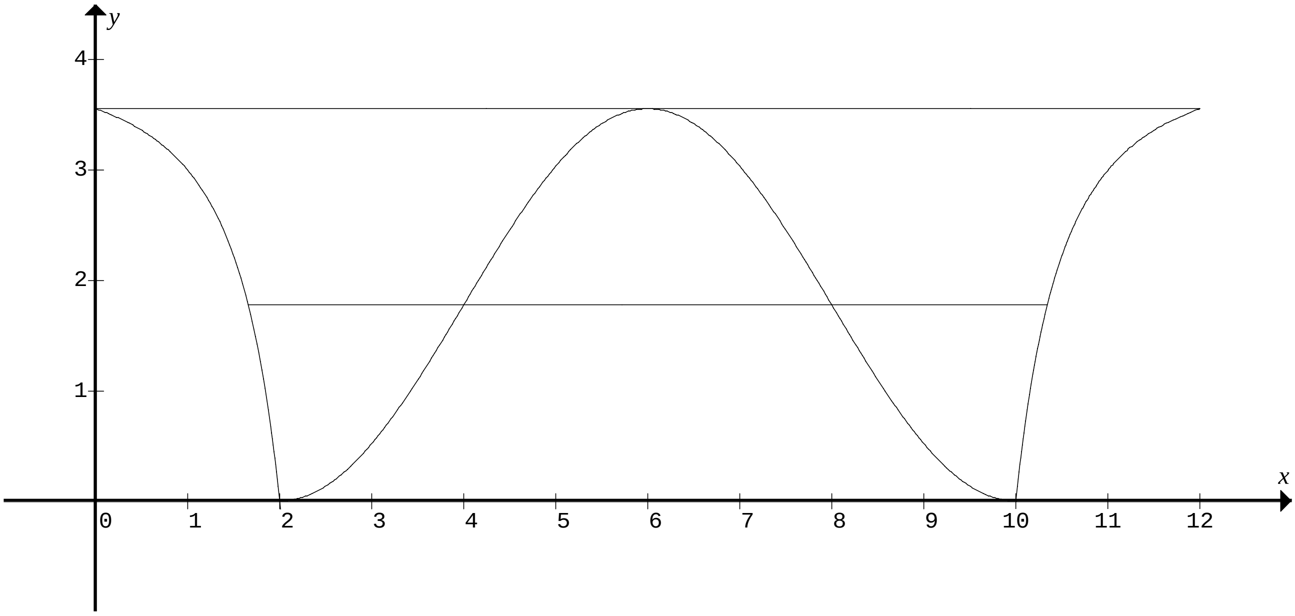
ii. Part of the cross-sectional area of the bridge is to be painted as shown in the diagram above. Write down definite integrals, which give an expression for the total area in square metres of the shaded region above. It may be given in terms of *b*, *n* and *c*.

2 marks

iii. Find the area of this shaded region.

1 mark

iv. A supporting beam is to be added to the structure, it is positioned at half the height and touches the curves *f* and *h* as shown in the diagram below. Find the length of the beam, giving your answer in metres, correct to three decimal places.



1 mark
Total 15 marks

Question 2

Every week Mr. and Mrs. J go to the local supermarket. They either go to the Coles or Safeway supermarket. If one week they go to Coles, then the probability that they go to Coles the following week is 0.65. If they go to Safeway one week, then the probability that they go Coles the following week is 0.55

a. During one particular month, Mr. and Mrs. J went to the supermarket four times and in the first week they went to Coles. During this particular month,

i. find the probability that they go to Coles once, give your answer correct to four decimal places.

1 mark

ii. find the probability that they go to Coles exactly twice, give your answer correct to four decimal places.

2 marks

iii. find the expected number of times that they go to Coles.

3 marks

b. find the steady state probability that they go to Coles.

1 mark

c. At a checkout in the supermarket, the probability that the barcode reader fails to read the barcode on a particular item on the first attempt is p and this is independent from item to item.

i. At one particular checkout Mr. and Mrs. J have 50 items in their shopping trolley, if $p = 0.08$, find the probability that the barcode reader fails to read the barcode on less than five items, if it is known to have failed to read the barcode on at least one item. Give your answer correct to four decimal places.

2 marks

ii. At another week, at a different checkout, when Mr. and Mrs. J had 70 items in their shopping trolley, an analysis showed that the barcode reader failed to read the barcode on items the first time with a mean of less than five, while the variance was 4.557. Find the value of p correct to two decimal places for this checkout.

2 marks

iii. At another instant, at another particular checkout out in the supermarket, when Mr. and Mrs. J had 40 items in their trolley, the barcode reader failed with a probability 0.47 to read the barcode on the first attempt on two or three items. Find the value of p for this checkout, giving your answer correct to four significant figures.

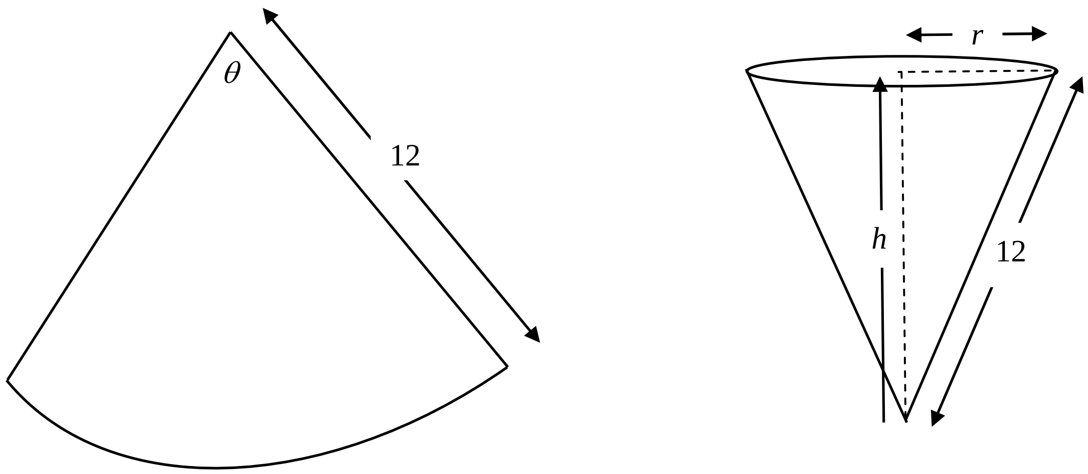
2 marks

d. Mr. and Mrs. J have found that the time spent shopping at the supermarket is normally distributed. They find that 20% of the time they take longer than 50 minutes at the supermarket, while 37% of the time they take less than 36 minutes. Find the mean and standard deviation of the times spent at the supermarket, give your answers correct to the nearest minute.

4 marks
Total 17 marks

Question 3

An ice-cream shop makes waffle cones, which are made from a thin flat mixture which forms into a sector of a circle with radius 12 cm and angle θ radians. The two straight edges are joined without overlap to form a right cone, which has a height of *h* cm and whose circular top section has a radius of *r* cm as shown below.



a. Show that the volume V cm^3 of the waffle cone is given by $V = \frac{72\theta^2}{\pi^2}\sqrt{4\pi^2 - \theta^2}$.

3 marks

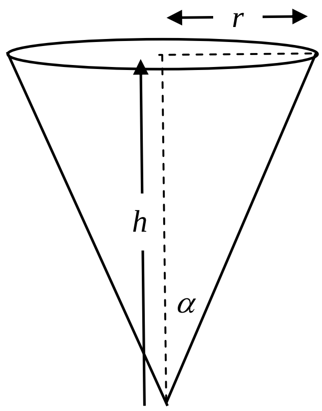
b. The waffle cone is to be made to have a maximum volume. Find $\frac{dV}{d\theta}$ and hence find the maximum volume of the waffle cone and the corresponding value of θ for which it occurs.

3 marks

c. A baby waffle cone is to be made to have a volume of exactly one half of the maximum volume of the waffle cone. Find the value of θ for a baby waffle cone, giving your answer correct to two decimal places.

2 marks

d. The ice-cream shop also serves waffles cones filled with chocolate mousse. These cones are such that their semi-vertex angle $\alpha = \tan^{-1}\left(\sqrt{2}\right)$ as shown in the diagram below. Unfortunately after a while, the mousse begins to drip out through the vertex at the bottom of the cone at a rate of $0.5\,\text{cm}^3/\text{sec}$. Find the rate in cm/sec at which the height of the chocolate mousse is falling, when the height of the mousse is 4 cm, measured from the vertex of the cone.

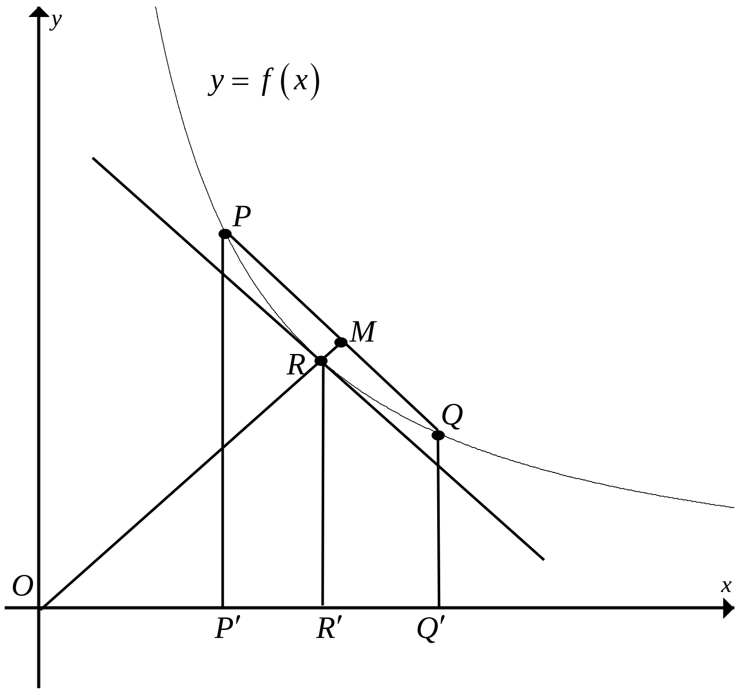


3 marks
Total 11 marks

Question 4

The diagram below shows part of the graph of the function $f:(0,\infty)\to R$ where $f(x)=\frac{1}{x}$.

Let $P(p, f(p))$ and $Q(q, f(q))$ where $q > p > 0$ be two points on the graph of $y = f(x)$. The points $P'(p,0)$ and $Q'(q,0)$ lie on the *x*-axis, while *M* is the mid-point of the line segment joining *P* and *Q*. The line through *OM*, where *O* is the origin crosses the function at the point $R(r, f(r))$ and the point $R'(r,0)$ also lies on the *x*-axis.



a. The equation of the tangent to the curve at the point *R* can be expressed as $y = mx + c$. Express *m* and *c* in terms of *r*.

2 marks

b. Find the co-ordinates of the point M and show that $r^2 = pq$.

3 marks

c. Explain why the tangent to the curve at R, is parallel to the line segment joining P and Q.

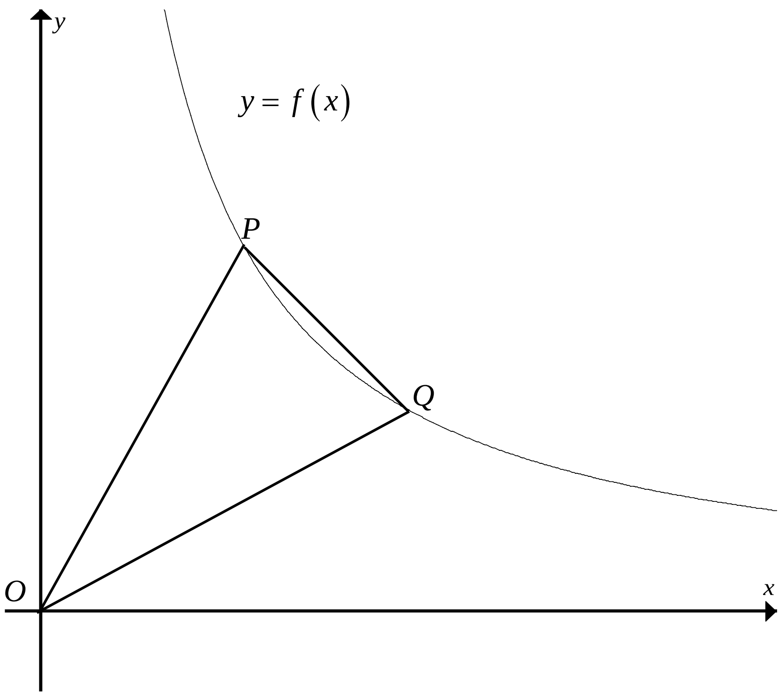
2 marks

d. Let A be the area bounded by the curve $y = f(x)$ and the points $P'Q'QP$. Express A in terms of p and q.

1 mark

e. Find in terms of *A*, the area bounded by the curve $y = f(x)$ and the points $P'R'RP$

3 marks



f. The area B bounded by the curve $y = f(x)$ and the line segments OP and OQ can be written as the sum of two definite integrals $B = \int_0^p g(x)\,dx + \int_p^q h(x)\,dx$.

Write down the rules for the functions $g(x)$ and $h(x)$.

2 marks

g. Express the area B in terms of the area A.

2 marks
Total 15 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	Volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}dx=\log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$$

$$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$$

$$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+h\,f'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

Transition Matrices $S_n=T^n\times S_0$

mean: $\mu=E(X)$

variance: $\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$

ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

SIGNATURE ____

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**