Year 2011

VCE

Mathematical Methods CAS

Trial Examination 1

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Victorian Certificate of Education
2011

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the linear simultaneous equations

$$-2x + ky = k + 2$$
$$(k+1)x - 6y = -10$$

where k is a real constant, find the value(s) of k, for which there is

i. a unique solution.

ii. infinitely many solutions.

iii. no solution.

4 marks

Question 2

Let $f(x) = \frac{1}{1-2x}$ be a function defined on its maximal domain. Find the inverse function f^{-1} stating its domain and range.

3 marks

Question 3

a. Differentiate $x\sin(2x)$ with respect to x.

1 mark

b. Hence find $\int x\cos(2x)\,dx$

2 marks

c. The function $f(x) = \begin{cases} 2\cos(2x) & \text{for } 0 \le x \le b \\ 0 & \text{otherwise} \end{cases}$

is a probability density function for the continuous random variable X.

i. Show that $b = \frac{\pi}{4}$.

1 mark

ii. Find the mean of $f(x)$.

2 marks

iii. Find the median of $f(x)$.

2 marks

Question 4

If $\frac{d}{dx}\left(\log_e\left(\frac{3x^2+4}{4x^2+3}\right)\right)=\frac{bx}{g(x)}$, find the value of b and the function $g(x)$.

3 marks

Question 5

The speed v, in metres per second, of an object moving in a straight line is given by a function of time t, in seconds, where $v(t)=\frac{72}{(3t+2)^2}$ where $t \geq 0$.

Find the distance travelled in metres by the object in the first 2 seconds.

3 marks

Question 6

Let $f : R \to R$ where $f(x) = x^3 - k^2 x$, where k is a positive real constant.

a. Find in terms of k, the subset of R, for which the graph of the function $f(x)$ is strictly increasing.

2 marks

b. If the area bounded by the graph of $f(x)$, the x-axis and $x = 0$ and $x = k$ is equal to 64 units, find the value of k.

2 marks

Question 7

Given the functions $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ where $f(x)=\tan(x)$ and $g:R^{+}\to R$ where $g(x)=\cos(x)$.

a. If the two functions intersect at $x=\alpha$, show that $\sin(\alpha)=\frac{1}{2}\left(\sqrt{5}-1\right)$

3 marks

b. Show that the functions intersect at right angles.

3 marks

Question 8

A binomial distribution of the random variable X, with six independent trials, is such that $\Pr(X = 3) = \Pr(X = 4)$. If p is the probability of a success on any trial, find the value of p.

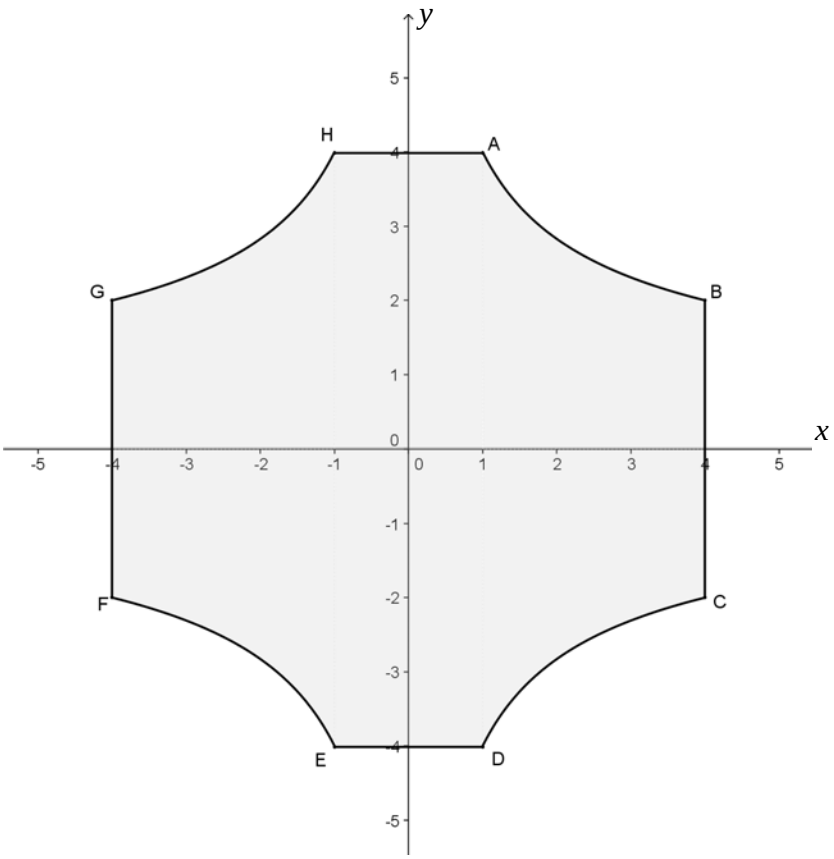
3 marks

Question 9

a. Evaluate $\int_1^4 \frac{4}{\sqrt{x}}\,dx$

1 mark

b. The diagram below shows a shield ABCDEFGH, the shield is symmetrical, with the curve CD being the reflection of AB in the *x*-axis, and the curve EF the reflection of CD in the *y*-axis. The function describing the curve AB is given by $f: [1,4] \to R$ where $f(x) = \frac{4}{\sqrt{x}}$.



i. Write down the **function** which describes the curve EF.

_____

_____

1 mark

ii. Find the total area of the shield.

_____

_____

1 mark

Question 10

The transformation $T : R^2 \to R^2$ of the plane defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$

maps the equation $y = \cos(x)$ into $y = -2\cos\left(\frac{\pi x}{3} + 1\right) + 4$.

Find the values of a, b, c, d, h and k.

3 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx$