Year 2011

VCE

Mathematical Methods CAS

Trial Examination 1



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Victorian Certificate of Education 2011

STUDENT NUMBER

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Figures							
Words							

MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the linear simultaneous equations

$$-2x + ky = k + 2$$

$$(k+1)x-6y = -10$$

where k is a real constant, find the value(s) of k, for which there is

- i. a unique solution.
- ii. infinitely many solutions.
- iii. no solution.

Question 2
Let $f(x) = \frac{1}{1-2x}$ be a function defined on its maximal domain. Find the inverse
function f^{-1} stating its domain and range.
3 mar
Question 3
a. Differentiate $x\sin(2x)$ with respect to x .
1 ma
b. Hence find $\int x \cos(2x) dx$

c.	The function $f(x) = \begin{cases} 2\cos(2x) & \text{for } 0 \le x \le b \\ 0 & \text{otherwise} \end{cases}$	
	is a probability density function for the continuous random variable X .	
i.	Show that $b = \frac{\pi}{4}$.	
ii.	Find the mean of $f(x)$.	1 mark
iii.	Find the median of $f(x)$.	2 marks

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If	$\frac{d}{dx} \left(\log_e \left(\frac{3x^2 + 4}{4x^2 + 3} \right) \right)$	$=\frac{bx}{g(x)}$, find the value of b and the function $g(x)$.

Question 5

The speed v, in metres per second, of an object moving in a straight line is given by a function of time t, in seconds, where $v(t) = \frac{72}{\left(3t+2\right)^2}$ where $t \ge 0$.

Find the distance travelled in metres by the object in the first 2 seconds.

Quest	
a.	: $R \to R$ where $f(x) = x^3 - k^2 x$, where k is a positive real constant. Find in terms of k , the subset of R , for which the graph of the function $f(x)$ is strictly increasing.
	2 marks
b.	If the area bounded by the graph of $f(x)$, the x-axis and $x = 0$ and $x = k$ is equal to 64 units, find the value of k .

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Given the functions $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ where $f(x)=\tan(x)$ and $g:R^+\to R$ where $g(x)=\cos(x)$.

a. If the two functions intersect at $x = \alpha$, show that $\sin(\alpha) = \frac{1}{2}(\sqrt{5} - 1)$

3 marks

b. Show that the functions intersect at right angles.

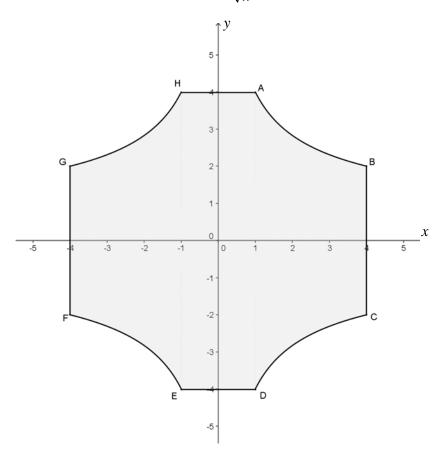
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A binomial distribution of the random variable X , with six independent trials, is such that $Pr(X=3) = Pr(X=4)$. If p is the probability of a success on any trial, find the value of
p.
3 marks
Question 9 a. Evaluate $\int_{1}^{4} \frac{4}{\sqrt{x}} dx$

1 mark

b. The diagram below shows a shield ABCDEFGH, the shield is symmetrical, with the curve CD being the reflection of AB in the *x*-axis, and the curve EF the reflection of CD in the *y*-axis. The function describing the curve AB is given by

$$f: [1,4] \rightarrow R$$
 where $f(x) = \frac{4}{\sqrt{x}}$.



i. Write down the **function** which describes the curve EF.

1 mark

ii. Find the total area of the shield.

1 mark

Question 10				
The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ of the plane defined by $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$				
maps the equation $y = \cos(x)$ into $y = -2\cos(\frac{\pi x}{3} + 1) + 4$.				
Find the values of a , b , c , d , h and k .				

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$ volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$ volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$ area of triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^{n}) = nx^{n-1}$$

$$\int x^{n} dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_{e}|x| + c$$

$$\int \frac{1}{x} dx = \log_{e}|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$$Pr(A) = 1 - Pr(A')$$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

 $\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ **Transition Matrices** $S_n = T^n \times S_0$

mean: $\mu = E(X)$ variance: $\operatorname{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$