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Mathematical Methods(CAS)

2011

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

If $\tan\theta = \dfrac{x}{x-1}$ and $\tan\phi = 1 - \dfrac{1}{x}$, then $\theta + \phi =$

A. 0

B. x

C. $\dfrac{\pi}{2}$

D. e

E. π

Question 2

The rule of function f is $f(x) = e^{\log_{\frac{1}{e}} \sqrt{x}}$. Function f is

A. an exponential function of x
B. a logarithmic function of x
C. a linear function of x
D. a polynomial function of x
E. a power function of x

Question 3

The equation(s) of the asymptote(s) of $y = \dfrac{x+1}{nx\sin(nx)\cos(nx)}$, where $n \in R$ and $x \in \left(-\dfrac{\pi}{n}, \dfrac{\pi}{n}\right)$, are

A. $x = -\dfrac{\pi}{n}$, $x = -\dfrac{\pi}{2n}$, $x = 0$, $x = \dfrac{\pi}{2n}$ and $x = \dfrac{\pi}{n}$

B. $x = -\dfrac{\pi}{2n}$, $x = 0$ and $x = \dfrac{\pi}{2n}$

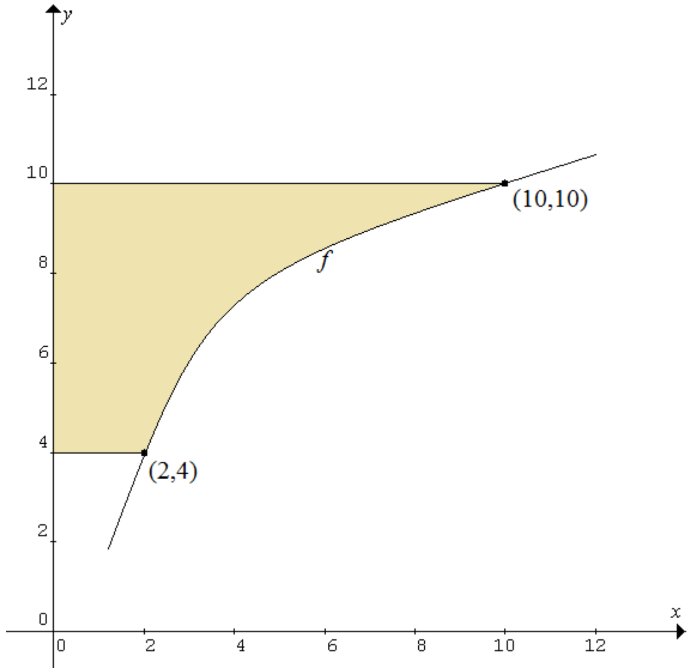
C. $x = 0$

D. $x = -\dfrac{\pi}{4n}$, $x = 0$ and $x = \dfrac{\pi}{4n}$

E. $x = -\dfrac{\pi}{2n}$, $x = -\dfrac{\pi}{4n}$, $x = 0$, $x = \dfrac{\pi}{4n}$ and $x = \dfrac{\pi}{2n}$

Question 4

The graph of function *f* passes through the points $(2,4)$ and $(10,10)$. The area of the shaded region is 28.



The value of $\int_2^{10} f(x)dx$ is

A. 60
B. 62
C. 64
D. 66
E. 68

Question 5

The area of the region bounded by the *x*-axis and the parabola $y = ax^2 - 1$ is given by

A. $\int_{-1}^{1}\left(ax^2 - 1\right)dx$

B. $\int_{-1}^{1}\left(1 - ax^2\right)dx$

C. $\int_{-\frac{1}{a}}^{\frac{1}{a}}\left(ax^2 - 1\right)dx$

D. $\int_{-\frac{1}{\sqrt{a}}}^{\frac{1}{\sqrt{a}}}\left(1 - ax^2\right)dx$

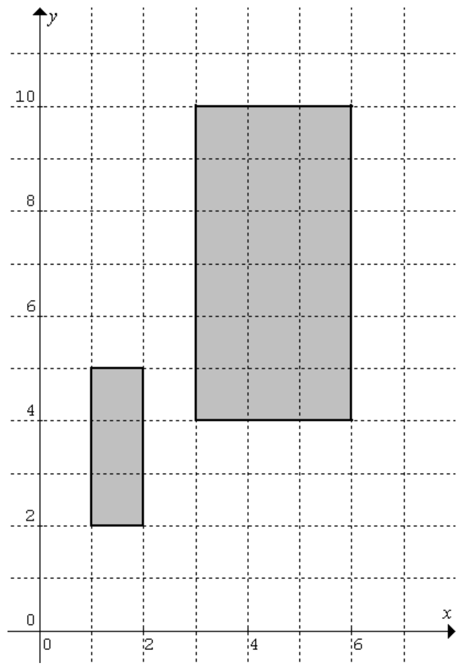
E. $\int_{-\sqrt{a}}^{\sqrt{a}}\left(ax^2 - 1\right)dx$

Question 6

Given $f: D \to R, f(x) = \dfrac{\sqrt{x^2 - a^2} + \sqrt{a^2 - x^2}}{x + a}$ and $a \in R$. D is

A. $\{x : x \neq -a\}$
B. $\{a\}$
C. R
D. $\{x : x < a\}$
E. $\{x : x > -a\}$

Question 7



Transformation T changes the large rectangle to the small rectangle. T is defined by

A. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

B. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

C. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

D. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

E. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

Question 8
Which one of the following transformations is **NOT** required to change $f(x)$ to $g(x)$, where $g(x)=-1+\frac{1}{2}f(3-2x)$?

A. A reflection in the *x*-axis
B. A reflection in the *y*-axis
C. A dilation in the *x* direction
D. A translation parallel to the *x*-axis
E. A translation parallel to the *y*-axis

Question 9
The sum of all possible solutions to the equation $\sin x+\cos 2x=0$ for $x\in[-\pi,\pi]$ is

A. $-\frac{2\pi}{3}$
B. $-\frac{\pi}{2}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$
E. $\frac{2\pi}{3}$

Question 10
Function *f* is defined by $f(x)=c-2(x-a)(x+b)^3$, where $a,b,c\in R$.
Let *n* be the number of *x*-intercepts that is possible for *f*. Which one of the following statements is true?

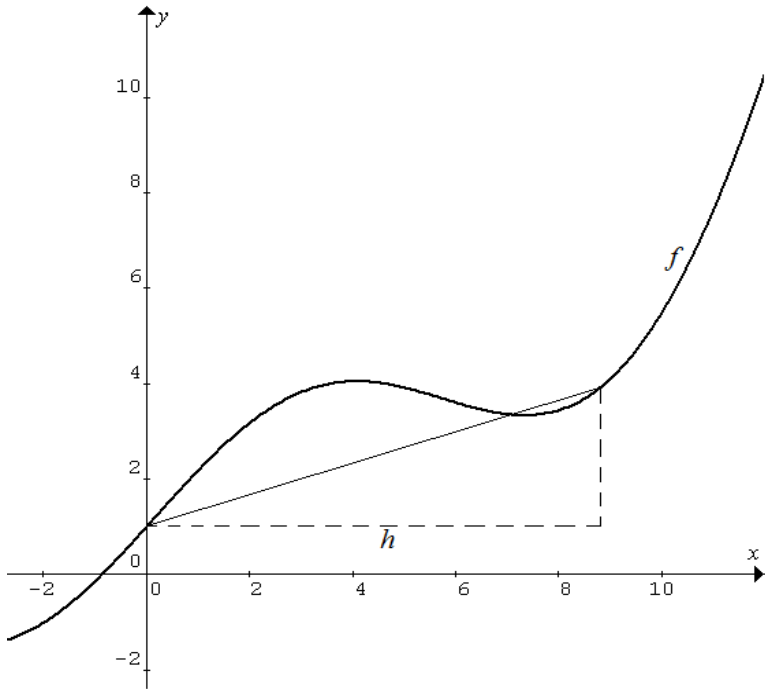
A. $0\le n\le 2$
B. $0<n\le 2$
C. $0\le n\le 3$
D. $0<n\le 3$
E. $0<n\le 4$

Question 11
Which one of the following functional equations is **NOT** satisfied by $f(x)=-x$?

A. $f(kx)=kf(x)$, where *k* is a real constant
B. $f(f(x))=x$
C. $f(x+y)=f(x)+f(y)$
D. $f(x-y)=f(x)-f(y)$
E. $f(xy)=f(x)f(y)$

Question 12

The graph (curve) of function *f* is shown below.



The estimated value of $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ is closest to

A. ∞
B. 3.2
C. 2.2
D. 1.2
E. 0.8

Question 13

Given $a > 0$ and $b > 0$, the intersection of $\{(x, y): y = |ax - b|\}$ and $\{(x, y): y = a|x| - b\}$ is

A. $\left\{\left(\frac{a}{b}, 0\right)\right\}$

B. $\left\{\left(0, -\frac{a}{b}\right)\right\}$

C. $\left\{\left(\frac{b}{a}, 0\right)\right\}$

D. $\left\{(x, y): y = a|x| - b, x < \frac{b}{a}\right\}$

E. $\left\{(x, y): y = ax - b, x \geq \frac{b}{a}\right\}$

Question 14

Given $f'(x)=\dfrac{1}{(1-2x)^2}$ and $\int f(x)dx = 1-2x-\dfrac{1}{4}\log_e(1-2x)$.

$f(x)=$

A. $-\dfrac{1}{1-2x}+2$

B. $\dfrac{1}{1-2x}-2$

C. $-\dfrac{1}{2(1-2x)}+2$

D. $\dfrac{1}{2(1-2x)}-2$

E. $-\dfrac{1}{6(1-2x)^3}+2$

Question 15

Function g is defined by $g(x)=\dfrac{2}{\cos^2(2x)}$. The average value of g in the interval $\left[0,\dfrac{\pi}{8}\right]$ is

A. $\dfrac{16}{\pi}$

B. $\dfrac{8}{\pi}$

C. 2

D. 1

E. $\dfrac{\pi}{16}$

Question 16

Function f is defined by $f(x)=(x-a)^2-b$, $x \in R$.

f^{-1}, the inverse function of f,

A. does not exist

B. is defined by $f^{-1}(x)=a\pm\sqrt{x+b}$, $x\in R$

C. is defined by $f^{-1}(x)=a\pm\sqrt{x+b}$, $x\geq -b$

D. is defined by $f^{-1}(x)=a\pm\sqrt{x-b}$, $x\in R$

E. is defined by $f^{-1}(x)=-a\pm\sqrt{x-b}$, $x\geq b$

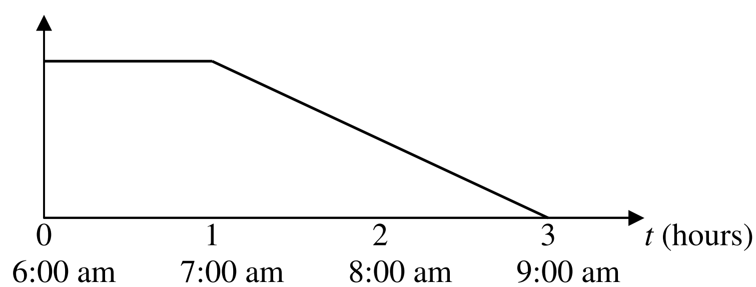
Question 17
A blue marble, a green marble and a red marble are placed in a bag. The marbles are drawn from the bag one at a time at random without replacement.
Given that a red marble may or may not be drawn in the first two draws, the probability that the last draw is a red marble is

A. $\frac{4}{27}$
B. $\frac{1}{6}$
C. $\frac{1}{3}$
D. $\frac{4}{9}$
E. $\frac{5}{9}$

Question 18
A \$1-coin and a \$2-coin are tossed. The probability of *both are heads or both are tails* is 0.5. Which one of the following statements is ***false***?

A. Both coins are unbiased.
B. Both coins are biased.
C. The \$1-coin is unbiased.
D. The \$2-coin is unbiased.
E. At least one of the coins is unbiased.

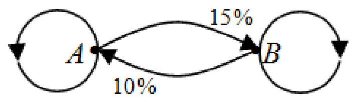
Question 19
A person always leaves home for work between 6:00 am and 9:00 am. The area under the following probability density function gives the probability that the person has left home for work between 6:00 am and 9:00 am.



The probability of *finding the person at home between 8:00 am and 9:00 am* is closest to

A. 0.125
B. 0.333
C. 0.400
D. 0.500
E. 0.667

Question 20
In an isolated country town each household does the weekly shopping at either Centre *A* or Centre *B*. A transition diagram is shown below.



This week 65% of the households shop at Centre *A*. The percentage of the households expected to be shopping at Centre *B* two weeks later is closest to

A. 44%
B. 45%
C. 46%
D. 47%
E. 48%

Question 21
The probability distribution for discrete random variable *X* is given by the probability function
$p:\{0.01, 0.10, 0.30, 0.50, 0.59\} \to R, p(x) = a + 3x - 5x^2$.
$\overline{X}$ is closest to

A. 0.20
B. 0.25
C. 0.30
D. 0.35
E. 0.48

Question 22
Random variable *X* has a normal distribution given by $N(12.3, 1.69)$.
If $\Pr(9.1 \le X < b) = 0.95$, then *b* is closest to

A. 16.2
B. 15.8
C. 14.9
D. 14.5
E. 13.6

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Consider function *f* defined by $f(x) = e^x - 2x$, $x \in R$.

a. ***Use calculus*** to find the exact coordinates of the stationary point(s) of *f*. 2 marks

b. ***Use calculus*** to determine the nature of the stationary point(s). 2 marks

c. ***Hence*** show that $e^x > 2x$ for $x \in R$. 2 marks

d. ***Hence*** show that $e^x \geq x^2 + 1$ for $x \geq 0$. 3 marks

e. Use $(x-1)^2$ to show $x^2 + 1 \geq 2x$ for $x \in R$. 1 mark

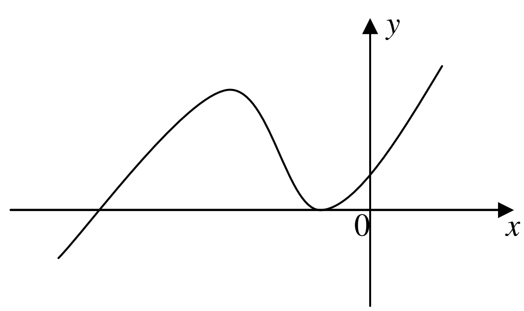
g. ***Use calculus*** to find the derivative of $\log_e\left(x^2+1\right)$ with respect to *x*. 1 mark

h. ***Hence*** find $\int_0^x \frac{2t}{t^2+1}\,dt$. 1 mark

i. ***Use part e and part h*** to show that $e^x \geq x^2+1$ for $x \geq 0$. 3 marks

Question 2

The graph of cubic polynomial *P* defined by $P(x)=x^3+6ax^2+6bx+4c$, where constants $a,b,c \in R$, is shown below. It has a turning point on the *x*-axis.



a. Differentiate $P(x)$ with respect to *x*. 1 mark

b. Show that the turning point on the x-axis is $\left(\dfrac{ab-c}{b-2a^2},0\right)$. 3 marks

Cubic polynomial $Q(x)=x^3+0.4x^2-3.36x-2.88$ has a turning point on the x-axis.

c i. ***Use part b*** to find the exact coordinates of this turning point. 2 marks

c ii ***Hence*** find the exact x-coordinates of the other x-axis intercept and the other turning point.
3 marks

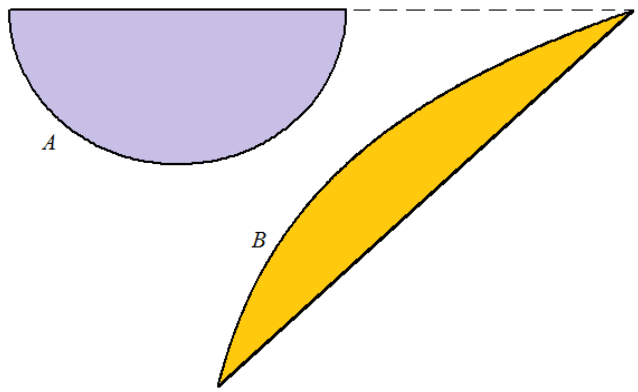
d. Find the area of the region bounded by $Q(x)$ and the x-axis (correct to 2 decimal places). 1 mark

Question 3

The following diagram shows the site plan of two buildings on horizontal ground. (Units are not required)

Curve *A* is a semi-circle. The coordinates of its centre is $(0,1)$ and the length of its diameter is 2.

Curve *B* has the equation $y = a\log_e x + b$, where $a, b \in R$. It passes through the points with coordinates $(1,0)$ and $(e,1)$.



a. Determine the cartesian equation of curve *B*. 1 mark

b. Determine the catesian equation of curve *A*. Express *y* in terms of *x*. 2 marks

c. Accurately draw the *x* and *y* axes on the diagram above. 1 mark

d. Find the exact area of the ground space bounded by Curve *A*, Curve *B*, $y = 0$ and $y = 1$. 3 marks e i. In terms of p determine the equation of the normal to Curve B at $x = p$, where $p \in [0.5, 2]$. 2 marks

ii. Determine the equation of the normal to Curve B, which passes through the centre of the semi-circle.
2 marks

iii. ***Hence, or otherwise,*** find the exact value of the closest separation between the two building.
2 marks

f i. Curve A and Curve B have the same gradient at $y = c$. Find the value of c (correct to 2 decimal places).
3 marks

ii. ***Hence, or otherwise,*** find the magnitude (correct to 2 decimal places) of the translation in the x-direction of either one of the buildings required to make them in contact with each other. 2 marks

Question 4
A manufacturer produces envelopes whose weight X is normally distributed with mean $\mu = 1.950$ grams and standard deviation $\sigma = 0.025$ grams. ***Write your answers to 3 decimal places unless stated otherwise.***

a i. If $\Pr(\mu - w < X < \mu + w) = 0.800$, find the value of w. 1 mark

ii. How many (nearest whole number) out of 1000 envelopes are outside the interval $(\mu - w, \mu + w)$?
1 mark

iii. Among those outside the interval $(\mu - w, \mu + w)$ in part ii, what is the probability that an envelope is within the interval $(\mu - 2\sigma, \mu + 2\sigma)$? 1 mark

The manufacturer also produces writing papers whose weight is also normally distributed with mean $\mu = 1.950$ grams and standard deviation $\sigma = 0.025$ grams. A letter (with no stamps) is overweight and has to pay more for postage if it weighs more than 7.875 grams.
(Hints: $E(X_1 + X_2 + ...) = E(X_1) + E(X_2) + ...$, $\sigma^2 = \sigma_1^{\,2} + \sigma_2^{\,2} + ...$)

b i. Find the mean and standard deviation of a letter consisting of an envelope and 3 writing papers made by the manufacturer.
2 marks

ii. Find the probability that the letter is overweight. 1 mark

iii. If a letter consisting of an envelope and 3 writing papers is sent once a fortnight, what is the probability that higher postage is required in more than 2 occasions in a year (26 fortnights)?
1 mark

iv. If a small sticky note (weighing 0.150 grams) is attached to a letter consisting of an envelope and 3 writing papers, what is the probability that no extra postage is required?

1 mark

Another manufacturer produces envelopes whose weight X has the distribution specified by probability density function f which is defined by

$$f(x)=\begin{cases} ke^{\frac{x}{2}}\sin(x), & 1.500 < x < 2.200 \\ 0, & elsewhere \end{cases}$$

c i. Find the value of k. 1 mark

ii. Find the average weight of the envelopes. 2 marks

iii. Find the median weight of the envelopes. 2 marks

iv. If an envelope is randomly selected among those envelopes whose weight is greater than the average weight, what is the probability that it's weight is also greater than the median weight?

2 marks

End of exam 2