INSIGHT

YEAR 12 *Trial Exam Paper*

2011

MATHEMATICAL METHODS (CAS)

UNIT 3

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	Number of questions to be answered	Number of marks
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.

- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 9 pages, with a separate sheet of miscellaneous formulas.

- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.

- Remove the formula sheet during reading time.

- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Question 1

Let $f(x) = x^2 - 3$ and $g(x) = \cos(x)$. Write down the rule for $(g(f(x)))$.

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<div align="right">1 mark</div>

Question 2

For the function $f : R^+ \to R,\ f(x) = 2e^{3x} - 1$, find

a.　　the rule for the inverse function f^{-1}.

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b.　　the domain of the inverse function f^{-1}.

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<div align="right">2 + 1 = 3 marks</div>

Question 3

a. Let $f(x) = e^{\sin(2x)}$. Find $f'(x)$.

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b. Let $y = x^2 \tan(x)$. Evaluate $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{4}$.

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$1 + 2 = 3$ marks

TURN OVER

Question 4

For the function $f : [-\pi, \pi] \to R,\ f(x) = |2\cos(2x) - 1|,$

a. Sketch the graph of the function f on the set of axes below. Label axes intercepts and endpoints with their coordinates.
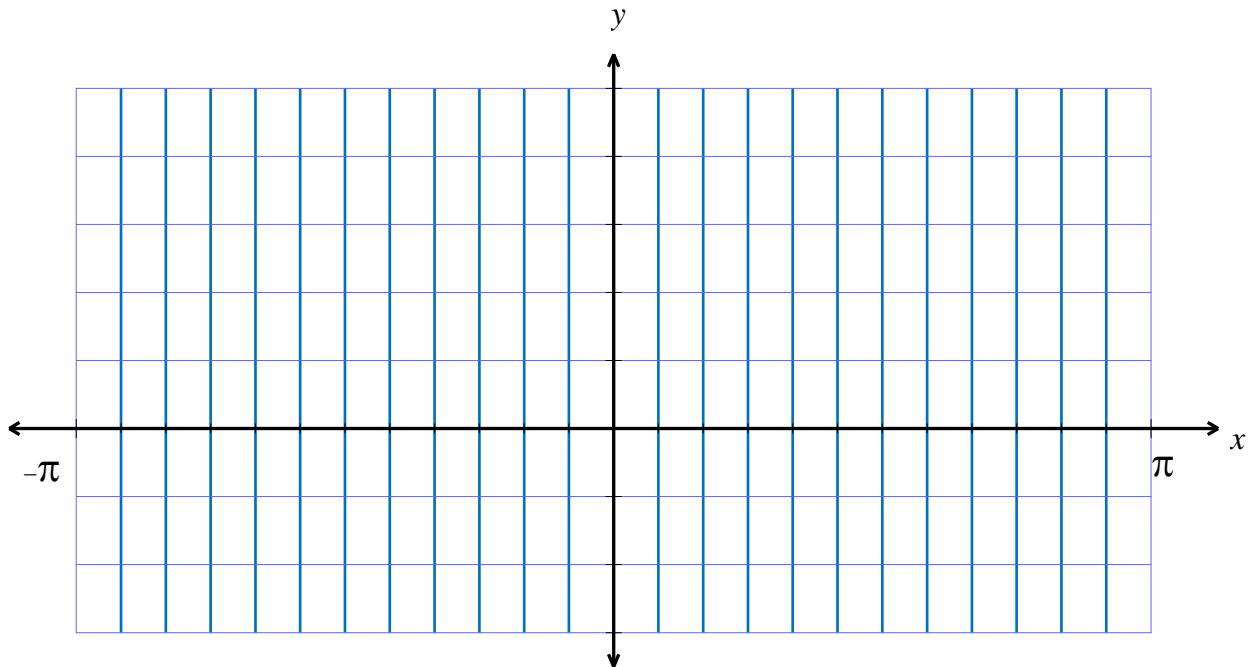
b. State the equation of the tangent to the curve at $x = \dfrac{3\pi}{4}$.

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$3 + 3 = 6$ marks

Question 5

The weights of the adult males of a species of Alaskan huskies are normally distributed, with a mean of 72 kg and a standard deviation of 3 kg. **Use the result that** $\Pr(Z < 1) = 0.84$, **correct to two decimal places**, to find

a.   the probability that a particular Alaskan husky weighs more than 75 kg.

_____

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_____

b.   the probability that an Alaskan husky weighs less than 69 kg if it is known that it weighs less than 72 kg.

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c.   Five Alaskan huskies are used to pull a sled through the snow. Find the probability that exactly three of them weigh more than 72 kg.

_____
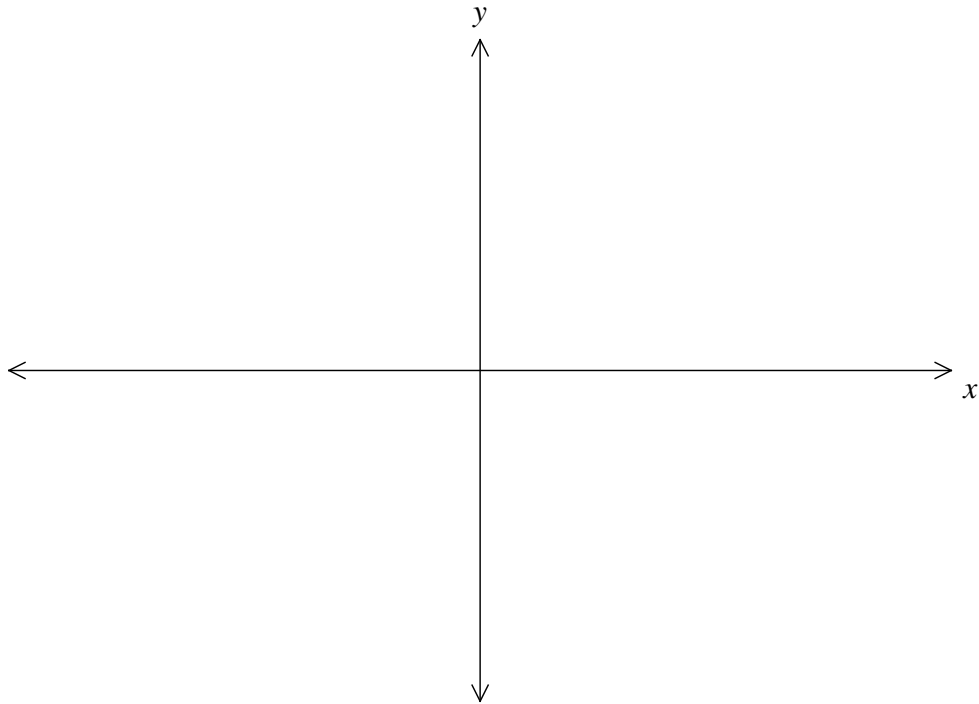
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$1 + 2 + 2 = 5$ marks

TURN OVER

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

a. Sketch the graph of f.

b. Find $\Pr(X < 3)$.

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c. If $\Pr(X \geq a) = \dfrac{1}{e^2}$, find the value of a.

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$1 + 2 + 2 = 5$ marks

Question 7

a. Find the general solution to the equation $\sin(x) = \sqrt{3}\cos(x)$.

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b. Find the average value of the function $y = \sin(2x)$ over the interval $\left[0, \dfrac{\pi}{8}\right]$.

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$2 + 3 = 5$ marks

TURN OVER

Question 8

Suppose that the probability of snow at a particular resort is dependent on whether or not it has snowed on the previous day. If it has snowed the previous day, then the probability of snow is 0.7. If it has not snowed the previous day, then the probability of snow is 0.1.

If it has snowed on a Thursday

a. What is the probability that it doesn't snow again until Sunday?

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b. What is the probability that it will snow in the long term?

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2 + 2 = 4 marks

Question 9

A **normal** to the curve $y = e^{x+1} - 1$ has the equation $y = -\dfrac{x}{e} + a$, where a is a real constant.

Find the value of a.

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4 marks

Question 10

For the function $f(x) = \dfrac{x+1}{x-1}$, show that $f(f(x)) = x$ for $x \in R \setminus \{1\}$.

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2 marks

Question 11

Find the values of m such that the system of linear simultaneous equations

$$mx + 12y = 24$$
$$3x + my = m$$

has a unique solution.

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2 marks

END OF QUESTION AND ANSWER BOOK