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2010 Trial Examination

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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SECTION 1 – Multiple-choice questions

Instructions for Section 1

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

Consider the following system of linear equations:

$2x + 8z = 26$

$4x - 4y - 14z = -38$

$8x - 4y + 2z = 14$

The set of solutions in terms of the parameter p, where $p \in R$, is:

A. $x = 13 + 4p,\ y = \frac{5}{2}(3 - t),\ z = p$

B. $x = 13 - 4p,\ y = \frac{45-15p}{2},\ z = p$

C. $x = 13 - 4p,\ y = \frac{45-15p}{2},\ z = 0$

D. $x = 13 - 4p$

E. $z = p$

Question 2

Which of the following is a true statement for the graph of $y = \frac{-2}{(x-1)^2} + 3$

A. The equation of the horizontal asymptote is $y = -3$.

B. As $x \to \infty,\ y \to 0$.

C. The $y-$intercept has coordinates $(0, 1)$

D. The equation of the vertical asymptote is $x = -1$

E. The $x-$intercept has coordinates $(0.22, 0)$

Question 3

Which of the following is not a true statement about the graph of $y = -(x-1)^2 - k^2$

A. There are no $x-$intercepts.

B. The equation of the axis of symmetry is $x = 1$

C. The turning point has coordinates $(1, -k^2)$

D. The $y-$intercept has coordinates $(0, -1 - k^2)$

E. For all real values of $x,\ y \leq -1 - k^2$

Question 4

Consider the function $f: R \to R,\ f(x) = |2e^x - 2|$. Which of the following statements is true?

A. $f'(x) = \begin{cases} 2e^x & x > 0 \\ -2e^x & x < 0 \end{cases}$

B. $f'(x) = |2e^x + 1|,\ x \in R$

C. $f'(x) = |2e^x|,\ x \in R \backslash \{1\}$

D. $f'(x) = \begin{cases} e^x & x > 0 \\ -e^x & x < 0 \end{cases}$

E. $f'(x) = \begin{cases} -2e^x & x > 0 \\ 2e^x & x < 0 \end{cases}$

Question 5

Grain is poured in a large silo. The volume, V cubic metres, of grain in the silo at time t hours is given by $V = \frac{2}{35}(t^2 - 0.14t + 3.7)$. The average rate of change of volume over the first 15 minutes, in cubic metres per hour, is closest to:

A. 12.8543
B. 0.0629
C. 0.1285
D. 0.0063
E. 6.2860

Question 6

If $\Pr(Z > c) = 0.25$, where Z is a standard normal random variable, then c is closest to:

A. 0.6745
B. -0.6745
C. 0.5987
D. 0.7734
E. -0.7734

TURN OVER

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Question 7

A polynomial function f has derivative f' with the following properties:

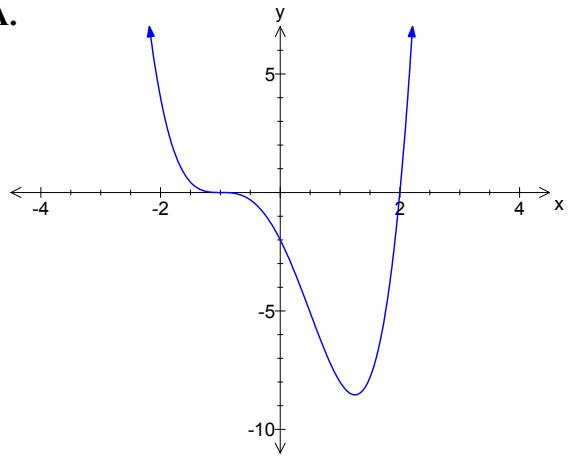
$f'(x) = 0$ for $x = -1$ and $x = 1.25$

$f'(x) > 0$ for $x > 1.25$

$f'(x) < 0$ for all other real values of x

Which of the following could be the graph of $y = f(x)$?

A.



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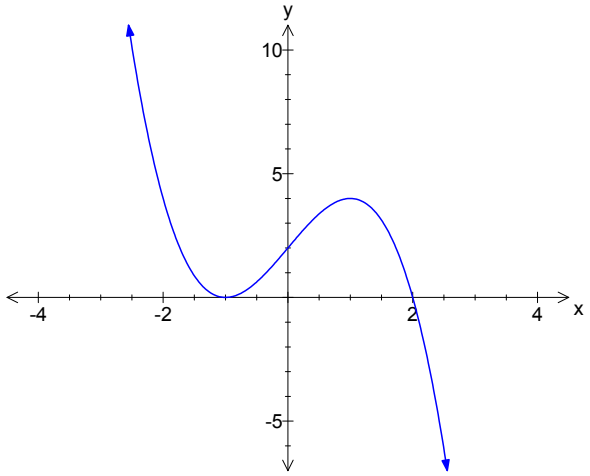
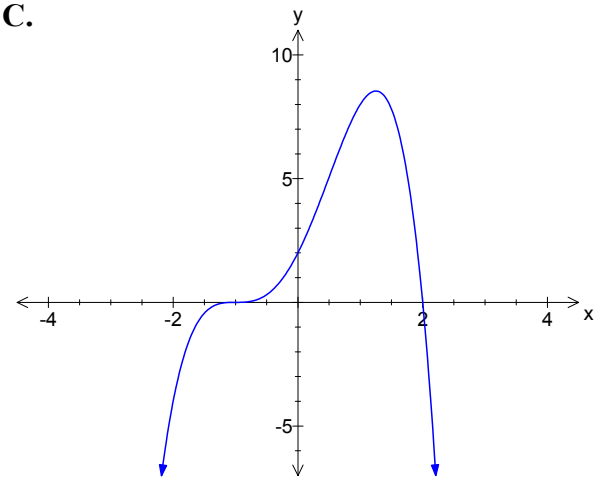
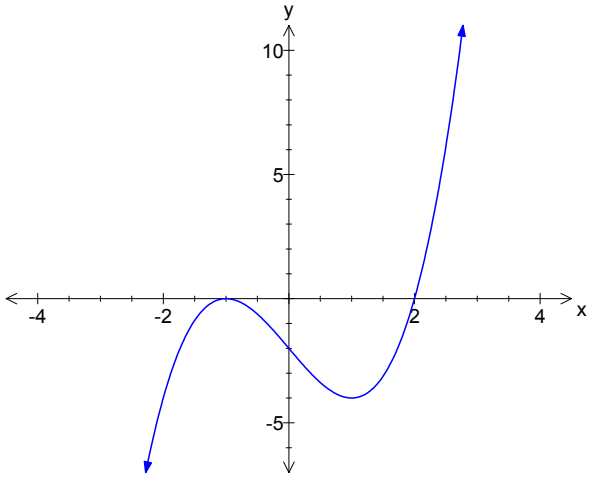


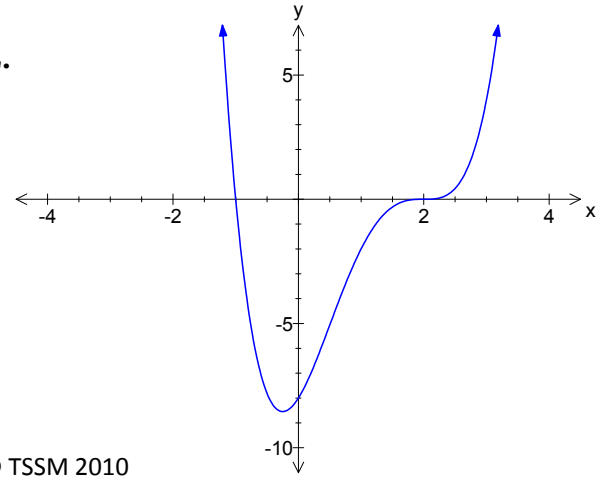
C.



D.



E.



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Question 8

If $f'(x) = (2x - 1)(-x + 4)$, then $f(x)$ could be:

A. $\frac{x(4x^2-27x+24)}{6}$

B. $\frac{2x^2}{3} - \frac{9x}{2} + 4x$

C. $-2x^3 + 9x^2 - 4$

D. $\frac{-2x^2}{3} + \frac{9x}{2} + 4x$

E. $\frac{-2x^3}{3} + \frac{9x^2}{2} - 4x$

Question 9

The following table represents a discrete probability distribution for a random variable, X.

x	1	2	3	4
$\Pr(X = x)$	$2m$	m	$4m$	$3m$

The expected value of X is:

A. $\frac{1}{10}$

B. $\frac{14}{10}$

C. $\frac{14}{5}$

D. 2

E. $\frac{8}{5}$

Question 10

If $Z \sim N(0, 1)$, then $\Pr(Z > 2.416)$ is:

A. 0.9922

B. 0.9241

C. 0.0784

D. 0.9925

E. 0.0078

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Question 11

The graph of a function lies above the $x-$ axis for $0 \leq x \leq 4$ and $\int_0^4 f(x)\,dx = 9$. The graph of f is dilated by a factor of 2 from the $x-$ axis, then translated 4 units down. The resulting function is $g(x)$. Find $\int_0^4 g(x)dx$.

A. 0.5
B. 0
C. 2
D. 2.5
E. 28

Question 12

The gradient of a curve at any point is given by $f'(x) = \sin\left(\frac{\pi x}{6}\right) + d$, where d is a constant. If the curve has a stationary point at $(3,\ 2)$, then its equation is:

A. $f(x) = \frac{6}{\pi}\cos\left(\frac{3\pi}{6}\right) + 5$
B. $f(x) = \frac{-6}{\pi}\cos\left(\frac{\pi x}{6}\right) - x + 5$
C. $f(x) = \frac{-6}{\pi}\sin\left(\frac{3\pi}{6}\right) + x - 5$
D. $f(x) = \frac{-6}{\pi}\cos\left(\frac{\pi x}{6}\right) + x + 5$
E. $f(x) = \frac{6}{\pi}\sin\left(\frac{\pi x}{6}\right) - x$

Question 13

It is found that 3 out of 15 balloons are defective. Thirty balloons are selected at random. The probability that 2 are defective is:

A. 0.0084
B. 0.0007
C. 1
D. 0.0337
E. 0.5

Question 14

If $X \sim N(10, 4)$ and $\Pr(X > x) = 0.47$, then x is closest to:

A. 9.8495
B. -10.1505
C. 10.1505
D. -9.8495
E. 0.5

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Question 15

The life span of light bulbs is normally distributed with an expected value of 36 months and a standard deviation of 3 months. The probability that a light bulb last for less than 27 months is closest to:

A. 0.0135

B. 0.0014

C. 0.9987

D. 0.9865

E. -0.9987

Question 16

Let $f(x) = \sqrt{2x},\ x \geq 0$ and $g(x) = (x+2)(x-1),\ x \in D$. Then the largest domain D such that $f \circ g$ exists is:

A. $R \backslash (-2, 1)$

B. R^+

C. $x > -0.5$

D. $R \backslash [-2, 1]$

E. $x < -0.5$

Question 17

An antiderivative of $3x\sqrt{x}$ with respect to x is:

A. $\frac{6\sqrt[5]{x^2}}{5}$

B. $\frac{6\sqrt{x^5}}{5}$

C. $\frac{6}{5\sqrt{x^5}}$

D. $\frac{5\sqrt{x^5}}{6}$

E. $5x^{\frac{5}{2}}$

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Question 18

If X is a random variable with a probability density function defined by

$$f(x) = \begin{cases} \frac{1}{k}(x^3 - 4) & 2 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

The value of k is:

A. $\frac{4}{561}$

B. 63

C. $\frac{1}{63}$

D. $\frac{561}{4}$

E. 25

Question 19

The left rectangle approximation using rectangles of width 1 to the area of the region enclosed by the curve with equation $y = \log_e(-x + 1)$, the x −axis and the line $x = -4$ is:

A. $10\log_e 12$

B. $2\log_e 2 + \log_e 5$

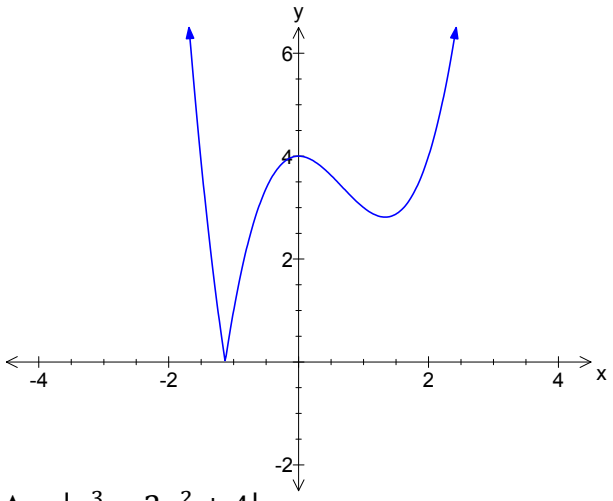
C. $2\log_e 2 + \log_e 3$

D. $\log_e 14$

E. $\log_e 120$

Question 20

The equation of the graph shown below is best describe by



A. $|x^3 - 2x^2 + 4|$

B. $|x^3 - 2x^2| + 4$

C. $x^3 - 2x^2 + 4$

D. $-|x^3 - 2x^2 + 4|$

E. $-x^3 - 2x^2 + 4$

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Question 21

If $y = \frac{1}{3}\log_e(5x + 2) - 1$, then the equation of the inverse is:

A. $\frac{1}{5}(e^{3x+1} - 2)$

B. $\frac{1}{5}(e^{3x+3} + 2)$

C. $\frac{1}{5}(e^{3x+3} - 2)$

D. $\frac{1}{5}(e^{3x-3} - 2)$

E. $5(e^{3x+3} - 2)$

Question 22

The function : $R \to R$, $f(x) = -5\sin\left(\frac{\pi x}{4}\right) + 1$, has period and range respectively:

A. $\frac{\pi}{2}$, $(-4, 6)$

B. $\frac{\pi}{2}$, $[-4, 6]$

C. 8, $[-2, 6]$

D. 2π, $(-2, 6)$

E. 8, $[-4, 6]$

END OF SECTION 1
TURN OVER

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SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

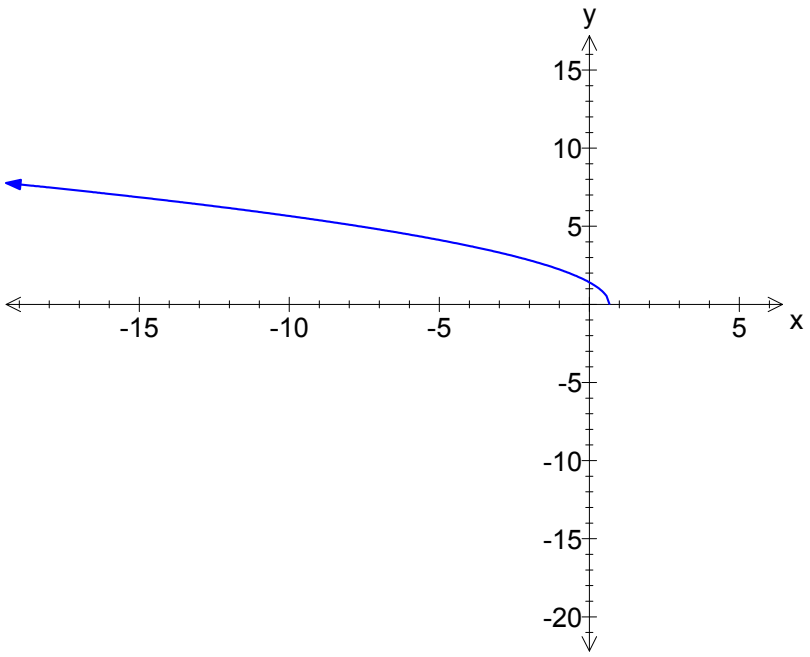
In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The diagram below shows the graph of the function $f: \left(-\infty, \frac{2}{3}\right] \to R, f(x) = \sqrt{2-3x}$.



a. Use calculus to show that the gradient of the tangent to the function $f(x)$ in terms of x, is $\frac{-3}{2\sqrt{2-3x}}$.

2 marks

b. Find the equation, $g(x)$, of the tangent at $x = \frac{119}{192}$. Give exact values.

2 marks

c. Show that $f o g$ does not exist.

2 marks

d. Find the combined function $g o f$.

3 marks

e. Sketch the graph of $g\big(f(x)\big)$ on the axes above, showing all key features.

2 marks

f. Sketch the graph of $h(x) = g\big(f(x)\big) + f(x)$ on the axes above, showing all key features.

2 marks

Total 13 marks

TURN OVER

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Question 2

A manufacturer makes candles with lengths that are normally distributed. It is known that 40% of all the candles are at least 49.62 mm and three-quarters of the candles are less than 51.37 mm.

a. Find the average length and the standard deviation of the candles made, correct to two decimal places.

_____

_____

_____

_____

_____

3 marks

b. The manufacturer would like each candle produced to have length between 47.15 and 49.87 mm. What is the probability, correct to four decimal places, that the length of a randomly selected candle is in this range?

_____

_____

_____

2 marks

The candles are packed and sold in boxes that each holds 25 candles. A candle fits into the box if its length is less than 51.42 mm.

c. What is the probability, correct to four decimal places, that a randomly selected candle produced by the manufacturer fits into a box?

_____

_____

2 marks

d. What is the probability, correct to four decimal places, that the length of a candle which fits into a box is between 47.15 and 49.87 mm.

_____

_____

_____

3 marks

The time in hours that it takes to manufacture a large number of candles on a particular day is a continuous random variable with probability density function given by

$$f(t) = \begin{cases} -0.5t + 2 & 2 \leq t \leq a \\ \\ 0 & \text{elsewhere} \end{cases}$$

e. Find the value of a.

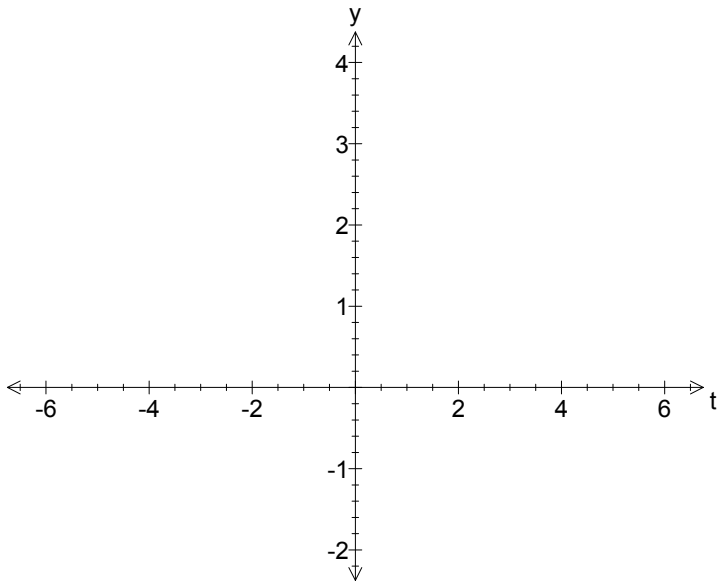
_____

_____

_____

1 mark

f. Sketch the probability density function, and label the maximum with its coordinates.



2 marks

TURN OVER

g. What is the probability, correct to four decimal places, that the manufacturer will take more than 140 minutes to manufacture the large amount of candles?

_____

_____

2 marks

h. What is the average time taken to manufacture the large amount of candles, correct to 4 decimal places?

_____

_____

_____

2 marks

i. What is the probability, correct to four decimal places, that the time to manufacture the candles is within two standard deviations of the mean?

_____

_____

_____

_____

3 marks

Total 20 marks

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Question 3

a. The gradient of a curve at any point is given by $\frac{dh}{dt} = \sin\left(\frac{\pi t}{6}\right)$. The depth of water in a tidal pool can be modelled by h, where h metres is the depth of the water and t is the number of hours after 6 a.m. on a given day. Find an expression for h in terms of t, if there is no water in the pool at 6 a.m.

_____

_____

_____

_____

3 marks

b. It is only safe to swim in the tidal pool when the water level is at the most two metres. Between what hours, over a 24-hour period, is it safe to swim? Give the time to the nearest minute.

_____

_____

_____

_____

2 marks

c. What is the exact average depth of water in the tidal pool over the 24-hour period?

_____

_____

_____

_____

2 marks

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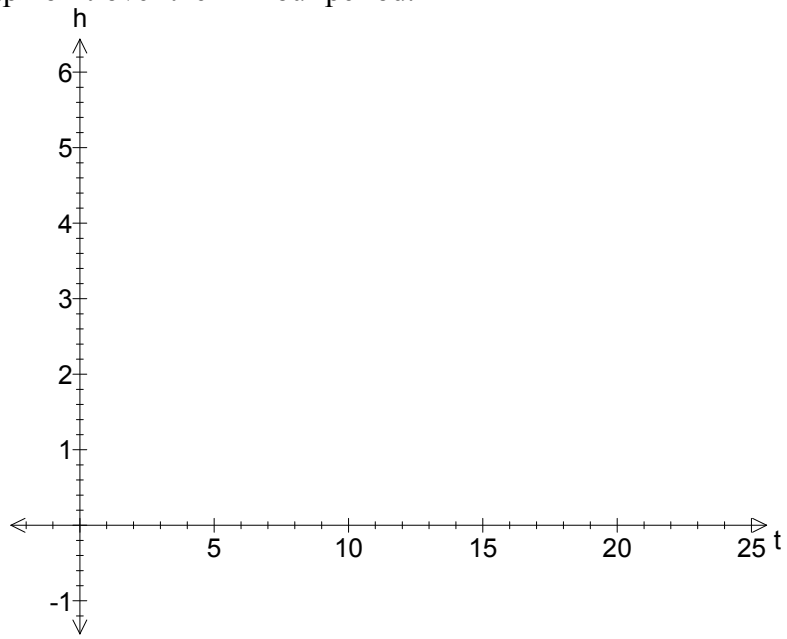
d. Sketch the graph of h and construct a rectangle whose area is the same as the area under the graph of h over the 24-hour period.



3 marks

e. Describe the transformations required to transform the graph of $y = \cos(t)$ into the graph of h.

_____

_____

_____

_____

2 marks

f. Find the general solution to the equation $0 = \frac{-6}{\pi}\cos\left(\frac{\pi t}{6}\right) + \frac{6}{\pi}$.

_____

_____

1 mark

g. Consider the functional equation $k(t + \pi) = k(t)$. Show that the function with rule $k(t) = \cos(2t)$ satisfies this functional equation. Give reasons for your answer.

_____

_____

2 marks

Total 15 marks

Question 4

The total surface area of a rectangular box with a square base and open at the top, is 675 cm^2.

a. Express h in term of x.

_____

_____

_____

_____

2 marks

b. **i**. Find the dimensions of the box so that its volume is a maximum, and hence state the volume of the box.

_____

_____

_____

_____

4 marks

ii. State the domain for this volume, correct to two decimal places.

_____

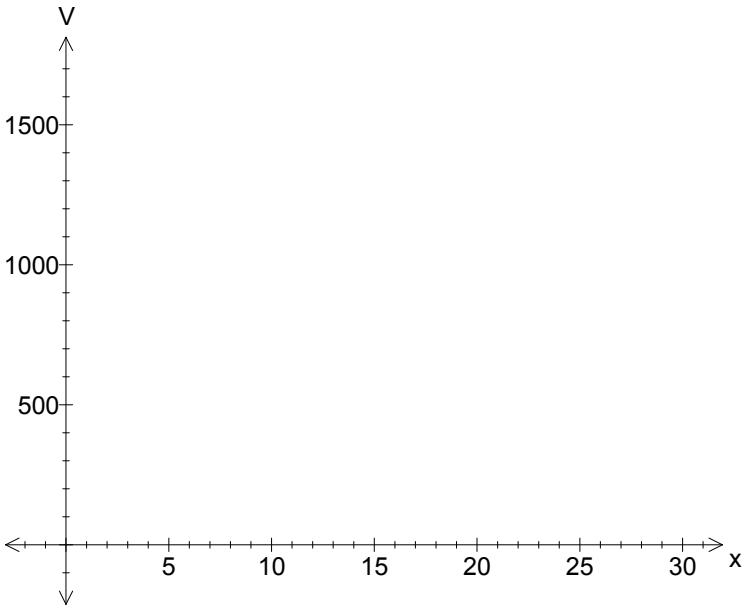
_____

_____

_____

2 marks

c. Sketch the graph of V against x for the domain found in Q4bii. Show key features correct to 2 decimal places.



2 marks

Total 10 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name:_____

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

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