2010 VCAA Math. Methods (CAS) Exam 1 Solutions

Q1a The product rule: $\frac{d}{dx}\left(x^3e^{2x}\right)=x^3\left(2e^{2x}\right)+\left(3x^2\right)e^{2x}$

$=x^2(2x+3)e^{2x}$

Q1b $f(x)=\log_e\left(x^2+1\right)$, $f'(x)=\frac{2x}{x^2+1}$, $f'(2)=\frac{4}{5}$

Q2a An antiderivative is $\int\cos(2x+1)dx=\frac{1}{2}\sin(2x+1)$

Q2b $\int_2^3\frac{1}{1-x}dx=\left[-\log_e|1-x|\right]_2^3=-\log_e(2)=\log_e\left(\frac{1}{2}\right)$, .: $p=\frac{1}{2}$

Q3a $f(x)=\frac{1}{x^2}$, $g(x)=f(f(x))=\frac{1}{\left(\frac{1}{x^2}\right)^2}=x^4$, $x\in R^+$

Q3b $g^{-1}(x)=x^{\frac{1}{4}}$, $x\in R^+$, $g^{-1}(16)=16^{\frac{1}{4}}=\left(2^4\right)^{\frac{1}{4}}=2$

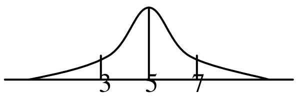
Q4a $amplitude=4$, $period=\frac{2\pi}{\frac{1}{3}}=6\pi$

Q4b $\frac{\sin(x)}{\cos(x)}=\frac{1}{\sqrt{3}}$, $\tan(x)=\frac{1}{\sqrt{3}}$, $x=-\frac{5\pi}{6}$, $\frac{\pi}{6}\in[-\pi,\pi]$

Q5a $\Pr(X>\mu)=0.5$

Q5b $\Pr(X>7)=\Pr(X<3)=\Pr\left(Z<\frac{3-5}{3}\right)=\Pr\left(Z<-\frac{2}{3}\right)$

.: $b=-\frac{2}{3}$



Q6

$\begin{bmatrix}x'\\y'\end{bmatrix}=\begin{bmatrix}3x-1\\2y+4\end{bmatrix}$

.: $x=\frac{x'+1}{3}$ and $y=\frac{y'-4}{2}$

Substitute into $y=2x^2+1$, simplify and remove the ’s,

$y=\frac{4}{9}x^2+\frac{8}{9}x+\frac{58}{9}$

.: $a=\frac{4}{9}$, $b=\frac{8}{9}$, $c=\frac{58}{9}$

Q7a $\int_0^5 ax(5-x)dx=1$, $\int_0^5\left(5ax-ax^2\right)dx=1$, $\left[\frac{5ax^2}{2}-\frac{ax^3}{3}\right]_0^5=1$

$\frac{125a}{2}-\frac{125a}{3}=1$, $a=\frac{6}{125}$

Q7b $\Pr(X<3)=\int_0^3\frac{6}{125}x(5-x)dx$

Q8 $p^2+p^2+\frac{p}{4}+\frac{4p+1}{8}=1$

Simplify: $16p^2+6p-7=0$

Factorise: $(2p-1)(8p+7)=0$ where $p>0$.: $p=\frac{1}{2}$

Q9a $\frac{d}{dx}\left(x^2\log_e(x)\right)=x^2\left(\frac{1}{x}\right)+(2x)\log_e(x)$ where $x>0$

.: $\frac{d}{dx}\left(x^2\log_e(x)\right)=x+2x\log_e(x)$

For use in Q9b, $x\log_e(x)=\frac{1}{2}\left(\frac{d}{dx}\left(x^2\log_e(x)\right)-x\right)$

Q9b $Area=\int_1^3 x\log_e(x)dx$

$=\int_1^3\frac{1}{2}\left(\frac{d}{dx}\left(x^2\log_e(x)\right)-x\right)dx=\frac{1}{2}\left[x^2\log_e(x)-\frac{x^2}{2}\right]_1^3$

$=\frac{9}{2}\log_e(3)-2$.: $a=\frac{9}{2}$, $b=3$, $c=-2$

Q10 $y=x^{\frac{1}{2}}+d$, $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$.

At $(9,c)$, $\frac{dy}{dx}=a$, the gradient of the tangent $y=ax-1$.

.: $\frac{1}{2\sqrt{9}}=a$, $a=\frac{1}{6}$.

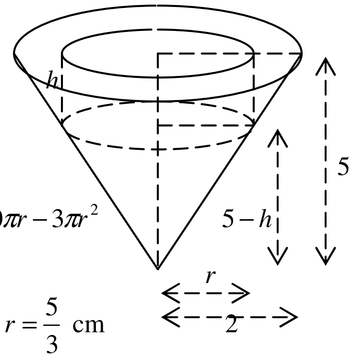
.: The tangent is $y=\frac{1}{6}x-1$, and $(9,c)$ is on the tangent

.: $c=\frac{1}{6}\times 9-1=\frac{1}{2}$

$\left(9,\frac{1}{2}\right)$ is on the curve $y=x^{\frac{1}{2}}+d$, .: $\frac{1}{2}=9^{\frac{1}{2}}+d$, $d=-\frac{5}{2}$

Q11a Similar triangles:

$\frac{5-h}{r}=\frac{5}{2}$.: $h=5\left(1-\frac{r}{2}\right)$



Q11b

$S=2\pi r\times 5\left(1-\frac{r}{2}\right)+2\pi r^2=10\pi r-3\pi r^2$

Q11c $\frac{dS}{dr}=10\pi-6\pi r=0$.: $r=\frac{5}{3}$ cm

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors