Student Name: _____

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

2009 Trial Examination

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*	*Suggested times (minutes)*
1	22	22	22	33
2	5	5	58	87
			Total 80	

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 24 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions for Section 1

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

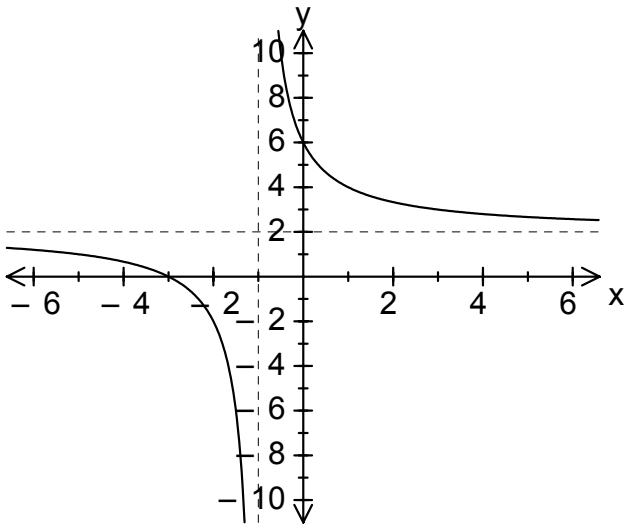
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The graph shown is given by the equation:



A. $y = \frac{1}{x-1} - 2$

B. $y = \frac{2}{x+1} + 2$

C. $y = \frac{2x+6}{x+1}$

D. $y = \frac{1}{x+1} - 2$

E. $y = \frac{x+1}{2x+2}$

Question 2

The derivative of $x\log_e(x^2 - x)$ with respect to x is given by:

A. $x(2x-1)\log_e(x^2 - x)$

B. $\log_e(x^2 - x) + \frac{2x-1}{x-1}$

C. $\frac{2x-1}{x^2 - x}$

D. $\frac{2x-1}{x-x}$

E. $x(x^2 - x) + 2x - 1$

Question 3

The **exact** range of the function $f:(e,5]\to R, f(x)=-2\log_e x$, is:

A. $[-2\log_e(5),-2)$

B. $(-2,-2\log_e(5)]$

C. $[-3.2,-2)$

D. $(-2,-3.2]$

E. $(-2\log_e(5),-2]$

Question 4

The inverse of the function $f:(-\infty,2]\to R, f(x)=(x-2)^2+3$ is:

A. $f^{-1}:[2,\infty)\to R, f^{-1}(x)=2-\sqrt{x-3}$

B. $f^{-1}:[3,\infty)\to R, f^{-1}(x)=2+\sqrt{x-3}$

C. $f^{-1}:[2,\infty)\to R, f^{-1}(x)=\sqrt{x}-1$

D. $f^{-1}:[3,\infty)\to R, f^{-1}(x)=2-\sqrt{x-3}$

E. $f^{-1}:[3,\infty)\to R, f^{-1}(x)=2+\sqrt{x-3}$

Question 5

If the graph of $y=g(x)$ is transformed by a reflection in the y axis, a dilation by a factor of 2 from the y axis and a translation 2 units in the positive direction of the x-axis, the resulting graph would have the equation:

A. $y=g(2-2x)$

B. $y=g\left(1-\frac{x}{2}\right)$

C. $y=-g\left(\frac{x}{2}-1\right)$

D. $y=g\left(2-\frac{x}{2}\right)$

E. $y=-g\left(\frac{x}{2}-2\right)$

TURN OVER

Question 6

The solution(s) of the equation $e^{2x} - e^{x} = 6$ is/are:

A. $\ln(6)$

B. $\ln(3), \ln(2)$

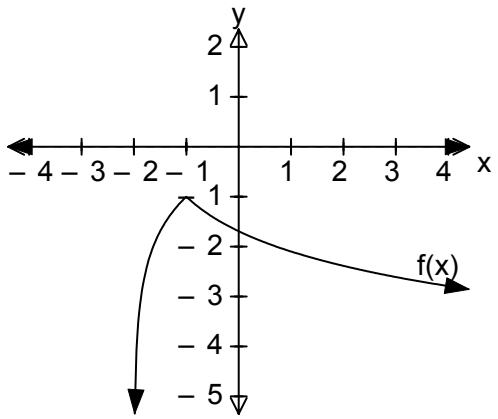
C. $\ln(3)$

D. 1

E. $\ln(3), -\ln(2)$

Question 7

The graph of the function f is shown below:



The equation of the function f is given by:

A. $f(x) = -\left|\log_e(x+1)\right| - 2$

B. $f(x) = \left|\log_e(x-2)\right| + 1$

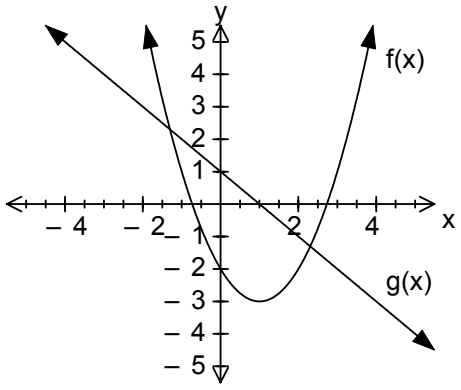
C. $f(x) = -\left|\log_e(x-2)\right| + 1$

D. $f(x) = -\left|\log_e(x+2)\right| - 1$

E. $f(x) = \left|\log_e(x+2)\right| - 1$

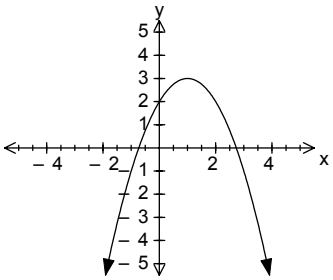
Question 8

The graphs of $f(x) = (x-1)^2 - 3$, and $g(x) = 1 - x$ are shown below

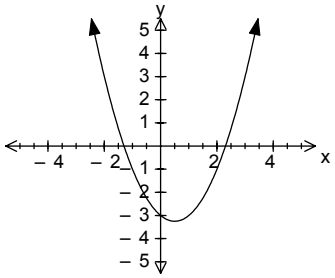


If $h(x) = f(x).g(x)$, then the graph of h is represented by:

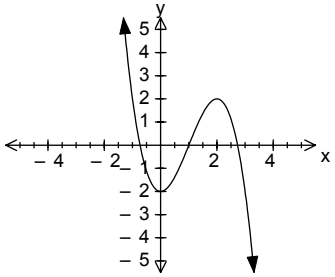
A.



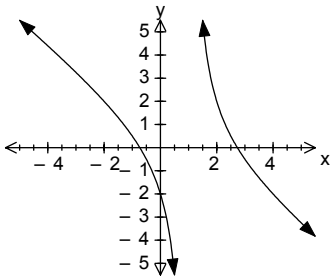
B.



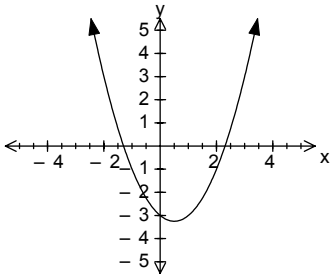
C.



D.



E.



TURN OVER

Question 9

A spherical ice-cube is slowly melting. The rate of change of the volume in cm^3 / cm, when the **diameter** is 2 *cm* is:

A. 16π

B. 8π

C. 4π

D. $\frac{64\pi}{3}$

E. $\frac{4\pi}{3}$

Question 10

If $\int_0^k (4x-5)dx = 3$ then k is:

A. $-\frac{1}{2}$, 3

B. $-\frac{1}{2}$

C. 3

D. 2

E. 2, 3

Question 11

If $\Pr(A|B) = 0.3$, $\Pr(A \cup B) = 0.6$ and $\Pr(B) = 0.4$, then $\Pr(A)$ is:

A. 0.20

B. 0.32

C. 0.82

D. 0.10

E. 0.48

Question 12

The value of E(X) for the following probability distribution is:

x	0	1	2	3
$\Pr(X = x)$	0.25	$2k$	0.45	$4k$

A. 1.00
B. 0.90
C. 2.30
D. 1.85
E. 1.60

Question 13

The chance of Daisy scoring a goal in netball is 0.85. If she has 30 shots at goal during a game the probability (correct to 4 decimal places) that she scores at least 27 goals is:

A. 0.8486
B. 0.6783
C. 0.1514
D. 0.7972
E. 0.3217

Question 14

$f(x)$ is a smooth, continuous curve such that :
$f(2) = 2, f(5) = 7, f(0) = 5, f'(2) = 0, f'(5) = 0, f'(x) > 0$ for $x \in (2,5) \cup (5,\infty)$ and
$f'(x) < 0$ for $x \in (-\infty,2)$. The general form of the equation of *f(x)* is:

A. $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
B. $f(x) = ae^{x-b} + c$
C. $f(x) = ax^3 + bx^2 + cx + d$
D. $f(x) = ax^2 + bx + c$
E. $f(x) = \dfrac{a}{x-b} + c$

TURN OVER

Question 15

If X is normally distributed with a mean $\mu = 1.5$ and a variance $= 0.0625$, then $\Pr(x > 1.6 | X < 1.8)$, correct to 4 decimal places is:

A. 0.0548
B. 0.2594
C. 0.2295
D. 1.0000
E. 0.3894

Question 16

X is a random variable with a probability density function defined by:

$$f(x) = \begin{cases} \frac{1}{2}\sin\left(\frac{x}{2}\right) & \pi \le x \le 2\pi \\ 0 & \textit{elsewhere} \end{cases}$$

The mode is given by:

A. $\frac{5\pi}{4}$
B. 2π
C. π
D. $\frac{3\pi}{2}$
E. $\frac{7\pi}{4}$

Question 17

The x intercepts for $y = 4\sin\left(\frac{x}{2}\right) + 2, x \in [-2\pi, 2\pi]$ are:

A. $\frac{-2\pi}{3}, \frac{-4\pi}{3}$
B. $\frac{-5\pi}{6}, \frac{-\pi}{6}$
C. $\frac{-5\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$
D. $\frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$
E. $\frac{-5\pi}{3}, \frac{-\pi}{3}$

Question 18

If $\int_{-1}^{3}(g(x))dx = -6$ then $\int_{-1}^{3}\left(5 - \frac{g(x)}{2}\right)dx$ is equal to:

A. 23
B. 7
C. 13
D. 8
E. 2

Question 19

If $f(x) = \frac{4}{3-2x}$, then $\int f(x+1)dx$ is equal to:

A. $-2\log_e|1-2x| + c$
B. $8\log_e|1-2x| + c$
C. $-4 + c$
D. $2\log_e|1-2x| + c$
E. $-8\log_e|1-2x| + c$

Question 20

The area bounded by the curve $f(x) = x^2 - 7x + 10, x \in R$ and the x and y axes in square units is :

A. $13\frac{1}{6}$
B. $4\frac{1}{2}$
C. $4\frac{1}{6}$
D. $8\frac{2}{3}$
E. $6\frac{5}{6}$

TURN OVER

Question 21

If Z is a standard normal random variable, and $\Pr(-z < Z < z) = 0.48$, then the value of z, correct to 4 decimal places is:

A. 0.6433
B. 0.0502
C. 0.3156
D. 0.3874
E. 0.7063

Question 22

A function has the rule $p(t) = 26\ln(3t+1)$. The average rate of change between $t = 3$ and $t = 5$ is **exactly** equal to:

A. $-13\ln(1.6)$
B. 6.11
C. 3.00
D. $13\ln\left(\frac{8}{5}\right)$
E. 7.80

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

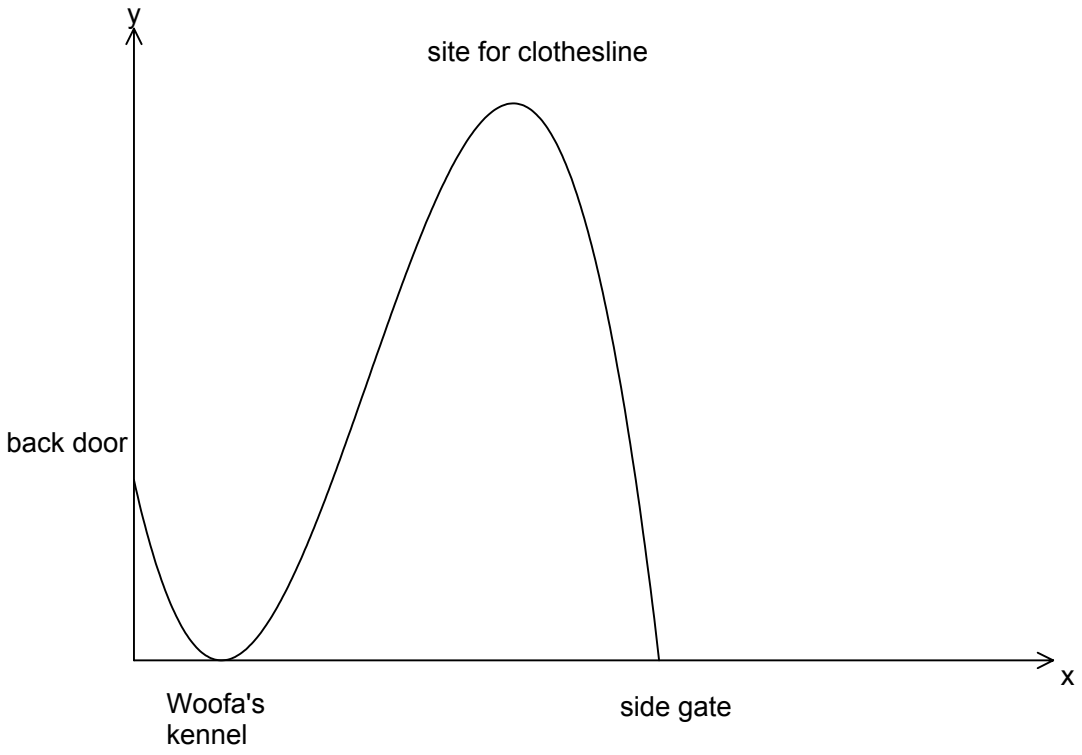
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Tom is landscaping his backyard. He decides to plot it on a grid, to help with the design. His back door is located at (0,3), Woofa's kennel is located at (1,0) and the side gate is located at (6,0).

He wants to construct a path joining the points in the shape of a cubic curve, so that one of the turning points occurs at Woofa's kennel as shown in the diagram.



TURN OVER

a. If the path has the general equation $y = a(x-b)^2(x-c)$, show that $a = \frac{-1}{2}$, $b = 1$, $c = 6$.

_____

_____

_____

2 marks

b. If the clothesline is to be located at the second turning point, use calculus to find the exact co-ordinates of the clothesline.

_____

_____

_____

3 marks

c. Tom marks out a straight line, $f(x)$, between the back door and the point on the path (4,9). Find the equation of this line.

_____

_____

_____

2 marks

d. The area between the line, $f(x)$, and the path, y, will be lawn. Use calculus to find the area of lawn that will be sown.

_____

_____

_____

2 marks

e. Tom is thinking of putting in a fence for a vegetable patch, so that it is a **normal** to the curve at $x = 2$. Find the equation of $g(x)$, representing this fence.

3 marks

f. Will this fence fit in the yard before the side gate? Justify your answer.

1 mark

Question 2

a. If a and b are positive integers such that $a^x = b^{1-x}$, show that $x = \dfrac{\ln(b)}{\ln(a) + \ln(b)}$.

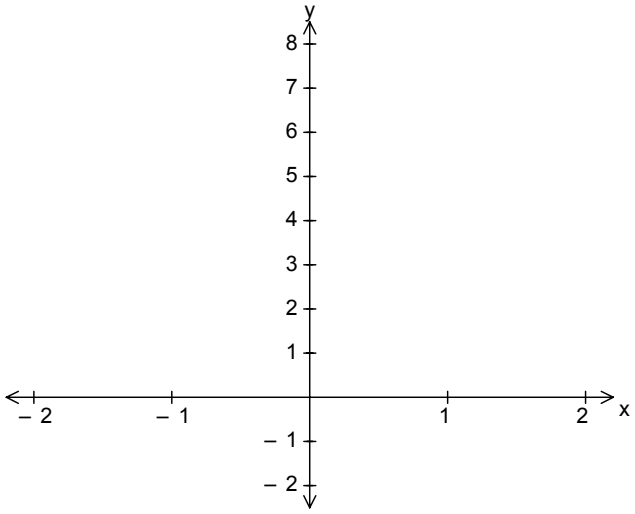
2 marks

b. Hence, show that the exact solution to the equation $9^x = 4^{1-x}$ is $x = \dfrac{\ln(2)}{\ln(6)}$.

2 marks

TURN OVER

c. Sketch $f(x) = 5^x$, and $g(x) = e^{1-x}$, on the same set of axes. Show asymptotes and axes intercepts.



2 marks

d. State the exact value of x at the point where the curves intersect.

_____

_____

2 marks

Question 3

The temperature in a butcher's cool room, t hours after 9*am,* is modelled by the equation:

$$T(t) = -5\cos\left(\frac{\pi}{12}(t+6)\right) + 4\,.$$

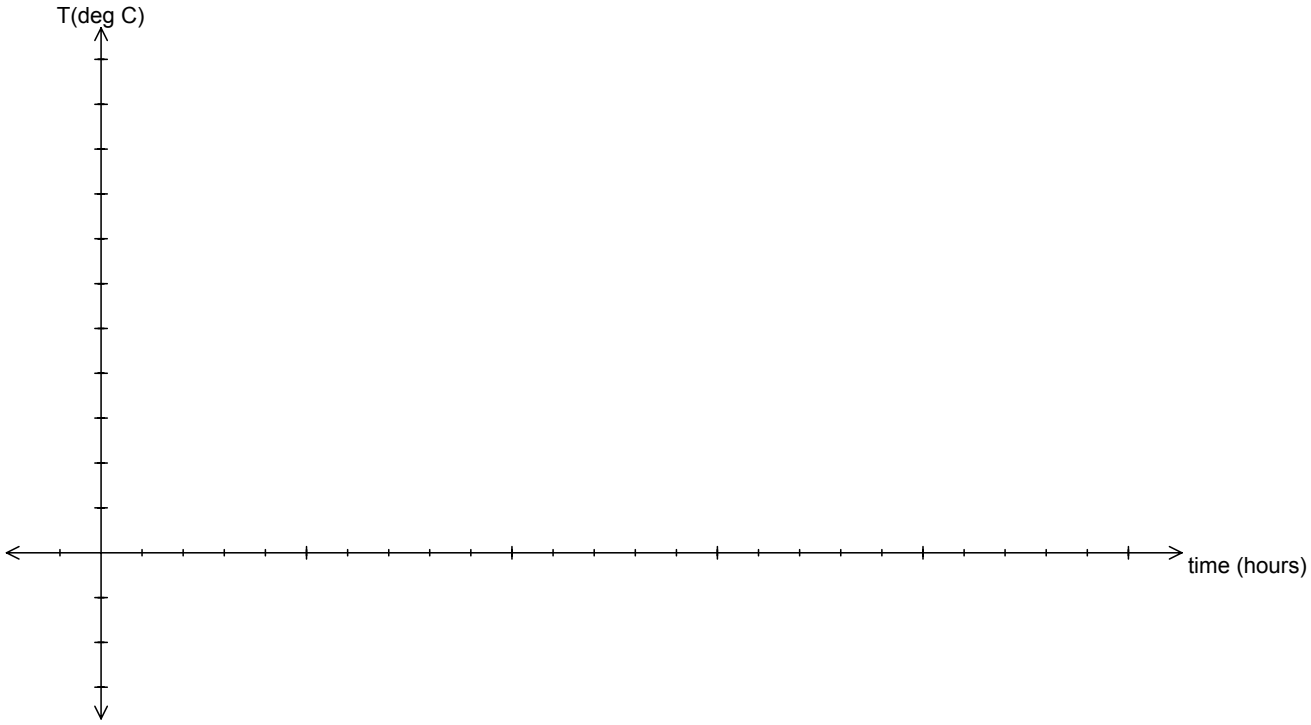
a. Find the temperature range in the cool room.

_____

_____

1 mark

b. Sketch the graph of the temperature in the cool room, over a 24 hour period. Give the axis intercepts (correct to 1 decimal place) and turning points.



3 marks

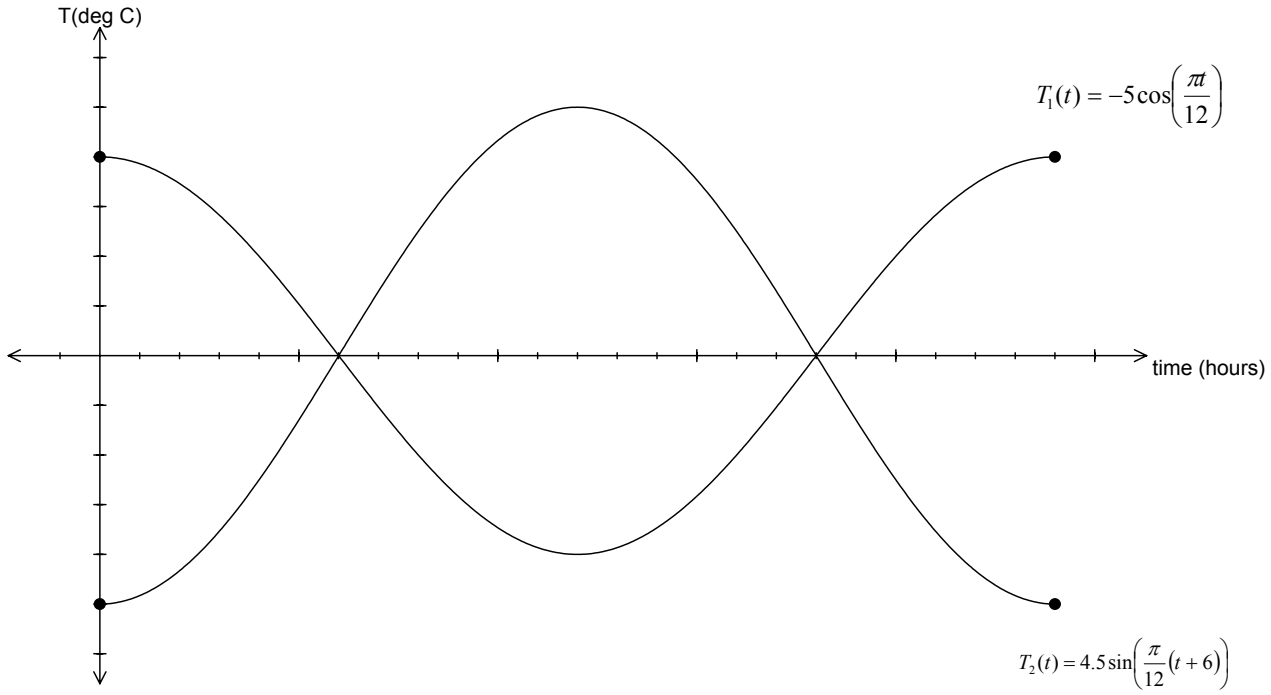
TURN OVER

This tempearature range is too great, so the butcher investigates a new cool room that has two cooling systems in operation. The temperature produced individually by each system is given by the following equations:

$$T_1(t) = -5\cos\left(\frac{\pi\ t}{12}\right) \text{ and } T_2(t) = 4\sin\left(\frac{\pi}{12}(t+6)\right)$$

The sum of these two functions, $T_3 = T_1 + T_2$ gives the overall temperature in the cool room t hours after 9 *am*. The graphs of T_1 and T_2 are shown below.

c. Use addition of ordinates to sketch the new graph, T_3, over a 24 hour period.



1 mark

d. What is the temperature range of the new cool room?

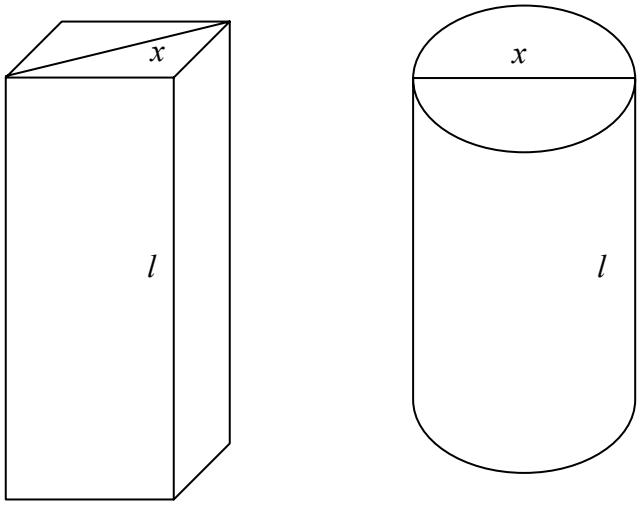
_____

_____

1 mark

Question 4

A company makes small cylindrical rods and small square end rods that have the same maximum width of the cross section, *x cm*. The rods must be the same length, *l*.



a. If a cylindrical rod has a volume of 100 cm^3, show that the length of the rod is given by $l = \frac{400}{\pi x^2}$

_____

_____

_____

2 marks

b. Find the exact volume of a square end rod.

_____

_____

_____

_____

2 marks

TURN OVER

It is known that in the process of manufacturing, 3% of the rods are faulty. After manufacturing, the rods are boxed in cartons of 100.

c. If a carton of rods is randomly selected, find the probability (to 4 decimal places) that no more than 2 rods are faulty in the carton.

_____

_____

_____

2 marks

d. Find the probability (to 4 decimal places) that at least nine out of ten cartons have no more than 2 faulty rods per carton.

_____

_____

_____

2 marks

Their customers order their cartons of rods monthly. The company has found that 65% of their customers order round end rods if their previous order was for round ends, whereas 75% of their customers order square end rods if their previous order was for square ends.

e. If a customer orders round ends in January, find the probability (correct to 4 decimal places) that they will order round ends in April.

_____

_____

_____

_____

_____

_____

3 marks

f. If a customer orders round ends in January, fid the probability (correct to 4 decimal places) that they will order square ends on at least 2 of their next 3 orders.

_____

_____

_____

_____

2 marks

The cost of producing the rods $\$C$ is a continuous random variable with a probability function given by $C(x) = \begin{cases} \frac{6}{5}\left(x^2 - x\right) & 1 \le x \le 2 \\ 0 & \textit{elsewhere} \end{cases}$

g.

i. Find the median cost of production.

_____

_____

_____

_____

3 marks

ii. Find the probability (correct to 4 decimal places) that the cost of production of the rods will be less than \$1.60.

_____

_____

_____

2 marks

TURN OVER

The weights W *g* of the rods are normally distributed with a mean of 108 *g*. It is known that 11% of the rods are under 107 *g*.

h.

i. Find the standard deviation, correct to 2 decimal places.

_____

_____

_____

_____

_____

3 marks

ii. Find the interval between which approximately 95% of the weights of the rods would lie. Give values correct to 2 decimal places.

_____

_____

1 mark

Question 5

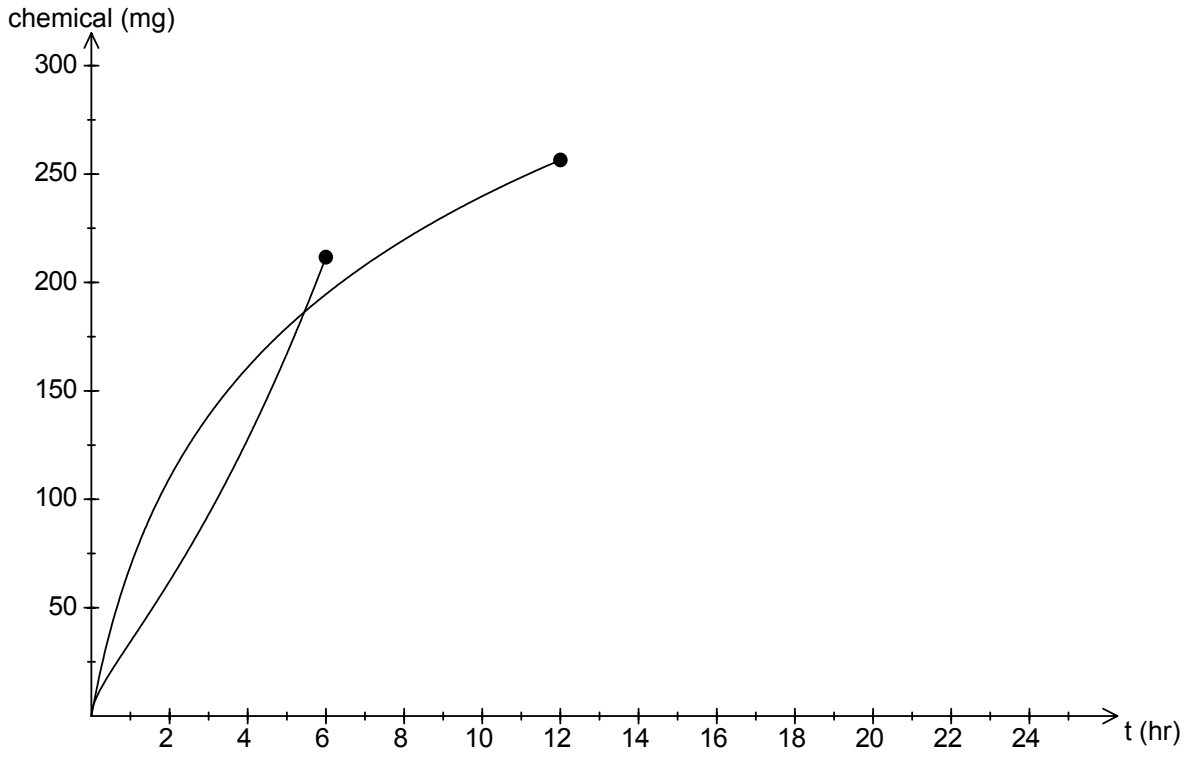
Two new drugs have been developed that help with pain relief for terminally ill patients. The tablets must be given together and they provide 24 hour relief to the patients. Tablet A is absorbed into the blood stream over twelve hours and it is cleared over the next twelve hours. Tablet B is absorbed into the blood stream over six hours and is cleared over the next six hours. This can be modelled by the following equations:

Tablet A $\begin{cases} A_1 = 100\ln(t+1) & 0 \le t \le 12 \\ A_2 = 100(25-t) & 12 < t \le 24 \end{cases}$ and Tablet B $\begin{cases} B_1 = 20(e^{\sqrt{t}} - 1) & 0 \le t \le 6 \\ B_2 = 20(e^{\sqrt{12-t}} - 1) & 6 < t \le 24 \end{cases}$

where A_1 A_2 B_1 B_2 represent the amount of active chemical in *mg* in the bloodstream after *t* hours.

The graphs of A_1 and B_1 are shown below.

a. On the same set of axes, sketch the graphs of A_2 and B_2.



1 mark

TURN OVER

b.

i. Describe the transformations that have changed equation A_1 to equation A_2.

_____

_____

1 mark

ii. Describe the transformations that have changed equation B_1 to equation B_2.

_____

_____

1 mark

c.

i. What is the maximum amount of tablet A absorbed into the bloodstream, correct to 2 decimal places?

_____

_____

1 mark

ii. What is the maximum amount of tablet B absorbed into the bloodstream, correct to 2 decimal places?

_____

_____

1 mark

Maximum relief for the patients is reached when the amount of tablet A and sublet B are added together, giving the greatest concentration of drugs in the bloodstream.

d. What is the greatest concentration of the two drugs in the bloodstream?

_____

_____

1 mark

e. How long after taking the tablets initially, until the maximum relief is achieved? Give your answer to the nearest minute.

_____

_____

1 mark

A period of very good relief is achieved when the level A is greater than 125 *mg* **and** the level of B is greater than 125*mg* at the same time.

f. How long does this period of very good relief last for, to the nearest minute?

_____

_____

2 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name:_____

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E