MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

2009 Trial Examination

SOLUTIONS

Question 1

a. $f'(x) = \frac{-2(3\sin(2x))(\sin(2x)) - 3\cos(2x)(2\cos(2x))}{\sin^2(2x)}$

$= \frac{-6(\sin^2(2x) + \cos^2(2x))}{\sin^2(2x)}$

$= \frac{-6}{\sin^2(2x)}$

M1 + A1
2 marks

b. $\therefore \int \frac{-6}{\sin^2(2x)}\,dx = \frac{3\cos(2x)}{\sin(2x)} + c$

$-3\int \frac{2}{\sin^2(2x)}\,dx = \frac{3\cos(2x)}{\sin(2x)} + c$

$\int \frac{2}{\sin^2(2x)}\,dx = \frac{-\cos(2x)}{\sin(2x)} + c$

$\therefore \int \left(\frac{2}{\sin^2(2x)} + 1\right)dx = -\frac{\cos(2x)}{\sin(2x)} + x + c$

M1 + A1
2 marks

Question 2

a. Let $x = 0$

$y = -3\log_e(2) - 1$

$= \log_e(2^{-3}) - 1$

$= \log_e\left(\frac{1}{8}\right) - \log_e(e)$

$= \log_e\left(\frac{1}{8e}\right)$

M1 + A1
2 marks

b. Let $x = -3\log_e(y+2) - 1$

$x + 1 = -3\log_e(y+2)$

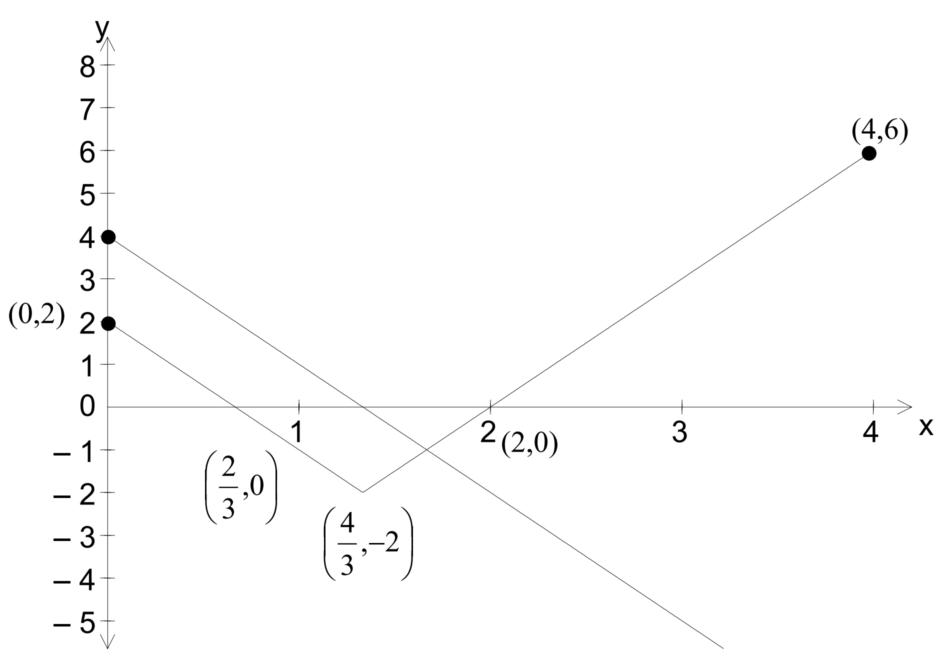
$y = e^{-\left(\frac{x+1}{3}\right)} - 2$

$\therefore f^{-1}(x) = e^{-\left(\frac{x+1}{3}\right)} - 2$

M1 + A1
2 marks

Question 3

a.



A1 + M1
2 marks

b. $h'(x)$ has domain $(0,4) \setminus \left\{\frac{4}{3}\right\}$ or $\left(0,\frac{4}{3}\right) \cup \left(\frac{4}{3},4\right)$

A1
1 mark

Question 4

a.

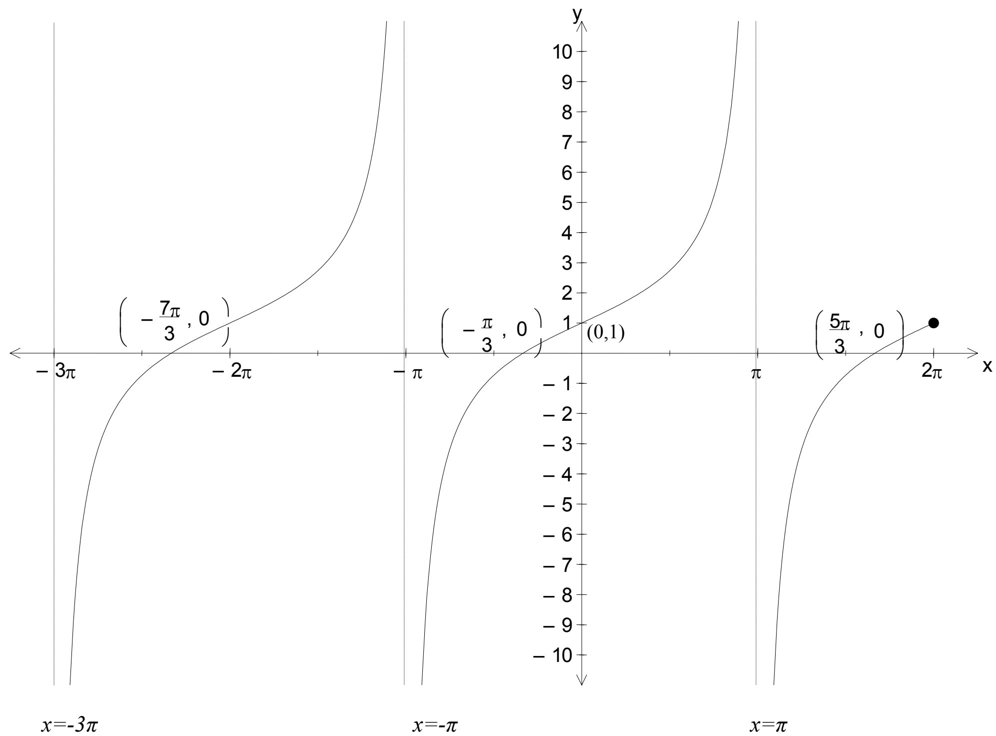
$$\tan\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{3}}, x \in \left[-\frac{3\pi}{2}, \pi\right]$$

$$\frac{x}{2} = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = -\frac{7\pi}{3}, -\frac{\pi}{3}, \frac{5\pi}{3}$$

M1 + A1
2 marks

b.



A3
3 marks

Question 5

a. $\int_{-1}^{1}\left(g(x)-f(x)\right)dx =$

$$=\int_{-1}^{1}\left((-x^3-x^2-x+1)-(x^2-1)\right)dx$$

$$=\int_{-1}^{1}\left(-x^3-2x^2+x+2\right)dx$$

$$=\left[\frac{-x^4}{4}-\frac{2x^3}{3}+\frac{x^2}{2}+2x\right]_{-1}^{1}$$

$$=\frac{-1}{4}-\frac{2}{3}+\frac{1}{2}+2-\left[\frac{-1}{4}+\frac{2}{3}+\frac{1}{2}-2\right]$$

$$=4-\frac{4}{3}$$

$$=2\frac{2}{3}\text{ units}^2$$

M2 + A1
3 marks

Question 6

$$f(x)=x^{\frac{-1}{2}},\ f'(x)=-\frac{1}{2}x^{\frac{-3}{2}}$$

$$f(4.1)\approx f(4)+0.1\times f'(4)$$

$$\approx\frac{1}{\sqrt{4}}+0.1\left(-\frac{1}{2}\right)(4)^{-\frac{3}{2}}$$

$$\approx\frac{1}{2}-\frac{1}{10}\times\frac{1}{2}\times\frac{1}{8}$$

$$\approx\frac{1}{2}-\frac{1}{160}$$

$$\approx\frac{79}{160}$$

M2 + A1
3 marks

Question 7

a. Median divides the data into halves. Therefore, the median is 2

A1
1 mark

b. $\Pr(X=5 \mid x>2) = \dfrac{\Pr(X=5)}{\Pr(X>2)}$

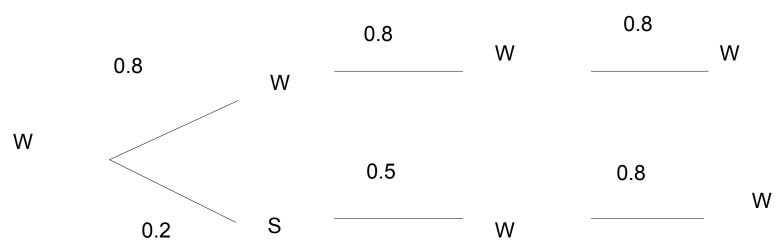
$= \dfrac{0.15}{0.4}$

$= \dfrac{3}{8}$

A1
1 mark

Question 8

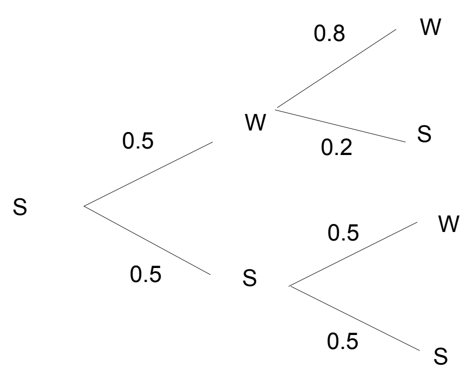
a.



$\Pr(\text{windy Sat and Sun} \setminus \text{windy Thurs}) = 0.512 + 0.080$
$= 0.592$

M1 + A1
2 marks

b.



$\Pr(\text{windy on at least Sat or Sun} \setminus \text{still on Fri}) = 0.5\times0.8 + 0.5\times0.2 + 0.5\times0.5$
$= 0.750$

M1 + A1
2 marks

Question 9

a. $1 = \int_1^2 \left(\frac{x}{2}\right) dx + \int_2^4 (k) dx$

$= \left[\frac{x^2}{4}\right]_1^2 + [kx]_2^4$

$= \frac{3}{4} + 2k$

$\therefore k = \frac{1}{8}$

M1+A1
2 marks

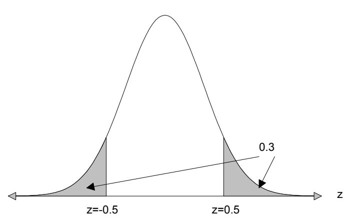
b. $\Pr(X \le 3) = \int_1^2 \left(\frac{x}{2}\right) dx + \int_2^3 \left(\frac{1}{8}\right) dx$

$= \frac{6+3-2}{8} = \frac{7}{8}$

A1
1 mark

Question 10

a. $0.5 = \frac{x-175}{10} \therefore x = 180$ this is 5 above the mean so $\Pr(X > 170) = 0.7$



M1+A1
2 marks

b. $\Pr(X > 180 \mid X > 175) = \frac{\Pr(X > 180)}{0.5} = \frac{0.3}{0.5} = \frac{3}{5}$

A1
1 mark

Question 11

a. $SA = \pi r^2 + 2\pi rh$

$200\pi = \pi(r^2 + 2rh)$

$\frac{200 - r^2}{2r} = h$

$\therefore h = \frac{100}{r} - \frac{r}{2}$

M1 + A1
2 marks

b. $V = \pi r^2 h$

$= \pi r^2 \left(\frac{200 - r^2}{2r}\right)$

$= 100\pi r - \frac{\pi r^3}{2}$

M1 + A1
2 marks

c. $\frac{dV}{dr} = 100\pi - \frac{3}{2}\pi r^2$

$0 = 100\pi - \frac{3}{2}\pi r^2$

$r^2 = \frac{200}{3} \Rightarrow r = \sqrt{\frac{200}{3}}$, rationalise

$r = \frac{10\sqrt{6}}{3} cm$

M1 + A1
2 marks