MATHEMATICAL METHODS (CAS)

Unit 2 – Written examination 1

2009 Trial Examination

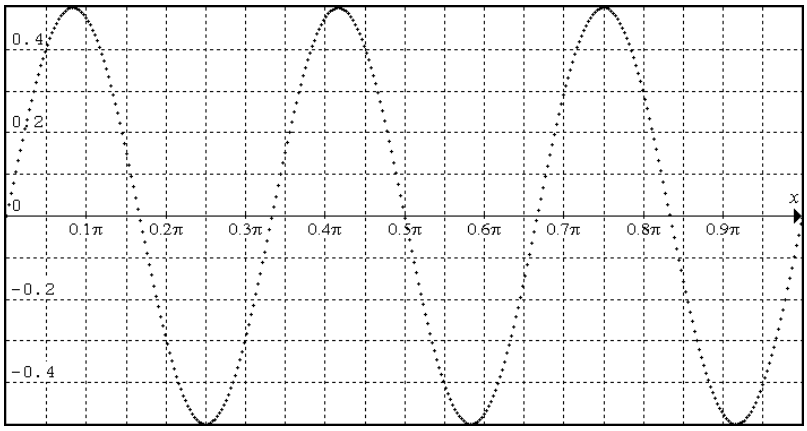
SOLUTIONS

Question 1

a. Amplitude = $\frac{1}{2}$, period: $p = \frac{2\pi}{6} = \frac{\pi}{3}$

A1
1 mark

b. Must show correct scale, shape and intercepts



A2
2 marks

c. range $[-0.5, 0.5]$

A1
1 mark

Question 2

a. $\cos(x)$ is negative in third quadrant. The value of $\cos(x)$ can be found using the right angled triangle or identities

$\sin^2(x)+\cos^2(x)=1$

$\cos^2(x)=1-\left(-\frac{2}{3}\right)^2$

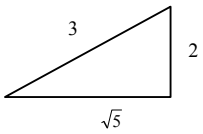
$\cos^2(x)=1-\frac{4}{9}$

$\therefore \cos(x)=\pm\frac{\sqrt{5}}{3}$

$\therefore \cos(x)=-\frac{\sqrt{5}}{3}\in\left[\pi,\frac{3\pi}{2}\right]$

or use Pythagoras Theorem: $b=\sqrt{9-4}$

$b=\sqrt{5}, and$



$\cos(x)=\frac{adj}{hyp}$

M1 + A1
2 marks

b.

$\sin(x)=-\frac{2}{3}$

$\cos(x)=-\frac{\sqrt{5}}{3}$

$\tan(x)=\frac{\sin(x)}{\cos(x)}=-\frac{2}{3}\times-\frac{3}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

M1 + A1
2 marks

Question 3

a. $p=\frac{\pi}{3}$

A1
1 mark

b. asymptotes $x=\pm\frac{\pi}{6}, x=\pm\frac{\pi}{2}, x=\pm\frac{5\pi}{6}$

A2
2 marks

Question 4

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

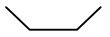
$$f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 7x - 7h + 1 - 2x^2 + 7x - 1}{h}$$

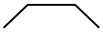
$$f'(x) = \lim_{h \to 0} \frac{h(4x + 2h - 7)}{h}$$

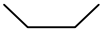
$$f'(x) = 4x - 7$$

M2 + A1
3 marks

Question 5

a. Stationary point at $x = -5$, gradient  local minimum

Stationary point at $x = -1$, gradient  local maximum

Stationary point at $x = 3$, gradient  local minimum

A2
2 marks

Question 6

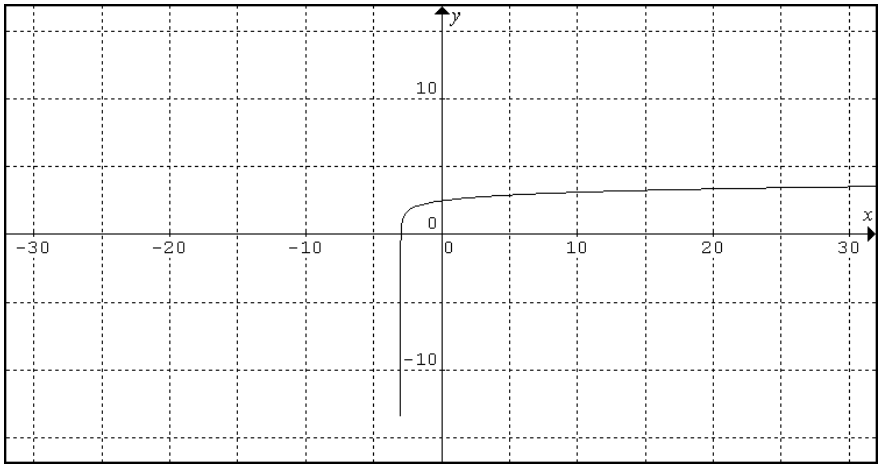
a. y – intercept, let $x = 0$, $y = 2 + \log_3 3 = 3$
(0, 3)
x – intercept, let $y = 0$, $0 = 2 + \log_3(x+3)$

$$x = 3^{-2} - 3 = -\frac{26}{9}$$

$$x = -2\frac{8}{9}$$

M2 + A1
3 marks

b. graph should show intercepts and asymptotes: VA $x = -3$



A2
2 marks

Question 7

a.

$$f'(x) = 4x^3 + 6x^2 - x + 9$$

$$f(x) = x^4 + 2x^3 - \frac{x^2}{2} + 9x + c$$

M1 + A1
2 marks

b. $y' = \dfrac{3x^3 + 5x^2 - 7x}{3x}$

$$y' = x^2 + \frac{5}{3}x - \frac{7}{3}$$

$$y = \frac{1}{3}x^3 + \frac{5}{6}x^2 - \frac{7}{3}x + c$$

M1 + A1
2 marks

Question 8

$$f(x) = \frac{1}{x^3} + \sqrt[3]{x^2} - 5x^{\frac{1}{2}}$$

$$f(x) = x^{-3} + x^{\frac{2}{3}} - 5x^{\frac{1}{2}}$$

$$f'(x) = -3x^{-4} + \frac{2}{3}x^{-\frac{1}{3}} - 5 \times \frac{1}{2}x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{3}{x^4} + \frac{2}{3\sqrt[3]{x}} - \frac{5}{2\sqrt{x}}$$

M1 + A1
2 marks

Question 9

$f(x) = x^3 + 3x^2 - x + 5$

$f'(x) = 3x^2 + 6x - 1$

$3x^2 + 6x - 1 = 8$

$3x^2 + 6x - 9 = 0$

$3\left(x^2 + 2x - 3\right) = 0$

$3(x+3)(x-1) = 0$

$x = -3, \quad x = 1$

$\therefore co-ords..(-3,8),\ (1,8)$

$eq1 : y_1 = 8x + 32$

$.....y_2 = 8x$

M3 + A1
4 marks

Question 10

$s = 100t - 5t^2$

$\frac{ds}{dt} = 100 - 10t$

$t = 3$

$\frac{ds}{dt_{t=3}} = 100 - 30 = 70m/s$

M1 + A1
2 marks

Question 11

a. $\begin{bmatrix} 0.2 & 0.3 \\ 0.8 & 0.7 \end{bmatrix}$

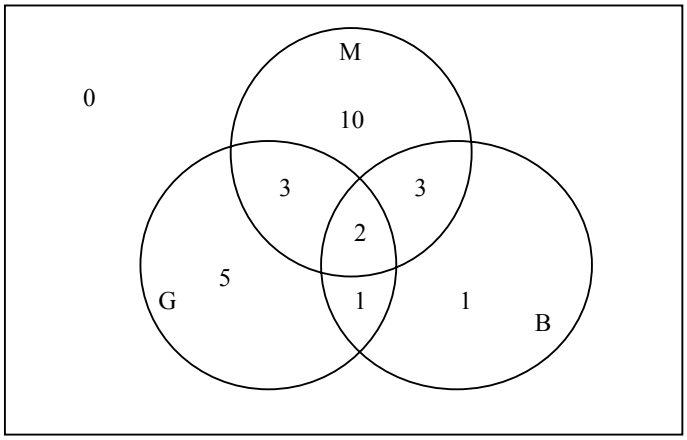
A1

b. $0.8 \times 0.3 = 0.24$
$0.7 \times 0.3 = 0.21$
$0.24 + 0.21 = 0.43$

M1 + A1
3 marks

Question 12

a.



M1 + A1
2 marks

b. Pr (B | G) = $\frac{\Pr(B \cap G)}{\Pr G} = \frac{3}{11}$

A1
1 mark

c. Pr (M and B only) = $\frac{1}{25}$

A1
1 mark