MATHEMATICAL METHODS (CAS)

Unit 1 – Written examination 2

2009 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: E

Explanation:

$$-\frac{15000}{3}$$
$$= -5000 L/h$$

Question 2

Answer: C

Explanation:

$$m_n = -\frac{1}{3}$$
$$y - 5 = -\frac{1}{3}(x + 6)$$
$$y = -\frac{1}{3}x + 3$$

Question 3

Answer: A

Explanation:

R \ the interval $(-3,7]$.

Question 4

Answer: B

Explanation:

Local maxima occurs on the x-axis when $y = 0, x = -\frac{3}{2}$ and $x = 1$

Question 5

Answer: E

Explanation:

Point of inflection at (3,1) and y intercept at (0, 2)

Question 6

Answer: D

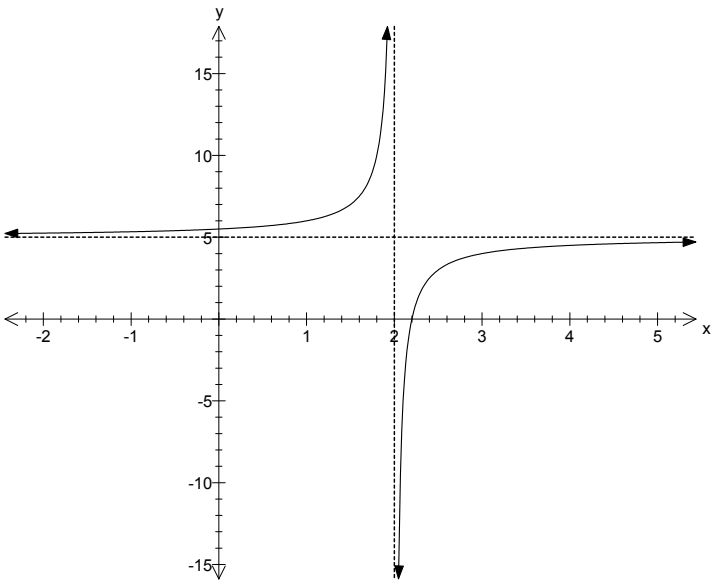
Explanation:

horizontal line cuts once, vertical line crosses twice or more.

Question 7

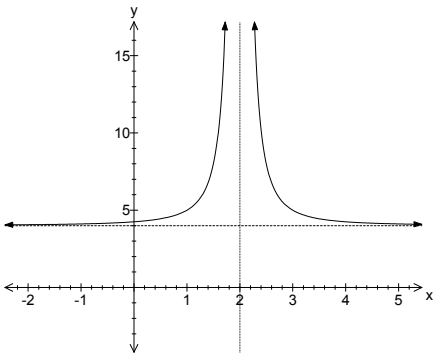
Answer: B

Explanation: from graph



Question 8

Answer: C
, y > 4



Question 9

Answer: A

Explanation:

$\Delta = 25 - 4 \times 2 \times 1 = 17 > 0$

Therefore 2 real solutions exist.

Question 10

Answer: D

Explanation:

$for..y, x = 0 \therefore y = 2$
$for..x, y = 0 \therefore x = 4, -7$

Question 11

Answer: E

Explanation:

if $P(a) = 0$ then $(x - a)$ is a factor therefore, $(x + 3)$ is the factor of $P(x)$

Question 12

Answer: E

Explanation:

gradient is positive **above** the x axis $x < -10$ or $x > -1$

Question 13

Answer: B

Explanation:

$$\frac{3\left(a^{-2}\right)^{-1}b}{\left(5ab^2\right)^3} \div \frac{9\left(a^2b^{-1}\right)^3}{25b^2}$$
$$= \frac{3a^2b}{125a^3b^6} \times \frac{25b^2}{9a^6b^{-3}}$$
$$= \frac{a^2b^3}{15a^9b^3}$$
$$= \frac{1}{15a^7}$$

Question 14

Answer: C

Explanation:

$$5^{-(2x-1)} = 5^3$$
$$-(2x-1) = 3$$
$$-2x+1 = 3$$
$$x = -1$$

Question 15

Answer: D

Explanation:

$$\frac{\log_2 7^2}{\log_2 7}$$
$$= \frac{2\log_2 7}{\log_2 7}$$
$$= 2$$

Question 16

Answer: B

Explanation:

$2 \times 2^{2x} - 9 \times 2^{x} + 4 = 0$

Let $y = 2^{x}$

$\Rightarrow 2y^{2} - 9y + 4 = 0$

$\Rightarrow (2y - 1)(y - 4) = 0$

$\Rightarrow y = \frac{1}{2}$ or $y = 4$

$2^{x} = 2^{-1}, \quad 2^{x} = 2^{2}$

$x = -1 \qquad x = 2$

Question 17

Answer: D

Explanation:

$(x - k)^{2} + (y - h)^{2} = r^{2}$

$k = -2, h = 1, r = \sqrt{16} = 4$

$(x + 2)^{2} + (y - 1)^{2} = 16$

Question 18

Answer: A

Explanation:

Start: $x = 0$, $y = 0$ and after 4 hours: $x = 4$, $y = 128$

$m = \frac{128 - 0}{4 - 0}$ therefore average rate = 32 km/h

Question 19

Answer: D

Explanation:

$(x^2-4)(9x^2-24x+16)$
$=9x^4-24x^3+16x^2-36x^2+96x-64$
$=9x^4-24x^3-20x^2+96x-64$

Question 20

Answer: E

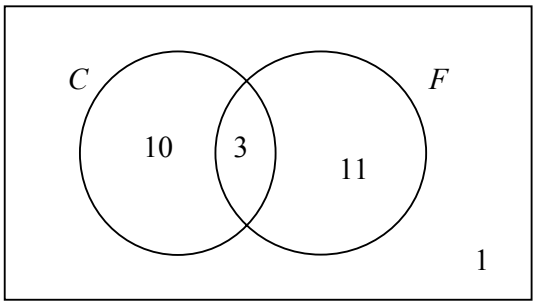
Explanation:

Find Pr $(A \cap B)$ first: Pr $(A \cap B)$ = Pr (A) × Pr $(B \mid A)$ = 0.2 × 0.35 = 0.07
Pr (B) = Pr $(A \cup B)$ – Pr(A) + Pr $(A \cap B)$ = 0.56 – 0.2 + 0.07 = 0.43

Question 21

Answer: A

Explanation:



Question 22

Answer: A

Explanation:

	R	*R'*	
P	21	27	48
P'	18	34	52
	39	61	100

SECTION 2 - Analysis Questions

Question 1

a. $f(x) = x^2 + 6x + 9 - 9 - 7 = (x+3)^2 - 16$

M1 + A1

2 marks

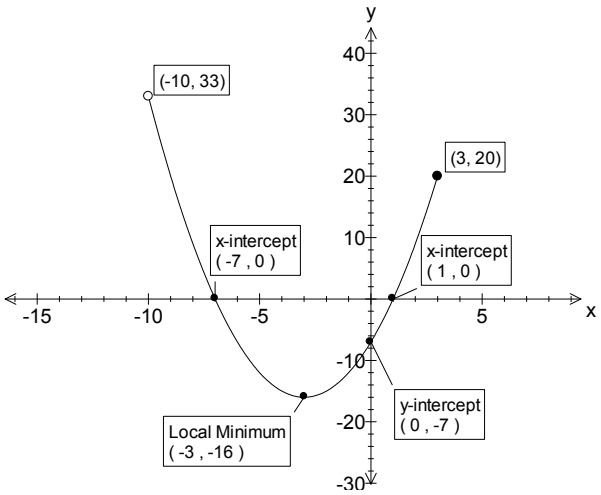
b. let $x = 0$: $y = -7$

let $y = 0$: $x = -7$, $x = 1$ A2

2 marks

c. Show turning point, intercepts and endpoints

Endpoints are (3,20) and (-10, 33), x – intercepts (-7, 0), (1, 0) and y –intercept (0, -7) A3

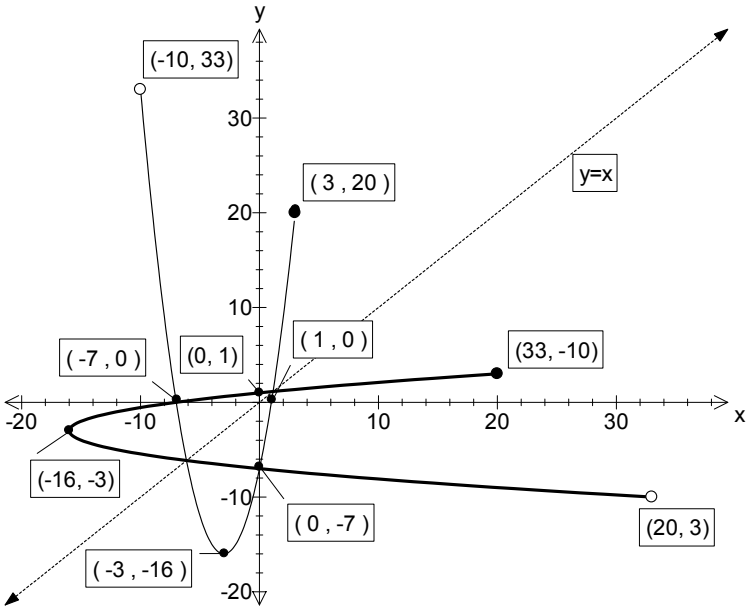


3 marks

d. range [-16,33) A1

1 mark

e. Sketch of inverse should show: Reflected TP, intercepts and endpoints



A2

2 marks

f. the largest domain $(-10,-3]$ makes it a one to one function

A1
1 mark

g. $f(x) = (x+3-3)^2 - 16 - 2 = x^2 - 18$

M1 + A1
2 marks
Total 13 marks

Question 2

a.

$s(1) = -3+18-36+21 = 0$

$$(x-1)\overline{)-3x^3+18x^2-36x+21}$$

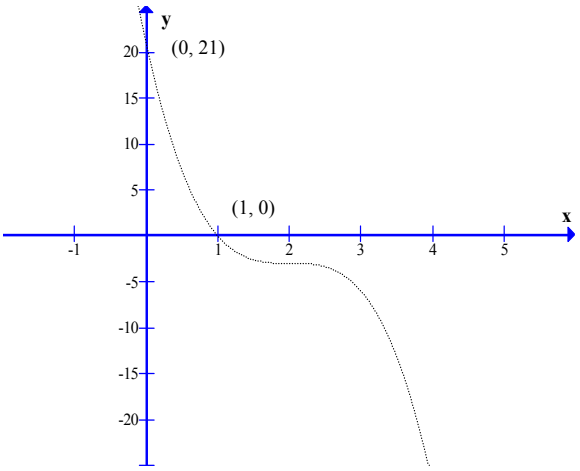
$$\begin{array}{l} \\ -3x^2+15x-21 \\ \hline -3x^3+18x^2-36x+21 \\ -3x^3+3x^2 \\ \hline 15x^2-36x \\ 15x^2-15x \\ \hline -21x+21 \\ -21x+21 \\ \hline 0 \end{array}$$

$-3x^2+15x-21 = -3(x^2-5x+7)$

$\therefore s(x) = -3(x-1)(x^2-5x+7)$

M3 + A1
4 marks

b. Show correct intercepts and shape
y –intercept (0, 21)
x –intercept (1,0)



A2
2 marks

c.

$d = \sqrt{(-2-0)^2 + (0-21)^2}$

$d = \sqrt{4+441} = \sqrt{445}$ must include correct rounding

$d = 21.10m$

M1 + A1
2 marks

d.

$midpo\text{int} = \left(\frac{0-2}{2}, \frac{21-0}{2}\right) = \left(-1, 10\frac{1}{2}\right)$

M1 + A1
2 marks

e.

$m = 0$

$y = 10\frac{1}{2}$

M1 + A1
2 marks

f.

Centre of the circle = (1+7+5,0) = (13,0)
Eqn of circle:

$(x-k)^2 + (y-h)^2 = r^2$

$k = 13, h = 0, r = \sqrt{25} = 5$

$\therefore (x-13)^2 + y^2 = 25$

M2 + A1
3 marks
Total 15 marks

Question 3

a.

i. 0 s < t < 20 s
ii. 40 s < t < 50 s
iii. 60 s < t < 70 s

A3
3 marks

b.

i. Bjiorn is stationary 50 m from the starting point about to go back to the initial position.
ii. He is moving away from the starting point moving with a constant velocity.

A2 + A1
3 marks

c.

i. $\frac{55-40}{30-20}=1.5m/\sec$

ii. $\frac{0-55}{70-52}=-3.06m/\sec$

A2

2 marks

d. $\frac{0-65}{70-60}=-6.5m/\sec$

A1

1 mark

Total 9 marks

Question 4

a.

$5^{2x}-20(5^x)-125=0$ Let $a=5^x \Rightarrow a^2-20a-125=0$

$(a+5)(a-25)=0$

$a=-5, a=25$

$5^x=-5$ (impossible) *or* $5^x=5^2$

$\therefore x=2$

M3 + A1

4 marks

b. Use CAS or:

$\log_{10} 3^{2x+1}=\log_{10} 12$

$(2x+1)\log_{10} 3=\log_{10} 12$

$2x-1=\frac{\log_{10} 12}{\log_{10} 3}$

$2x-1=2.2619$

$\therefore x=1.6309$

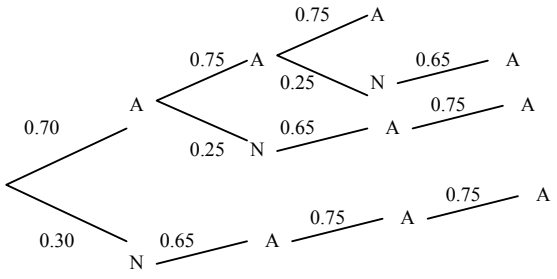
M3 + A1

4 marks

Total 8 marks

Question 5

a. must realise that some of the branches are not needed



4 correct branches with correct values

A4
4 marks

b. $\Pr(AA) = 0.7 \times 0.75$
$= 0.525$

M1 + A1
2 marks

c. $\Pr(AAN) + \Pr(ANA) + \Pr(NAA) = 0.7 \times 0.75 \times 0.25 + 0.7 \times 0.25 \times 0.65 + 0.30 \times 0.65 \times 0.75 = 0.39$

M1+A1
2 marks

d.

	P	***P'***	
GM	0.55	0.25	0.8
GM'	0.1	0.1	0.2
	0.65	0.35	1

A2
2 marks

e. $\Pr(P \cap GM') = 0.1$

A1
1 mark

f. $\Pr(\text{no As} \mid \text{no As in GM}) = \dfrac{\Pr(GM' \cap P')}{\Pr(GM')}$
$= \dfrac{0.10}{0.20}$
$= 0.5$

M1 + A1
2 marks
Total 13 marks