THE SCHOOL FOR EXCELLENCE (TSFX)
UNIT 4 MATHEMATICAL METHODS 2009

WRITTEN EXAMINATION 2

Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

This examination has two sections: Section 1 (multiple-choice questions) and Section 2 (extended-answer questions).

You must complete both parts in the time allocated. When you have completed one part continue immediately to the other part.

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory **DOES NOT** need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.

Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

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Please ensure that the paper size on your printer is selected as **A4** and that you select “**None**” under “Page Scaling”.

SECTION 1 – MULTIPLE CHOICE QUESTIONS

Instructions for Section 1

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers. You should attempt every question.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

The graph of a function has a horizontal asymptote with equation $y = a$ and a vertical asymptote with equation $x = -b$, where a and b are constants. The equation of the function could be

A. $y = \dfrac{4}{x-b} + a$

B. $y = \dfrac{4}{b-x} - a$

C. $y = \dfrac{1}{x+b} - a$

D. $y = \dfrac{1}{x+b} + a$

E. $y = \dfrac{1}{x+a} - b$

QUESTION 2

The quadratic function $f(x) = 3x^2 + 18x + 1$ will have an inverse function for the domain

A. $(-\infty, 3]$

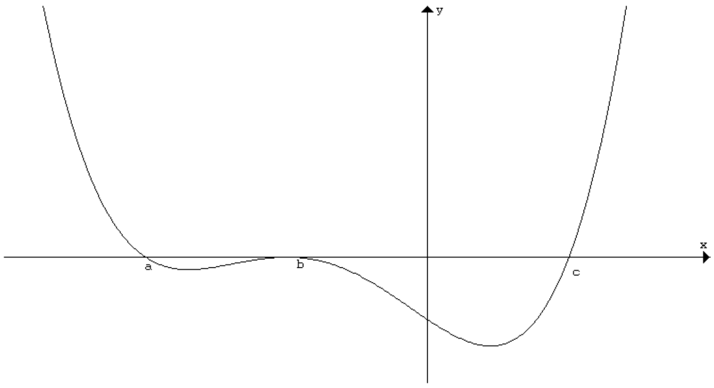
B. $(-\infty, 0]$

C. $(-\infty, -4]$

D. $[-26, 3]$

E. $[-4, \infty)$

QUESTION 3



The most likely equation for the graph shown above is

A. $y=(x+a)(x+b)^2(x-c)$

B. $y=-(x+a)(x+b)^2(x-c)$

C. $y=(x-a)(x-b)^2(x+c)$

D. $y=(x-a)(x-b)^2(x-c)$

E. $y=-(x-a)(x-b)^2(x-c)$

QUESTION 4

Consider the two relations $h: R/\{-1\} \to R$ where $h(x)=\dfrac{1}{|x+1|}+1$ and

$g:(1,\infty) \to R$ where $g(x)=\log_e(x-1)$

The composite function $g\left(h(x)\right)$ is defined as

A. $-\log_e\left(|x+1|\right)$ for $x \in R$

B. $-\log_e\left(|x+1|\right)$ for $(1,\infty)$

C. $\log_e\left(|x+1|\right)$ for $x \in R^+$

D. $\log_e\left(\dfrac{1}{x+1}\right)$ for $x \in R/\{-1\}$

E. $-\log_e\left(|x+1|\right)$ for $x \in R/\{-1\}$

QUESTION 5

If $a^{2x} - 5a^{x} + 4 = 0$ where a is a positive real constant, then

A. $x = 0$ only

B. $x = 0$ and $x = 1$

C. $x = 1$ and $x = 4$

D. $x = 0$ and $x = 4$

E. $x = 0$ and $x = \log_a 4$

QUESTION 6

The maximal domain for which the expression $\log_e(x^2) - \log_e(1 - x)$ is defined is

A. R^+

B. $R/\{0\}$

C. $(-\infty, 0) \cup (0, 1)$

D. $(-\infty, 0] \cup (0, 1]$

E. $(0, 1)$

QUESTION 7

$3^{4\log_3(x-1)+2}$ may be simplified to

A. $9(x-1)^4$

B. $6(x-1)^4$

C. $2 + (x-1)^4$

D. $9 + (x-1)^4$

E. $4x - 2$

QUESTION 8

The largest set of real values of a for which $\left|a^2 - 4a\right| \geq 4$ is

A. $a \geq 2 + 2\sqrt{2}$ and $a \leq 2 - 2\sqrt{2}$ only

B. $a = 2$ only

C. $a \geq 2 + 2\sqrt{2}$, $a = 2$ and $a \leq 2 - 2\sqrt{2}$

D. $a \geq 4$ and $a \leq 0$

E. $a \geq -2 + 2\sqrt{2}$ and $a \leq -2 - 2\sqrt{2}$ only

QUESTION 9

The equation $2\sin^2(\theta) = 3 - 3\cos(\theta)$ for $-\pi \leq \theta \leq \pi$ has solution(s)

A. $\frac{\pi}{3}$ only

B. $-\frac{\pi}{3}$, $\frac{\pi}{3}$ only

C. $-\frac{\pi}{3}$, 0, $\frac{\pi}{3}$

D. $-\pi$, 0, π

E. $-\frac{\pi}{6}$, 0 , $\frac{\pi}{6}$

QUESTION 10

The function $f(x) = a\sin(x) - b\sqrt{3}\cos(x)$ will have a minimum turning point at $x = \frac{\pi}{3}$ if

A. $a = 3b$ and $a < 0$

B. $a = 3b$ and $a > 0$

C. $a = -3b$ and $a > 0$

D. $a = -3b$ and $a < 0$

E. $a = -b$ and $a < 0$

QUESTION 11

If $y = \frac{\log_e(2x)}{x}$ then $\frac{dy}{dx} =$

A. $\frac{1-2\log_e(2x)}{2x^2}$

B. $\frac{1-\log_e(2x)}{x^2}$

C. $\frac{\log_e(2x)-1}{x^2}$

D. $\frac{1-\log_e(2x)}{4x^2}$

E. $\frac{1-x\log_e(2x)}{x^2}$

QUESTION 12

The equation of the **normal** to the curve with equation $y = x^3 - 4x^2 + 7x - 5$ at $x = 2$ is

A. $y - 3x + 5 = 0$

B. $3y - x - 1 = 0$

C. $3y + x + 5 = 0$

D. $y + 3x - 7 = 0$

E. $3y + x - 5 = 0$

QUESTION 13

If $\int_1^3 f(x)\, dx = 10$ then $\int_3^1 (5 - 3f(x))dx$ is equal to

A. -25

B. -20

C. 0

D. 20

E. 25

QUESTION 14

The graph of the function $f(x)$ has a minimum stationary value at the point with co-ordinates $(2\sqrt{3}, -1)$. The function $g(x) = 5f(-x+1)$ will have the corresponding stationary value of

A. a minimum at $(-2\sqrt{3}+1, -5)$

B. a maximum at $(-2\sqrt{3}+1, -5)$

C. a minimum at $(2\sqrt{3}-1, -5)$

D. a maximum at $(2\sqrt{3}-1, -5)$

E. a minimum at $(-2\sqrt{3}-1, -5)$

QUESTION 15

$\int\left(\frac{2}{1-2x}+e^{3x+1}\right)dx$ is equal to

A. $\frac{4}{(1-2x)^2}+3e^{3x+1}+c$

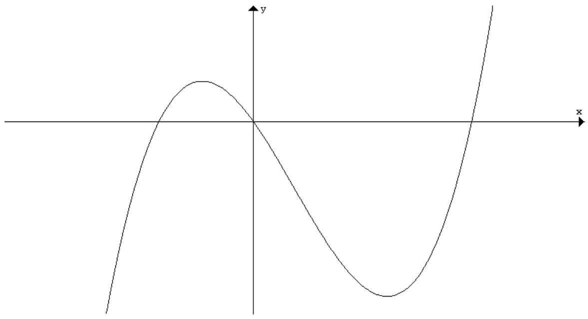
B. $-\frac{4}{(1-2x)^2}+3e^{3x+1}+c$

C. $-\log_e|2x-1|+\frac{1}{3}e^{3x+1}+c$

D. $\log_e|1-2x|+\frac{1}{3}e^{3x+1}+c$

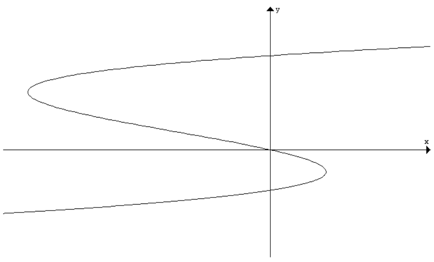
E. $2\log_e|1-2x|+\frac{1}{3}e^{3x+1}+c$

QUESTION 16
The graph of a function $y = f(x)$ is shown here.



The graph of $f'(x)$ could be

A.



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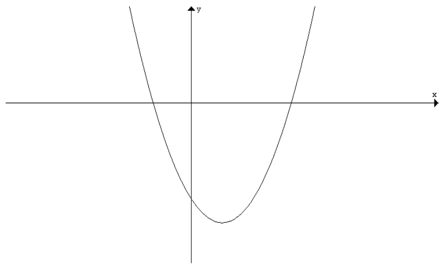

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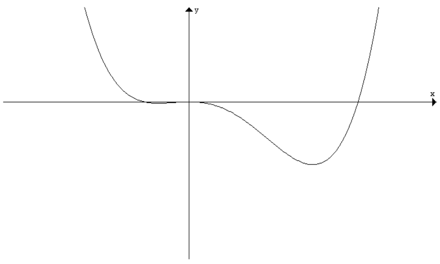

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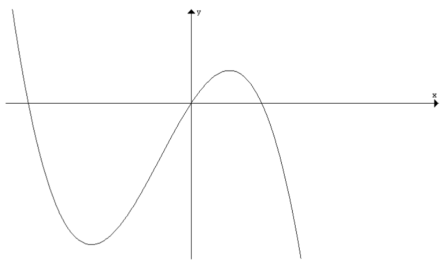
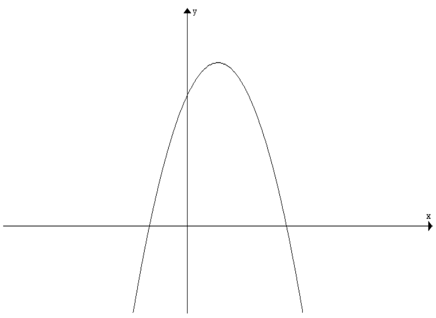


E.



QUESTION 17

The average rate of change of the function $f(x) = (2x-1)e^{3x}$ over $[0, 2]$ is

A. $(8x-3)e^{3x}$

B. $(6x-1)e^{3x}$

C. $\dfrac{3e^6-1}{2}$

D. $\dfrac{3e^6+1}{2}$

E. $3e^6+1$

QUESTION 18

An experiment consists of drawing a number at random from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
Let A = {1, 2, 3, 4, 5} , B = {2, 4, 6, 8, 10} , C = {3, 6, 9} and D = {1, 2, 5, 10}.
For which of the following pairs of events are the events independent?

A. A and B

B. A and C

C. B and C

D. A and D

E. B and D

QUESTION 19

Past records indicate that if it rains on a particular day there is an 80% chance that it will rain the next day. If it does **not** rain on a particular day, the probability that it will rain the next day is ***p***%. It rained on Monday and the probability that it rained on the following Wednesday was 65%. The value of ***p*** is

A. 5

B. 10

C. 15

D. 20

E. 35

QUESTION 20

A Binomial Probability Distribution has a mean of 5 and a variance of 4.
Correct to 4 decimal places, the value of $\Pr(\mu - \sigma \le X \le \mu + \sigma)$ is closest to

A. 0.7927

B. 0.8909

C. 0.6800

D. 0.6827

E. 0.9789

QUESTION 21

The weight of 3-year old cats is normally distributed with a mean of 5 kg and a standard deviation of 0.5 kg. The probability that a 3-year old cat weighs greater than 4 kg **given that** it weighs less than 5 kg is closest to

A. 0.1080

B. 0.4124

C. 0.4772

D. 0.9544

E. 0.9545

QUESTION 22

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} ax & 0 \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

where a and b are positive real numbers with $b > a$. If the ***median*** is $\frac{4}{3}$ then

A. $a = \frac{\sqrt{3}}{4}$, $b = \frac{64}{3}$

B. $a = \frac{9}{16}$, $b = \frac{4\sqrt{2}}{3}$

C. $a = \frac{3}{4}$, $b = \sqrt{\left(\frac{8}{3}\right)}$

D. $a = 1$, $b = \sqrt{2}$

E. $a = \frac{16}{9}$, $b = \frac{4\sqrt{2}}{3}$

SECTION 2 – EXTENDED ANSWER QUESTIONS

Instructions For Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

The graph of the function $f(x) = (x + a)(x - b)^2 + 2$ where a and b are real constants is shown below. It is known that $a > b$.

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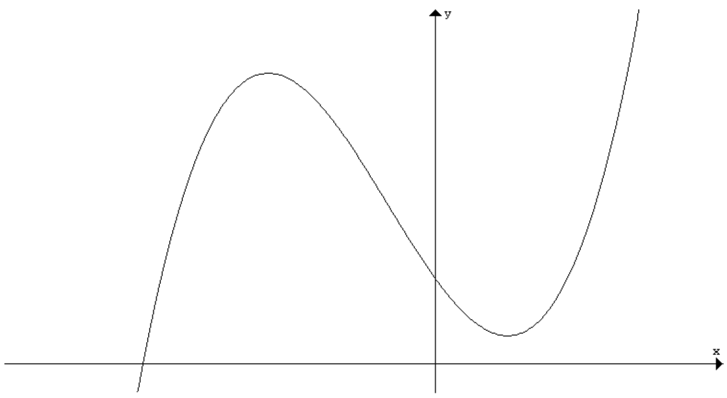


a. Given that $f'(1) = f'\left(-\frac{7}{3}\right) = 0$, use calculus to show that $a = 4$ and $b = 1$.

4 marks

b. Find the coordinates of the turning point of the graph of $y = f(x)$ at $x = 1$.

1 mark

c. If the coordinates of the other turning point of the graph of $y = f(x)$ is $\left(-\frac{7}{3}, c\right)$, find the value of c correct to 2 decimal places.

1 mark

d. Find the real values of m for which the equation $f(x) = m$ has three distinct solutions. (Non-integer values are to be given correct to 2 decimal places).

2 marks

The graph of $y = f(x)$ is dilated by a factor of k from the *y*-axis to form another function $g(x)$ so that the **horizontal** distance between the two turning points is 10 units.

e. Find this value of k.

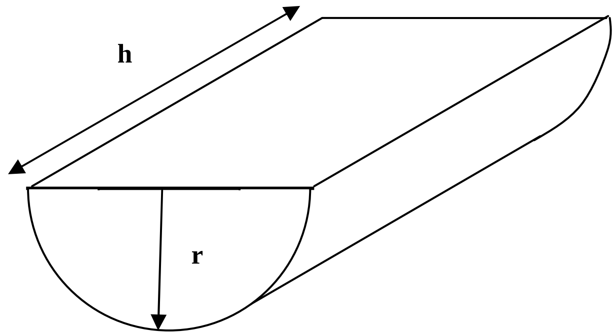
2 marks

f. Hence, or otherwise, write down the coordinates of the turning points of the graph with equation $y = g(x)$. (Non-integer values are to be given correct to 2 decimal places).

2 marks

Total = 12 marks

QUESTION 2



A quantity of precious metal whose volume is $500\,cm^3$ is melted and moulded into a shape of length ***h*** centimetres and uniform semi-circular cross section of radius ***r*** centimetres, as shown in the diagram.

a. Find an equation for A**,** the total surface area, in terms of h and r.

2 marks

b. Hence show that $A = \pi r^2 + \dfrac{1000(2+\pi)}{\pi r}$.

2 marks

c. Using calculus, find the value of r, correct to 2 decimal places, for which this surface area is a minimum.

2 marks

d. Write down the minimum surface area, correct to 2 decimal places.

1 mark

To make this precious metal into a beautiful piece of jewellery, it is decided to cover the surface area with gold leaf. The flat surface area is easier to cover and costs $***p*** per cm^2 whereas the curved surface area is more difficult to coat with gold leaf and costs $***q*** per cm^2, where ***p*** and ***q*** are constants with ***q*** > ***p***.

e. Using the information that you used in parts **a.** and **b.**, show that an expression for the ***total cost*** (in dollars) of covering this piece of jewellery in terms of ***r***, ***p*** and ***q*** is given by $C = \boldsymbol{p}\left(\pi r^2 + \frac{2000}{\pi r}\right) + \frac{1000}{r}\boldsymbol{q}$.

2 marks

f. Find the value of r in terms of ***p*** and ***q*** for which this cost is a minimum.

3 marks

g. Hence, or otherwise, if $\boldsymbol{p} = 15$ and $\boldsymbol{q} = 35$ write down

i. the minimum cost (correct to the nearest dollar).

1 mark

ii. the radius (in centimetres, correct to 2 decimal places).

1 mark

h. For safety reasons, the height of the jewellery piece must **not** be less than 10 centimetres in length. Find the minimum cost to cover the surface area with gold leaf, if $\boldsymbol{p} = 15$ and $\boldsymbol{q} = 35$. Give your answer correct to the nearest dollar.

2 marks

Total = 16 marks

QUESTION 3

Speeds of vehicles recorded by a speed-camera along a Victorian country road are normally distributed. Over a one month period, 20% of cars exceeded 94 km/h and 1% of cars travelled at more than 122 km/h.

a. Set up two equations which would enable you to find the mean ***p*** and standard deviation ***q*** of these recorded speeds. Non-integer values are to be rounded to 3 decimal places.

2 marks

b. Solve the equations to show that the values of ***p*** and ***q*** are 78.1 and 18.9 km/h, respectively. Find the percentage of cars recorded travelling at less than the speed limit of 80 km/h. Give your answer correct to the nearest percentage.

2 marks

c. Of the drivers exceeding 80 km/h on this country road, what is the probability that they were travelling at more than 100km/h? Give your answer correct to 3 decimal places.

3 marks

d. Drivers on this country road are fined if they exceed the speed limit. The penalties are shown in the table below.

Recorded speed of car	**Amount of penalty**	**Probability**
Below 80 km/h	zero	
From 80 km/h to under 100 km/h	\$220	
From 100 km/h to under 110 km/h	\$440	
Over 110 km/h	\$500	

Fill in the probability column, giving each value correct to 2 decimal places.

2 marks

e. Calculate the expected value of the penalty for speeding drivers, correct to the nearest ten dollars.

2 marks

f. In a particular hour, the speed camera on this country road recorded 48 vehicles exceeding 100 km/h. How many cars passed the speed camera during that hour?

2 marks

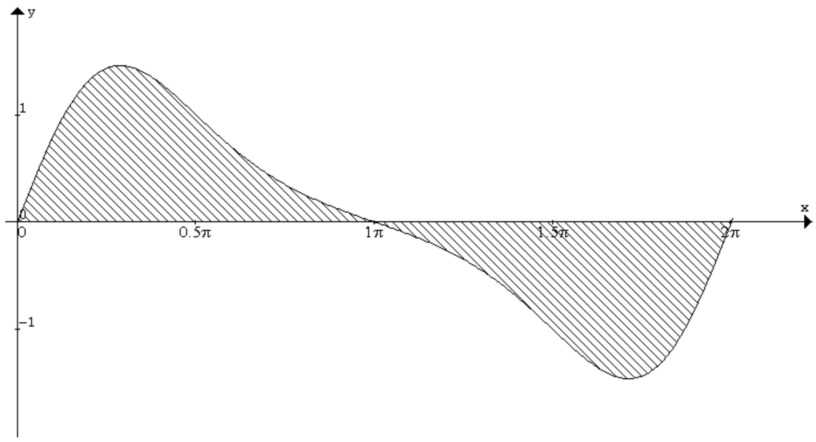
g. Six drivers who passed this speed camera stopped further along this country road for a cup of coffee. What is the probability (correct to 4 decimal places) that at least two of them were fined for speeding?

3 marks

Total = 16 marks

QUESTION 4

A flower-bed is to be planted in a garden. The boundary of this flower-bed is the *x*- axis and the curve with equation $f(x)=\sin(x)e^{\cos(x)}$ for $0 \le x \le 2\pi$ where x is measured in metres.



a. **i.** Using calculus, find the derivative of $e^{\cos(x)}$.

2 marks

ii. Hence, find the ***exact*** area of the garden bed.

2 marks

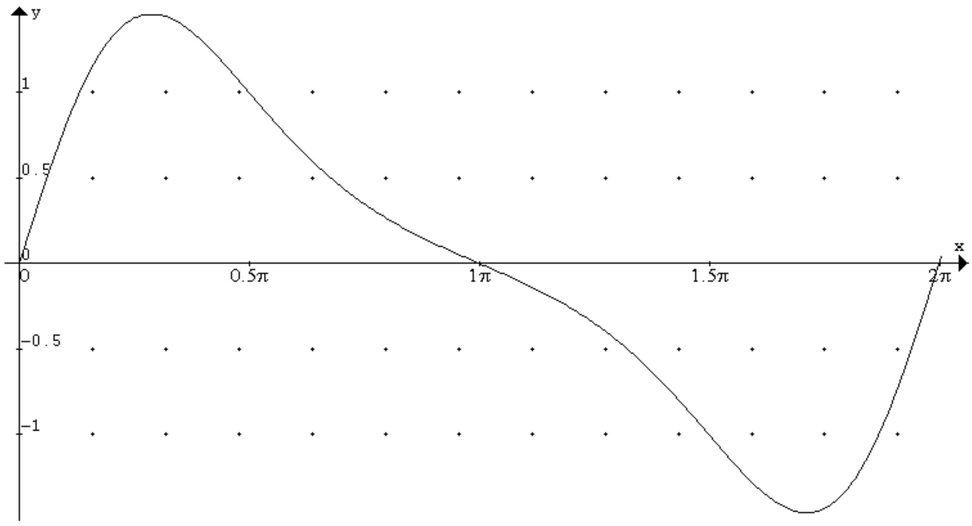
b. **i.** Find $f'(x)$.

2 marks

ii. Hence show algebraically that $f'(x) = 0$ if $\cos(x) = \dfrac{\sqrt{5}-1}{2}$.

2 marks

The boundary of the flower-bed whose equation is $f(x) = \sin(x)e^{\cos(x)}$ is shown on the diagram below. It is decided to install a watering system for this flower-bed. The area covered by this watering system is contained by the graph of the composite function $y = f(g(x))$ where $g(x) = \sqrt{(x^2+1)}$ for $x \geq 0$ and the *x*- axis (similar to that of the garden bed).



c. Write down the equation of $f(g(x))$.

1 mark

d. **i.** Using your calculator, sketch the graph of $y = f(g(x))$ for $0 \leq x \leq 2\pi$ on the grid above, giving any intercepts to 2 decimal places.

ii. Write down the coordinates of any points of intersection between the two curves, giving answers correct to 2 decimal places.

2 + 1 = 3 marks

e. **i.** This watering system does not cover the entire area of the garden. Write down the definite integrals that would represent the area **not** covered.
(It is not necessary to evaluate this area.)

2 marks

Total = 14 marks

End of Paper