THE SCHOOL FOR EXCELLENCE (TSFX)

UNIT 4 MATHEMATICAL METHODS 2009

WRITTEN EXAMINATION 1

Reading Time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are **NOT** permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

All written responses must be in English.

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PRINTING SPECIFICATIONS

Please ensure that the paper size on your printer is selected as **A4** and that you select "**None**" under "Page Scaling".

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Instructions

- Answer **all** questions in the spaces provided.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

(a) Find values of the constants a and b such that $\frac{d}{dx}\left(x^2 e^{-3x}\right) = \frac{ax^2 + bx}{e^{3x}}$.

(b) If $f(x) = \frac{x^2 + 1}{x^2 - 1}$, find $f'(2)$.

3 + 3 = 6 marks

QUESTION 2

(a) The polynomial equation $P(x)$ is given by the equation $P(x) = x^4 + x^3 + ax^2 - x + b$ where a and b are integers. If $P(1) = 0$ and $P(-2) = 12$, write down two equations which, when solved simultaneously, show that $a = 1$ and $b = -2$.

(b) If $(x+1)$ is also a factor of $P(x)$, completely factorise $P(x)$ for $x \in \boldsymbol{R}$.

2 + 2 = 4 marks

QUESTION 3

The equation $\log_4(x) - \log_4(x-4) = 1$ has the solution $x = \frac{a}{b}$ where a and b are integers and the fraction is written in its simplified form. Find the values of a and b.

3 marks

QUESTION 4

On a particular day in a garden hot-house, the temperature y (in degrees Celsius) can be modelled by the equation $y = 25 - 10\cos\left(\frac{\pi}{8}t\right)$ where t is the time in hours after 12 midnight.

(a) Write down the period of this function.

(b) What is the temperature at 12:00 noon?

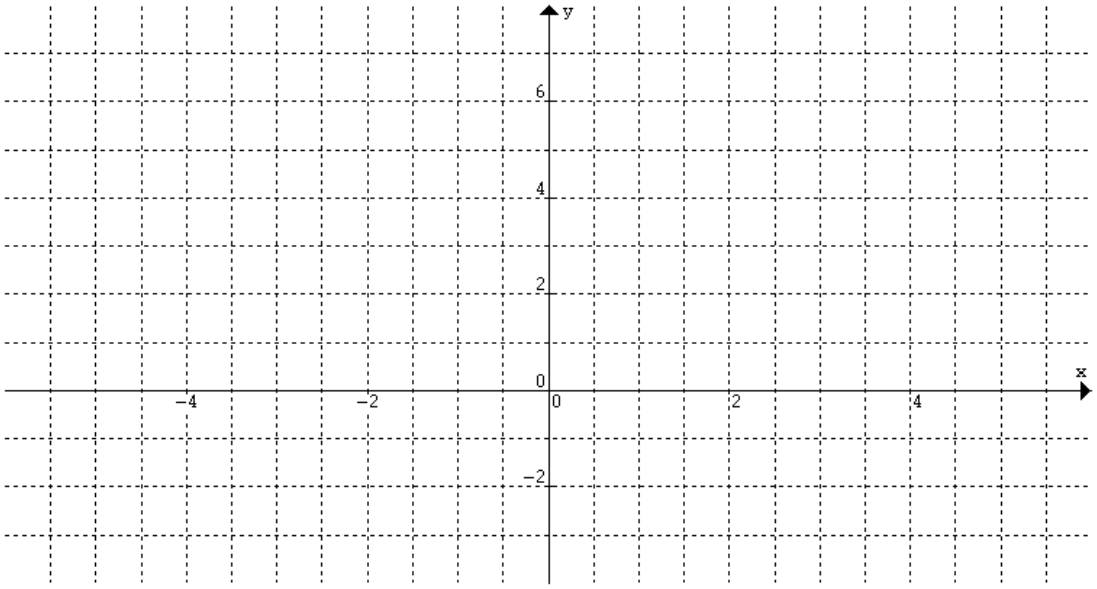
(c) What is the maximum temperature and at what time does this maximum temperature **first** occur?

1 + 1 +1 = 3 marks

QUESTION 5

(a) Describe all the transformations (in order) required to change the graph with equation $y = \sqrt{x}$ to the graph with equation $y = 3 - \sqrt{(4-2x)}$.

(b) Sketch the graph of $y = 3 - \sqrt{(4-2x)}$ on the axes below, showing the endpoint and intercept coordinates.



(c) Find the rule and domain of the inverse of $f(x) = 3 - \sqrt{(4-2x)}$.

2 + 2 + 3 = 7 marks

QUESTION 6

A spherical weather balloon is released into the atmosphere. As it rises, its volume increases at a constant rate of $16\ \pi\ cm^3 / minute$. At what rate is its radius increasing when the radius is 20 cm?

3 marks

QUESTION 7

X is distributed with a mean of 10 and a variance of 2.0. If $Y = 4X - 5$, find the mean and variance of Y.

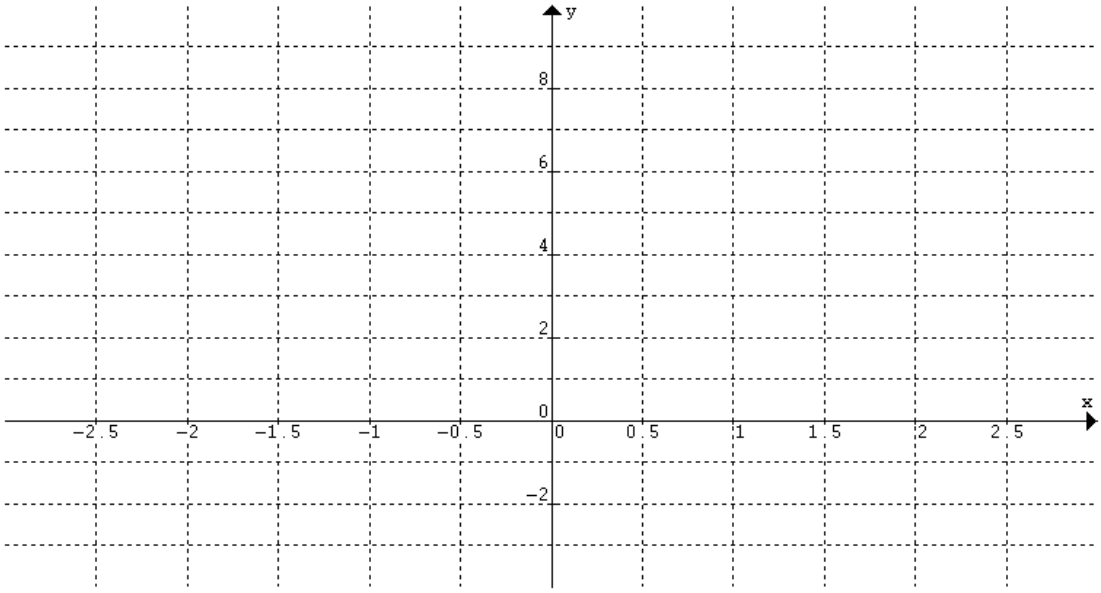
3 marks

QUESTION 8

A hybrid function is given by the equation

$$f(x) = \begin{cases} 3x + |5x| & -1 \leq x \leq 1 \\ 0 & \textit{elsewhere} \end{cases}$$

(a) Sketch the graph of $f(x)$ on the axes provided.



$f(x)$ is converted into a probability density function $g(x)$ by introducing a positive constant k so that $g(x)$ is given by the equation

$$g(x) = \begin{cases} k(\,3x + |5x|\,) & -1 \leq x \leq 1 \\ 0 & \textit{elsewhere} \end{cases}$$

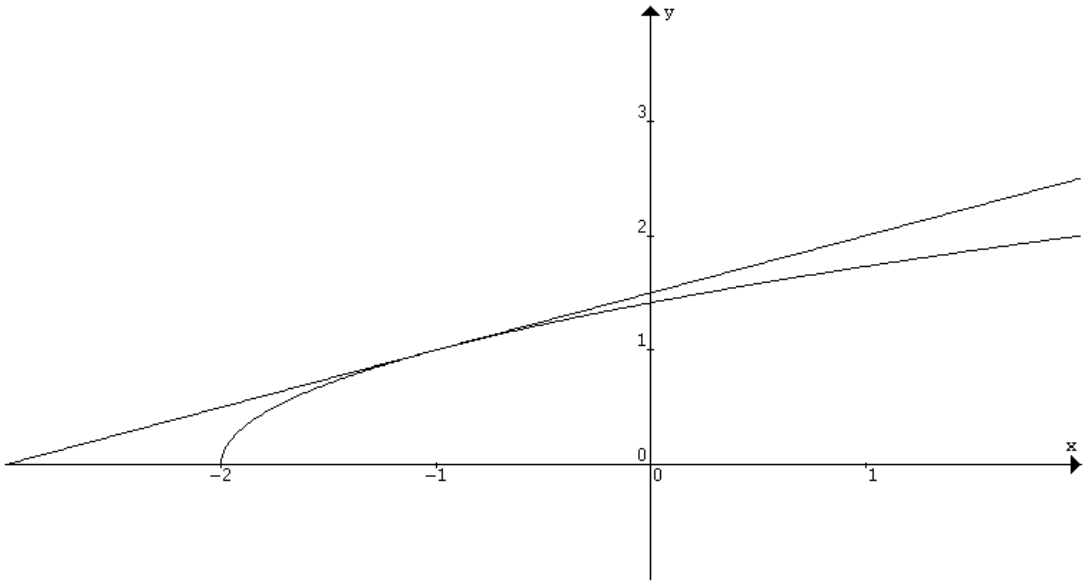
(b) Find the value of k which makes $g(x)$ a probability density function.

(c) Find the mean of $g(x)$.

3 + 1 + 2 = 6 marks

QUESTION 9

The graphs of the function $f(x) = \sqrt{(x+2)}$ and its tangent at $x = -1$ are shown on the diagram below.



(a) Find the equation of the tangent to the curve $f(x) = \sqrt{(x+2)}$ at the point where $x = -1$ giving your answer in the form $ay = bx + c$ where a, b and c are constants.

(b) Find the area enclosed between the tangent at $x = -1$, the curve $f(x)$ and the x – axis.

3 + 2 = 5 marks

END OF QUESTION AND ANSWER BOOK