Year 2009

VCE

Mathematical Methods CAS

Trial Examination 2

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Victorian Certificate of Education
2009

STUDENT NUMBER

Figures									Letter
Words									

MATHEMATICAL METHODS CAS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 29 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

If $f(x)=g(x)\log_e(2x)$, $g\left(\frac{e}{2}\right)=\frac{e}{2}$ and $g'\left(\frac{e}{2}\right)=1$ then $f'\left(\frac{e}{2}\right)$ is equal to

A. 1

B. $\frac{e}{2}+1$

C. $\frac{e}{2}$

D. 2

E. 3

Question 2

Given the two linear simultaneous equations
$2x+(p+1)y=4$ and $px+6y=q$, then

A. if $p=3$ and $q=6$ there is an infinite number of solutions.

B. if $p=3$ and $q=6$ there is no solution.

C. if $p=3$ and $q\in R$ there is a unique solution.

D. if $p=-4$ and $q\in R$ there is a unique solution.

E. if $p=-4$ and $q\neq 6$ there is more than one solution.

Question 3

The average value of the function with the rule $f(x) = x^3 + e^{2x}$ between $x = 0$ and $x = 2$ is equal to

A. $\dfrac{7+e^4}{2}$

B. $\dfrac{7+e^4}{4}$

C. $12+2e^4$

D. $5+e^4$

E. $\dfrac{11+2e^4}{2}$

Question 4

The transformation $T: R^2 \to R^2$, which maps the curve with equation $y = x^4$ to the curve with equation $y = (3x+6)^4 - 2$, could have the rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

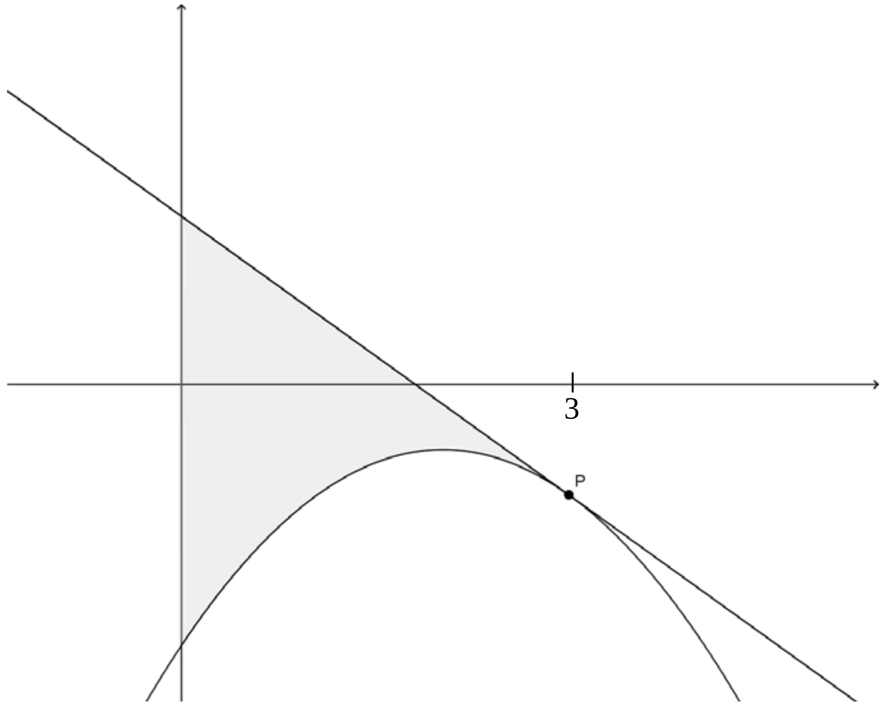
B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

Question 5



The diagram above show the graph of $y = -x^2 + 3x - 3$ and the tangent to the graph at the point *P*, where $x = 3$. The tangent has the equation $y = mx + c$. The shaded area *A* is the area between the graph, the tangent and the *y*-axis. Which of the following is **FALSE**?

A. $c = 6$

B. $m = -3$

C. $-3m + 6 = -3$

D. $m < 0$ and $c > 0$

E. $A = \int_0^3 \left(x^2 + (m-3)x + (c+3) \right) dx$

Question 6

If $f(x)=\frac{1}{x^2}$ then the best approximation to $\frac{1}{3.99^2}$ is given by

A. $f(4)-0.01f'(4)$

B. $f(4)+0.01f'(4)$

C. $f(2)-0.01f'(4)$

D. $f(2)+0.01f'(2)$

E. $f(2)-0.01f'(2)$

Question 7

Which of the following is **NOT** a measure of the area bounded by the graph of $y=x^2-a^2$ and the *x*-axis?

A. $\left|\int_{-a}^{a}\left(x^2-a^2\right)dx\right|$

B. $2\int_{-a}^{0}\sqrt{a^2-x}\,dx$

C. $2\int_{0}^{a}\left(a^2-x^2\right)dx$

D. $\int_{-a}^{a}\left(a^2-x^2\right)dx$

E. $2\left|\int_{-a^2}^{0}\sqrt{x+a^2}\,dx\right|$

Question 8

If a is a real positive constant, then the inverse of the function
$f : R^- \to R$, $f(x) = x^2 + a$ is

A. $f^{-1} : R \to R$, $f^{-1}(x) = \dfrac{1}{x^2 + a}$

B. $f^{-1} : R \to R$, $f^{-1}(x) = \sqrt{a} - \sqrt{x}$

C. $f^{-1} : (a, \infty) \to R$, $f^{-1}(x) = -\sqrt{x - a}$

D. $f^{-1} : (a, \infty) \to R$, $f^{-1}(x) = \sqrt{x - a}$

E. $f^{-1} : (0, \infty) \to R$, $f^{-1}(x) = \sqrt{x} - \sqrt{a}$

Question 9

If $f(x) = f(-x)$ and $\int_{-6}^{6} f(x)\,dx = 10$, then $\int_{0}^{6} \left(2f(x) - 1\right) dx$ is equal to

A. 4

B. 14

C. 7

D. $10 - x$

E. $20 - x$

Question 10

The graph of the function $f(x) = x^2 \log_e |x|$ has

A. a local maximum at $x = 0$ and local minimums at $x = \pm \dfrac{1}{\sqrt{e}}$

B. local minimums at $x = \pm \dfrac{1}{\sqrt{e}}$

C. a local minimum at $x = 0$

D. a point of inflexion at $x = 0$

E. a point of inflexion at $x = 0$ and local minimums at $x = \pm \dfrac{1}{\sqrt{e}}$

Question 11

Given the function $f(x) = b + \frac{a}{x-b}$ where *a* and *b* are non-zero real constants,
and $y = f^{-1}(x)$ is the inverse function.
Which of the following statements is **FALSE?**

A. The graph of $y = f(x)$ has the line $x = b$ as a vertical asymptote and the line $y = b$ as a horizontal asymptote.

B. The domain and range of both f and f^{-1} are $R \backslash \{b\}$.

C. The inverse function $f^{-1} = f$.

D. The graph of $y = f(x)$ passes through $\left(0, b - \frac{a}{b}\right)$ and the graph of $y = f^{-1}(x)$ passes through $\left(b - \frac{a}{b}, 0\right)$.

E. The graphs of $y = f(x)$ and $y = f^{-1}(x)$ always intersects on the line $y = x$ at the points $\left(b \pm \sqrt{a}, b \pm \sqrt{a}\right)$.

Question 12

A certain curve has its gradient given by $2\cos\left(\frac{x}{2}\right)$. If the curve crosses the *x*-axis at $x = \frac{5\pi}{3}$, then it crosses the *y*-axis at

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. $\frac{\sqrt{3}}{2}$

E. 2

Question 13

A curve has a vertical asymptote at $x = a$ and a horizontal asymptote at $y = b$. Which one of the following could be the equation of the curve ?

A. $y = \dfrac{x-b}{x+a}$

B. $y = \dfrac{x-b}{x-a}$

C. $y = \dfrac{b}{x-a}$

D. $y = \dfrac{bx}{x-a}$

E. $y = b + \dfrac{1}{x+a}$

Question 14

Let $f:[0,\pi] \to R$, $f(x) = 2\cos\left(\dfrac{x}{2}\right) - 2$. The graph of f is transformed by a reflection in the *x*-axis, followed by a dilation of factor 2 from the *y*-axis, then a dilation by a factor of 3 from the *x*-axis. The resulting graph is defined by

A. $g:[0,\pi] \to R$, $g(x) = 6 - 6\cos(x)$

B. $g:\left[0,\dfrac{\pi}{3}\right] \to R$, $g(x) = 4 + 4\cos\left(\dfrac{3x}{2}\right)$

C. $g:[0,3\pi] \to R$, $g(x) = 4 - 4\cos\left(\dfrac{x}{6}\right)$

D. $g:[0,2\pi] \to R$, $g(x) = 6 - 6\cos\left(\dfrac{x}{4}\right)$

E. $g:[0,2\pi] \to R$, $g(x) = 4 - 4\cos\left(\dfrac{3x}{2}\right)$

Question 15

If $\Pr(A)=a$, $\Pr(B)=b$ and $\Pr(A\cap B)=p$ where $0<a<1$, $0<b<1$ and $0<p<1$, then $\Pr(A'\cap B')$ is equal to

A. $1-p$

B. $1-(a+b)$

C. $1-(a+b)-ab$

D. $2-(a+b)-(1-a)(1-b)$

E. $1+p-(a+b)$

Question 16

The function $f:(-\infty,a)\to R$ with the rule $f(x)=x^4-4x^2$, will have an inverse provided that

A. $a<2$

B. $a>0$

C. $a<\sqrt{2}$

D. $a<-\sqrt{2}$

E. $a>-\sqrt{2}$

Question 17

Anthony and Betty play n games of table tennis. The probability that Anthony wins any game is 0.4, no game can be a draw. If the probability that Betty wins no games is less than 0.01, then the minimum number of games they played is equal to

A. 5

B. 6

C. 7

D. 9

E. 10

Question 18

If X is a normally distributed random variable with mean μ and variance $\frac{9\mu^2}{4}$,

then $\Pr(X > 2\mu)$ is closest to

A. 0.023

B. 0.252

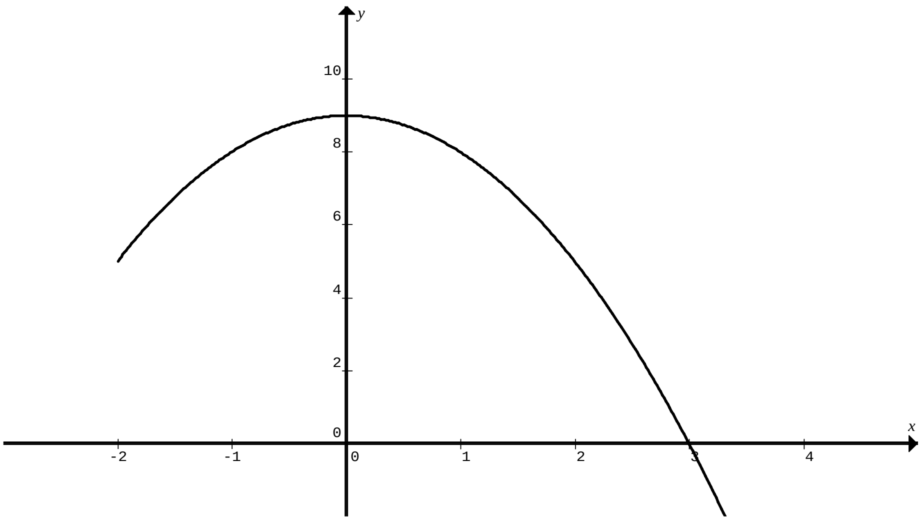
C. 0.504

D. 0.672

E. 0.748

Question 19

Shown below is the graph of a function *f*.



Let $g(x) = \int_0^x f(t)\,dt$, then

A. $g(0)=1 \quad g'(0)=0 \quad g'(3)=0$

B. $g(0)=1 \quad g'(0)=0 \quad g'(3)=1$

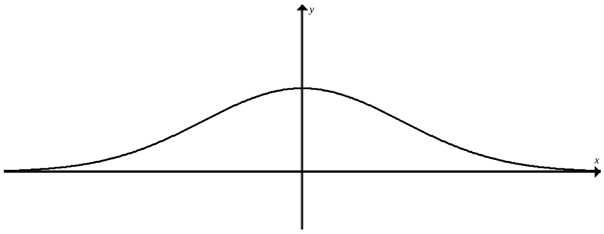
C. $g(0)=0 \quad g'(0)=9 \quad g'(3)=0$

D. $g(0)=0 \quad g'(0)=9 \quad g'(3)=18$

E. $g(0)=0 \quad g'(0)=0 \quad g'(3)=0$

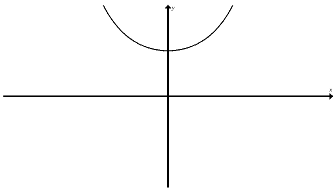
Question 20

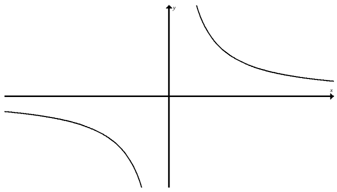
The graph of $f\left(g\left(x\right)\right)$ is shown.



The graphs of $y = f\left(x\right)$ and $y = g\left(x\right)$ are best represented respectively as

A.





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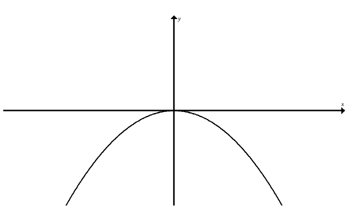
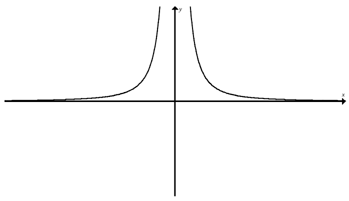
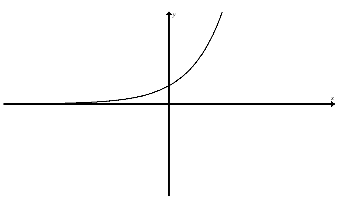
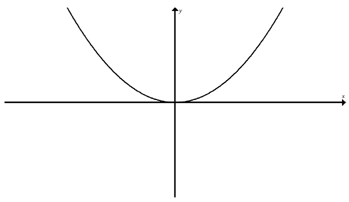


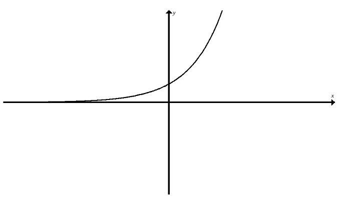


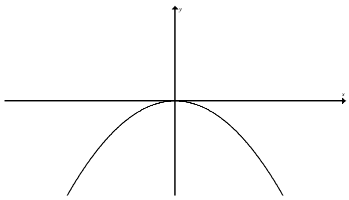
C.



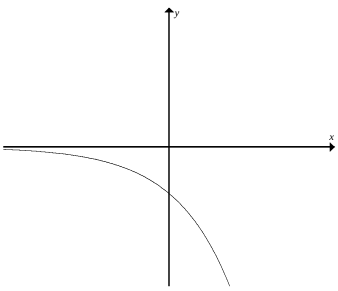


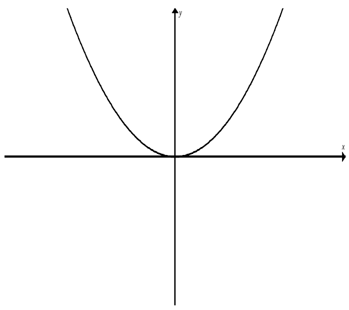
D.





E.





Question 21

A regular hexagon, has each side length being L cm. The area A of the hexagon is given by $\frac{3\sqrt{3}}{2}L^2$. If each of the six sides of the hexagon increases at a rate of $\sqrt{3}$ cm/s, then the rate in cm^2/s at which the area of the hexagon is increasing, when the side lengths are $\sqrt{3}$ cm, is given by

A. $9\sqrt{3}$

B. 9

C. $6\sqrt{3}$

D. $\frac{9\sqrt{3}}{2}$

E. $\frac{27}{2}$

Question 22

The random variable X has the following probability distribution.

X	-2	-1	1	2
$\Pr(X=x)$	$\frac{a}{2}$	a	b	$\frac{b}{2}$

Which of the following statements is **FALSE?**

A. $3(a+b)=2$

B. $E(X)=\sum x\Pr(X=x)=2(b-a)$

C. $E(X^2)=2$

D. $\text{var}(X)=2-4b^2+8ab-4a^2$

E. $E\left(\frac{1}{X}\right)=\frac{1}{2(b-a)}$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function $f: R \to R$, $f(x) = x^3 - 3x^2 + cx + d$, where c and d are real numbers.

a. The coordinates of the turning point on the graph of $y = f(x)$ are $(-1, 5)$ and (u, v).

i. Show that in this case $c = -9$, $d = 0$ $v = -27$ and determine the value of u.

2 marks

ii. For what values of d does the graph of $y = x^3 - 3x^2 - 9x + d$ cross the x-axis at three distinct points?

2 marks

b. For what values of c and d does the graph of $y = x^3 - 3x^2 + cx + d$ have two distinct turning points?

2 marks

c. If the graph of $y = f(x)$ is translated p units to the left away from the y-axis, it becomes the graph of $y = x^3$. Find the values of p, c and d in this case.

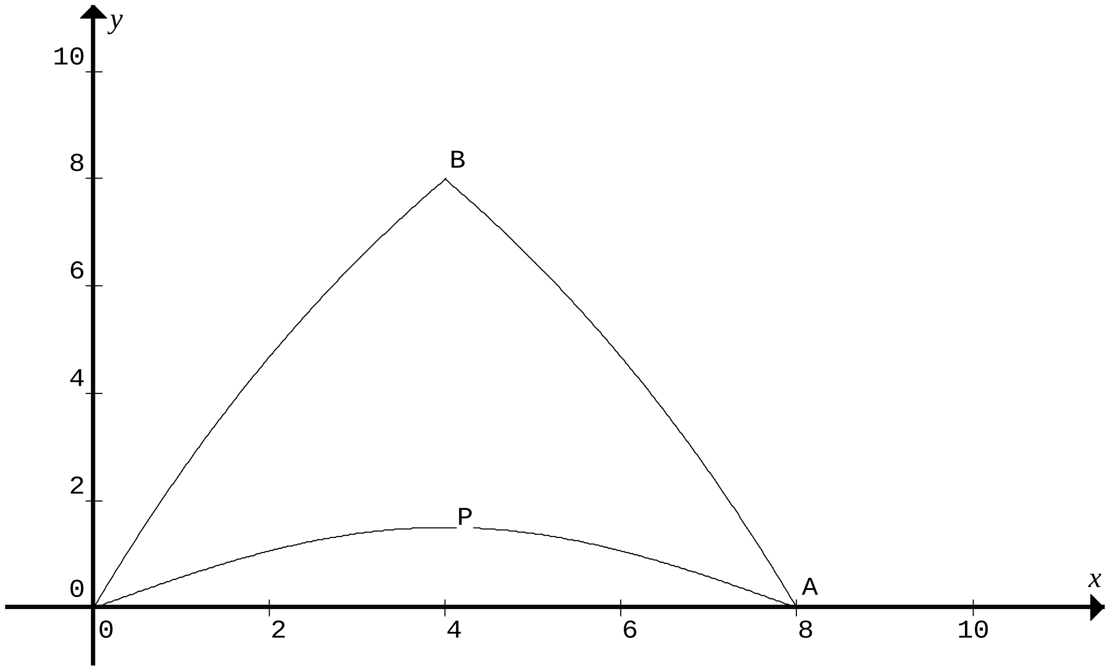
2 marks

d. Let A be the area bounded by the graph of $y = x^3 - 3x^2 + cx + d$, the coordinate axes and $x = 2$, and that $y \geq 0$ for $0 \leq x \leq 2$. If this area is approximated by four equally spaced left rectangles, the area is 10 square units. If this area is approximated by four equally spaced right rectangles, the area is 6 square units. The exact area bounded by the graph of $y = x^3 - 3x^2 + cx + d$, the coordinate axes and $x = 2$ is 8 square units. Determine the values of c and d.

5 marks
Total 13 marks

Question 2

The diagram below shows a "triangular" shade cloth, which is designed to block the direct sunlight onto a children's playground. The cloth lies in a horizontal plane and has vertical posts erected at points O, A and B. The point O is the origin and the co-ordinates of the points A , P and B are $(8,0)$, $(4,1.5)$ and $(4,8)$ respectively. The axes are shown on the drawing and the units are in metres.



The curve OPA has the form $y = a\sin(nx)$.

a. Explain why $a = 1.5$ and $n = \frac{\pi}{8}$.

1 mark

The curve OB has the form $y = 16\left(1 - e^{-kx}\right)$.

b. Find the value of *k*.

1 mark

c. The curve BA is the reflection of the curve OB in the line $x = 4$.

i. Write down **two** transformations, which take the curve OB into the curve BA.

1 mark

ii. Hence write down a **function** in terms of *k*, which describes the curve BA.

2 marks

d. i. Write down a definite integral, in terms of k, which gives the total area of the shade cloth.

1 mark

ii. If the area of the shade cloth can be expressed in the form $p+\frac{q}{k}+\frac{r}{\pi}$, find the values of p, q and r.

4 marks
Total 10 marks

Question 3

At a cinema, the time T minutes, that customers have to wait in order to buy their tickets has a probability density function given by

$$f(t) = \begin{cases} bt^2 & \text{for} \quad 0 \le t \le 4 \\ c(8-t) & \text{for} \quad 4 < t \le 8 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $b = \dfrac{3}{160}$ and $c = \dfrac{3}{40}$.

3 marks

b. Sketch the graph of $y = f(t)$ on the axes provided, clearly labelling the scale.

1 mark

c. Find the probability that a customer waits more than six minutes to buy a ticket.

2 marks

d. Find the mean time that customers have to wait in order to buy their tickets, give your answer correct to one decimal place.

2 marks

e. Find the median time, correct to two decimal places, that customers have to wait in order to buy their tickets.

2 marks

The running times of movies shown at the cinema are normally distributed with a mean of 94 minutes, with a standard deviation of 10 minutes.

f. Find the probability that the running time of a movie is more than 109 minutes, give your answer correct to three significant figures.

1 mark

g. A certain cinema complex has four different movies playing. Find the probability that at least two of the movies have a running time of more than 109 minutes, give your answer correct to three significant figures.

2 marks

Pete goes to the movies once every week. He likes to see only action or comedy movies. If he sees an action movie one week, the probability that he sees an action movie the following week is 0.45, while if he sees a comedy one week, the probability that he sees an action movie the following week is 0.35. Suppose he has just seen an action movie.

h. What is the probability that of the next three movies he sees, exactly two are comedies, give your answer correct to three decimal places.

2 marks

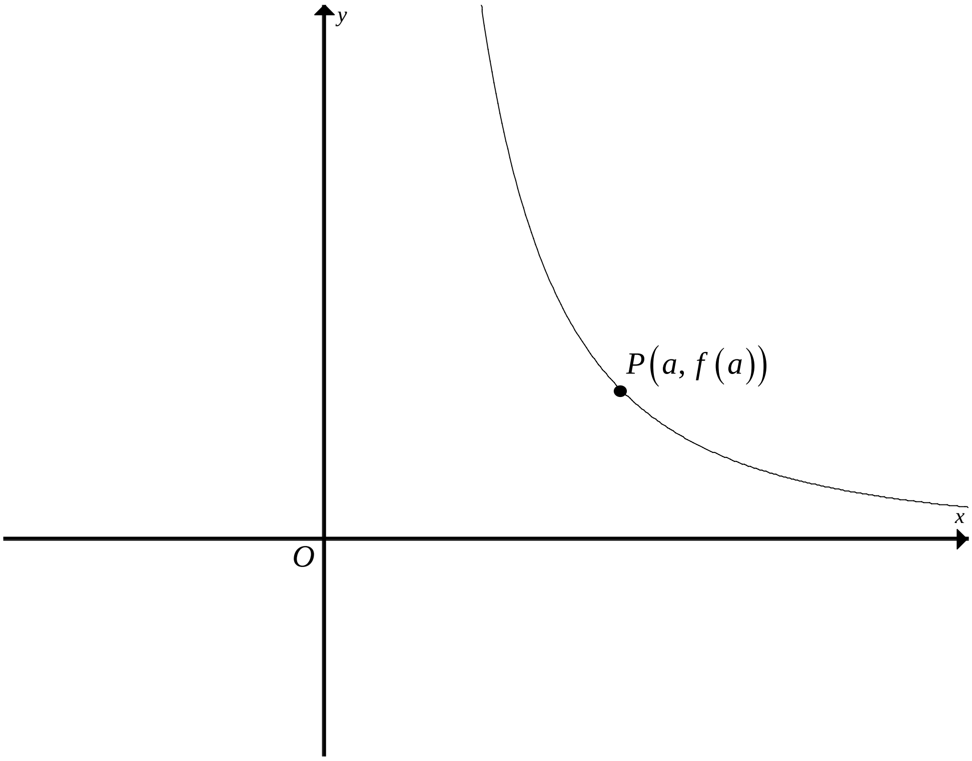
i. In the long term, what percentage of the movies that Pete sees are action movies? Give your answer correct to one decimal place.

1 mark
Total 16 marks

Question 4

The diagram below shows part of the graph of the function $f:(0,\infty)\to R$ where $f(x)=\dfrac{4}{x^2}$.

Let $P(a, f(a))$ where $a>0$ be a point on the graph of $y=\dfrac{4}{x^2}$.



a. Show that the distance s from the origin O to the point P is given by $s=\dfrac{\sqrt{16+a^6}}{a^2}$.

2 marks

b i. Find, the exact value of a, for which the distance s is a minimum.
Verify that it is a minimum.

3 marks

ii. Find the minimum distance, give an exact answer.

1 mark

c.i. Find in terms of a, the equation of the normal to the curve $y = f\left(x\right)$ at the point P.

2 marks

ii. Find the value of a, for which the normal passes through the origin.

1 mark
Total 9 marks

Question 5

Given the function $g: R \to R$, $g(x) = \sqrt{3}\sin(2x) + \cos(2x)$

a. Find the general solution of $g(x) = 0$.

1 mark

b. State the smallest positive value of T for which $g(x+T) = g(x)$.

1 mark

Given the function $f: [0, 2\pi] \to R$, $f(x) = \sqrt{3}\sin(2x) + \cos(2x)$.

c. Find $\{ x : f(x) = 0 \}$.

1 mark

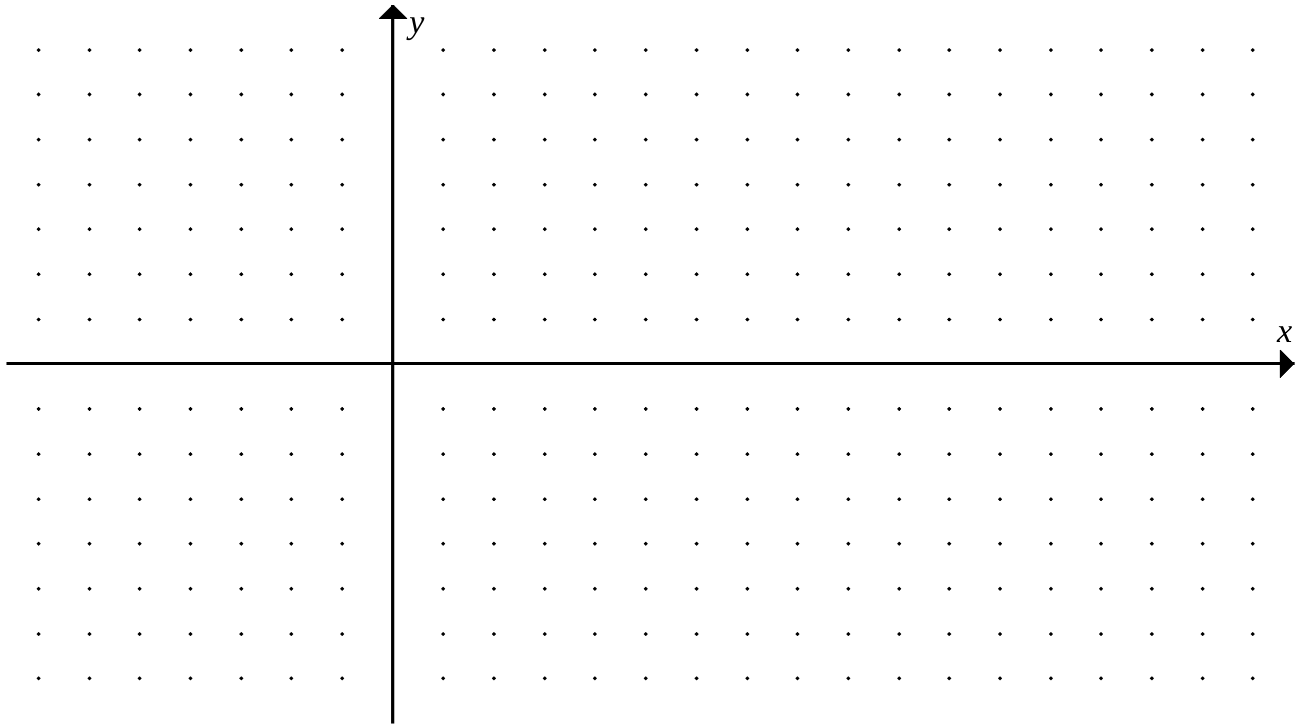
d.i. Find $\{ x : f'(x) = 0 \}$.

2 marks

ii. Find the exact coordinates of the maximum and minimum turning points on the graph of $y = f(x)$.

1 mark

e. Sketch the graph of $y = f(x)$ on the axes below, clearly labelling the scale.



2 marks

f. If $f(x) = A\sin(2(x+\alpha))$ state the values of A and α.

2 marks
Total 10 marks

END OF EXAMINATION

EXTRA WORKING SPACE

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	Volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2\,p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)\,dx$

ANSWER SHEET

STUDENT NUMBER

										Letter
Figures										
Words										

SIGNATURE ____

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**

Year 2009

VCE

Mathematical Methods CAS

Solutions

Trial Examination 2

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SECTION 1

ANSWERS

1	A	B	C	**D**	E
2	**A**	B	C	D	E
3	A	**B**	C	D	E
4	A	B	C	D	**E**
5	A	B	**C**	D	E
6	**A**	B	C	D	E
7	A	**B**	C	D	E
8	A	B	**C**	D	E
9	**A**	B	C	D	E
10	A	**B**	C	D	E
11	A	B	C	D	**E**
12	A	B	**C**	D	E
13	A	B	C	**D**	E
14	A	B	C	**D**	E
15	A	B	C	D	**E**
16	A	B	C	**D**	E
17	A	**B**	C	D	E
18	A	**B**	C	D	E
19	A	B	**C**	D	E
20	A	B	C	**D**	E
21	**A**	B	C	D	E
22	A	B	C	D	**E**

SECTION 1

Question 1 **Answer D**

$f(x)=g(x)\log_e(2x)$ differentiating using the product rule

$$f'(x)=g'(x)\log_e(2x)+\frac{g(x)}{x}$$

$$f'\left(\frac{e}{2}\right)=g'\left(\frac{e}{2}\right)\log_e(e)+\frac{2}{e}g\left(\frac{e}{2}\right)$$

$$f'\left(\frac{e}{2}\right)=1\text{x}1+\frac{e}{2}\text{x}\frac{2}{e}=2$$

Question 2 **Answer A**

$2x+(p+1)y=4$ and $px+6y=q$, in matrix form as $\begin{bmatrix}2 & p+1\\ p & 6\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ q\end{bmatrix}$

The determinant

$$\begin{vmatrix}2 & p+1\\ p & 6\end{vmatrix}=12-p(p+1)=12-p^2-p=-\left(p^2+p-12\right)=-(p-3)(p+4)$$

so when $p=3$ and $p=-4$ there is no unique solution. If $p=3$ the equations become

(1) $2x+4y=4$ and (2) $3x+6y=q$

$3\text{x}(1)$ $6x+12y=12$ and $2\text{x}(2)$ $6x+12y=2q$ when $q=6$ the two equations are the same equation, so when $p=3$ and $q=6$ there is an infinite number of solutions.

Question 3 **Answer B**

$f(x)=x^3+e^{2x}$ average value is $\frac{1}{2}\int_0^2\left(x^3+e^{2x}\right)dx=\frac{7+e^4}{4}$

Question 4 **Answer E**

$y=x^4$ into $y=(3x+6)^4-2$ or $y+2=(3x+6)^4$

$y=y'+2$ and $x=3x'+6$ become

$x'=\frac{x}{3}-2$ and $y'=y-2$ in matrix form

$$T\left(\begin{bmatrix}x\\ y\end{bmatrix}\right)=\begin{bmatrix}\frac{1}{3} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}-2\\ -2\end{bmatrix}$$

Question 5 **Answer C**

Let $y_1 = mx + c$ and $y_2 = -x^2 + 3x - 3$, the tangent to the graph at the point *P*, where $x = 3$. $\frac{dy_2}{dx} = -2x + 3$ $\left.\frac{dy_2}{dx}\right|_{x=3} = -3 = m$ so **B.** is true.

At $x = 3$ $y_2 = -9 + 9 - 3 = -3$ $P(3,-3)$ is on the tangent,

$y_1 = mx + c \quad \Rightarrow -3 = -9 + c \quad \Rightarrow c = 6$ so **A**. is true, also **D.** is true.

The area $A = \int_a^b (y_1 - y_2)\,dx \quad a = 0 \quad b = 3$, so that $A = \int_0^3 \left(x^2 + (m-3)x + (c+3)\right)dx$

E. is true, **C.** is false.

Question 6 **Answer A**

$f(x+h) \approx f(x) + hf'(x)$ with $f(x) = \frac{1}{x^2} \quad x = 4 \quad h = -0.01$

so that $\frac{1}{3.99} \approx f(4) - 0.01 f'(4)$

Question 7 **Answer B**

The required area is below the *x*-axis, so taking the absolute value, makes the area positive. **A.** is true $\left|\int_{-a}^{a}\left(x^2 - a^2\right)dx\right|$ this is also equal to **D.** which is true $\int_{-a}^{a}\left(a^2 - x^2\right)dx$,

by symmetry **C.** is true $2\int_0^a \left(a^2 - x^2\right)dx$. The graph of $y = x^2 - a^2$, this crosses the *y*-axis at $-a^2$, now the inverse function is $x = y^2 - a^2 \;\Rightarrow y^2 = x + a^2 \;\Rightarrow\; y = \sqrt{x + a^2}$, the area bounded by the curve and the *y*-axis is $2\left|\int_{-a^2}^{0}\sqrt{x + a^2}\,dx\right|$, so that **E.** is true, **B.** is false.

Question 8 **Answer C**

$f: \quad y = x^2 + a \qquad \text{dom } f = R^- \quad \text{ran } f = (a, \infty)$

$f^{-1} \quad x = y^2 + a$ transposing

$y^2 = x - a \quad y = \pm\sqrt{x - a}$ but ran $f^{-1} = R^-$ dom $f^{-1} = (a, \infty)$ so we must take the negative, $f^{-1} : (a, \infty) \to R$, $f^{-1}(x) = -\sqrt{x - a}$

Question 9 **Answer A**

$f(x)=f(-x)$, $f(x)$ is an even function, and $\int_{-6}^{6} f(x)\,dx=10$, then $\int_{0}^{6} f(x)\,dx=5$

$\int_{0}^{6}\left(2f(x)-1\right)dx=2\int_{0}^{6} f(x)\,dx-[x]_0^6=2\text{x}5-(6-0)=4$

Question 10 **Answer B**

The function is not defined when $x=0$,
all of **A, C, D.** and **E.** are false,
The function is an even function,
symmetrical about the *y*-axis.

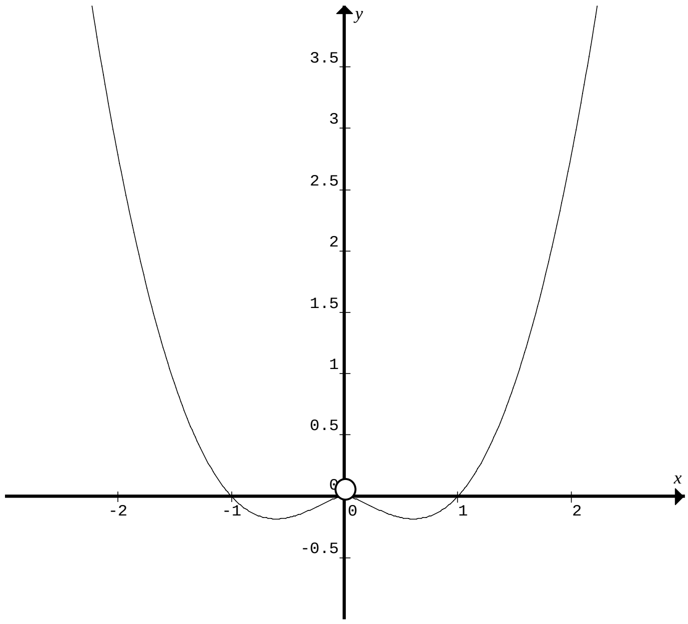
$y=x^2\log_e(x)$

$\frac{dy}{dx}=2x\log_e(x)+x=x\left(2\log_e(x)+1\right)$

for turning points, $\frac{dy}{dx}=0$, since $x\neq 0$

$\log_e(x)=-\frac{1}{2}$ $\quad x=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$,

the graph has minimums at $x=\pm\frac{1}{\sqrt{e}}$



Question 11 **Answer E**

$f:\quad y=b+\frac{a}{x-b}$

$f^{-1}\quad x=b+\frac{a}{y-b}\qquad\Rightarrow x-b=\frac{a}{y-b}\qquad\Rightarrow y-b=\frac{a}{x-b}$

$f^{-1}(x)=y=b+\frac{a}{x-b}$ so $f^{-1}=f$

The domain and range of both f and f^{-1} are $R\setminus\{b\}$.

Since $a\neq 0$ and $b\neq 0$, the graph of $y=f(x)$ passes through $\left(0,\ b-\frac{a}{b}\right)$

and the graph of $y=f^{-1}(x)$ passes through $\left(b-\frac{a}{b},0\right)$.

All of **A. B. C. D.** are true, however **E.** is false
The graph of $y=f(x)$ and $y=f^{-1}(x)$ always intersects on the line $y=x$ at the points $\left(b\pm\sqrt{a},b\pm\sqrt{a}\right)$ only if $a>0$.

Question 12 **Answer C**

$\frac{dy}{dx} = 2\cos\left(\frac{x}{2}\right) \Rightarrow y = \int 2\cos\left(\frac{x}{2}\right)dx = 4\sin\left(\frac{x}{2}\right) + c$ to find *c*, use $y\left(\frac{5\pi}{3}\right) = 0$

$0 = 4\sin\left(\frac{5\pi}{6}\right) + c = 2 + c = 0 \quad \Rightarrow c = -2$

$y = 4\sin\left(\frac{x}{2}\right) - 2$ now when $x = 0$ $y = 4\sin(0) - 2 = -2$

Question 13 **Answer D**

$y = \frac{bx}{x-a} = \frac{bx - ab + ab}{x-a} = b + \frac{ab}{x-a}$ has $y = b$ as a horizontal asymptote and $x = a$ as a vertical asymptote.

Question 14 **Answer D**

Let $f:[0,\pi] \to R$, $f(x) = 2\cos\left(\frac{x}{2}\right) - 2$. The period is $T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

The graph of *f* is transformed by a reflection in the *x*-axis, the rule is

$g(x) = 2 - 2\cos\left(\frac{x}{2}\right)$, we only have one-quarter of a cycle

now a dilation of factor 2 from the *y*-axis, replace x with $\frac{x}{2}$

$g:[0,2\pi] \to R$, $g(x) = 2 - 2\cos\left(\frac{x}{4}\right)$ since we must have one-quarter of a cycle,

the new domain is $[0,2\pi]$

then a dilation by a factor of 3 from the *x*-axis, multiply *y* by 3

the equation becomes $g:[0,2\pi] \to R$, $g(x) = 6 - 6\cos\left(\frac{x}{4}\right)$

Question 15 **Answer E**

$\Pr(A' \cap B') + b - p = 1 - a$ or

$\Pr(A' \cap B') + a - p = 1 - b$

$\Pr(A' \cap B') = 1 + p - (a + b)$

	A	A'	
B	p	$b - p$	b
B'	$a - p$	?	$1 - b$
	a	$1 - a$	

Question 16 **Answer D**

$f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x$

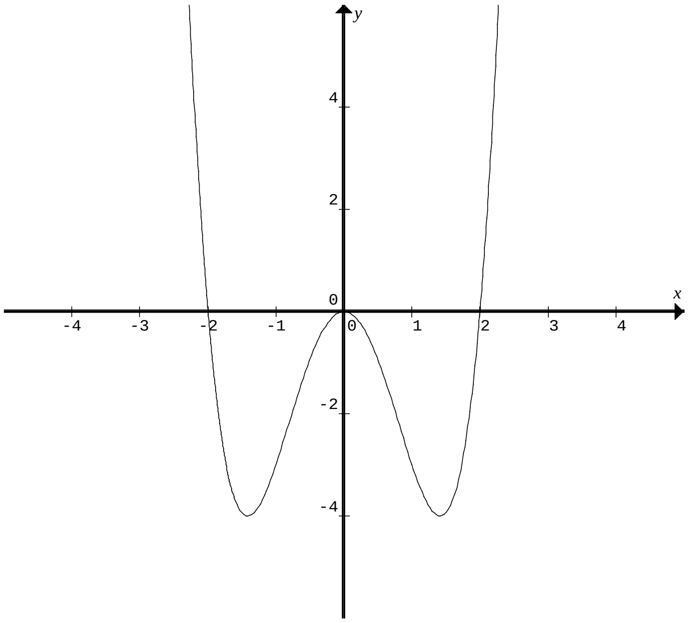
$f'(x) = 4x(x^2 - 2)$

turning points at

$x = 0$, $x = \pm\sqrt{2}$

for the function to be one-one, we require

$a < -\sqrt{2}$



Question 17 **Answer B**

$X \stackrel{d}{=} Bi(n = ?, p = 0.6)$ Betty winning a game.

$\Pr(X = 0) = 0.4^n \le 0.01$

$n \log_e(0.4) \le \log_e(0.01)$

$n \ge \dfrac{\log_e(0.01)}{\log_e(0.4)} = 5.02$ so $n = 6$

Question 18 **Answer B**

$X \stackrel{d}{=} N\left(\mu_X = \mu, \sigma_X^2 = \dfrac{9\mu^2}{4}\right)$

$\Pr(X > 2\mu) = \Pr\left(Z > \dfrac{2\mu - \mu}{\frac{3}{2}\mu}\right) = \Pr\left(Z > \dfrac{2}{3}\right) = 0.252$

Question 19 **Answer C**

Let $g(x) = \int_0^x f(t)\,dt$ then $g'(x) = f(x)$

$g(0) = 0$ $g'(0) = f(0) = 9$ $g'(3) = f(3) = 0$

Question 20 **Answer D**

Option **D.** has $f(x) = e^x$ $g(x) = -x^2$ and $f(g(x)) = e^{-x^2}$
Which is the graph required.

Question 21 **Answer A**

$A = \frac{3\sqrt{3}}{2} L^2$ $\frac{dA}{dL} = 3\sqrt{3}L$ given $\frac{dL}{dt} = \sqrt{3}$ cm/s

$\frac{dA}{dt} = \frac{dA}{dL} . \frac{dL}{dt} = 3\sqrt{3}\, L \times \sqrt{3} = 9L$

$\left.\frac{dA}{dt}\right|_{L=\sqrt{3}} = 9\sqrt{3}$ cm^2/s

Question 22 **Answer E**

Since $\sum \Pr(X = x) = 1 \Rightarrow \frac{a}{2} + a + b + \frac{b}{2} = \frac{3a}{2} + \frac{3b}{2} = 1 \Rightarrow 3(a+b) = 2$ **A.** is true

$E(X) = \sum x \Pr(X = x) = -2 \times \frac{a}{2} - a + b + 2 \times \frac{b}{2} = -2a + 2b = 2(b-a)$ **B.** is true

$E(X^2) = \sum x^2 \Pr(X = x) = (-2)^2 \times \frac{a}{2} + (-1)^2 a + (1)^2 b + (2)^2 \times \frac{b}{2} = 2a + a + b + 2b = 3(a+b) = 2$

C. is true, since **A.** is true.

$\text{var}(X) = E(X^2) - (E(X))^2 = 2 - 4(b-a)^2 = 2 - 4b^2 + 8ab - 4a^2$ **D.** is true

E. is false, $E\left(\frac{1}{X}\right) = \sum \frac{1}{x} \Pr(X = x) = -\frac{a}{4} - a + b + \frac{b}{4} = \frac{5}{4}(b-a)$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a.i $f(x) = x^3 - 3x^2 + cx + d$

$f'(x) = 3x^2 - 6x + c$

but $x = -1$ is a turning point so $f'(x) = (x+1)(3x-9) = 3(x+1)(x-3)$

Expanding gives $c = -9$,

also $u = 3$, A1

$f(-1) = 5 = -1 - 3 - c + d = -4 + 9 + d$ so that

$d = 0$ and $f(3) = v = 27 - 27 - 27 = -27$ $v = -27$ A1

ii. The graph of $y = x^3 - 3x^2 - 9x$ has a maximum value of 5, and a minimum value of -27, and crosses the *x*-axis at three distinct points. The graph of $y = x^3 - 3x^2 - 9x + d$ will therefore cross the *x*-axis at three distinct points, provided that $d \in (-5, 27)$ or $-5 < d < 27$ A2

b. $f(x) = x^3 - 3x^2 + cx + d$

$f'(x) = 3x^2 - 6x + c$ **,** for two distinct turning points, we require

$\Delta = 36 - 12c > 0$ M1

$c < 3$ and $d \in R$ A1

c. $f(x+p) = (x+p)^3 - 3(x+p)^2 + c(x+p) + d$

$f(x+p) = x^3 + x^2(3p-3) + x(3p^2 - 6p + c) + p^3 - 3p^2 + cp + d = x^3$

therefore $3p - 3 = 0$ $\Rightarrow p = 1$

and $3p^2 - 6p + c = 0$ since $p = 1$ $c = 6 - 3 = 3$

$c = 3$ A1

and $p^3 - 3p^2 + cp + d = 0$

so $d = -1$ A1

alternative method, if $y = (x-1)^3 = x^3 - 3x^2 + 3x - 1$ $\rightarrow$ $y = x^3$

so that $p = 1$ $c = 3$ and $d = -1$

d. $A=\int_a^b f(x)\,dx \quad a=0 \quad b=2 \quad h=\frac{1}{2} \quad n=4 \quad f(x)=x^3-3x^2+cx+d$

$(1)\quad L=10=h\left[f(0)+f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)\right]$ M1

$(2)\quad R=6=h\left[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)\right]$

$(1)\Rightarrow\quad 20=f(0)+f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)$

$(2)\Rightarrow\quad 12=f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)$ subtracting gives M1

$8=f(0)-f(2)=d-(8-12+2c+d)=4-2c$

$2c=-4$

$c=-2$ A1

Now $\int_0^2\left(x^3-3x^2-2x+d\right)=2d-8=8$ A1

$d=8$ A1

Question 2

a. the amplitude is 1.5, so that $a=1.5$

one-half cycle is 8, so that $T=\frac{2\pi}{n}=16 \quad\Rightarrow\ n=\frac{\pi}{8}$ A1

b. $y=16\left(1-e^{-kx}\right)$ passes through the origin $O(0,0)$ and $B(4,8)$

$8=16\left(1-e^{-4k}\right) \quad\Rightarrow\quad \frac{1}{2}=1-e^{-4k}$

$e^{-4k}=\frac{1}{2} \qquad e^{4k}=2$

$4k=\log_e(2)$

$k=\frac{1}{4}\log_e(2)$ A1

c.i reflect in the *y*-axis
translate 8 units, to the right, away from the *y*-axis A1
or translate 8 units, to the right parallel to the *x*-axis.

ii. $f:[4,8]\to R\ ,\ \ f(x)=16\left(1-e^{k(x-8)}\right)$ A2
must give domain.

d.i $A=2\int_0^4\left(16\left(1-e^{-kx}\right)-\frac{3}{2}\sin\left(\frac{\pi x}{8}\right)\right)dx$ A1

ii. $A=2\left[16x+\frac{16}{k}e^{-kx}+\frac{12}{\pi}\cos\left(\frac{\pi x}{8}\right)\right]_0^4$ each term A1

$A=2\left[\left(64+\frac{16}{k}e^{-4k}+\frac{12}{\pi}\cos\left(\frac{\pi}{2}\right)\right)-\left(0+\frac{16}{k}+\frac{12}{\pi}\cos(0)\right)\right]$ but $e^{-4k}=\frac{1}{2}$ M1

$A=2\left[64+\frac{8}{k}-\frac{16}{k}-\frac{12}{\pi}\right]$

$A=128-\frac{16}{k}-\frac{24}{\pi}$

$p=128\quad q=-16$ and $r=-24$ A2

Question 3

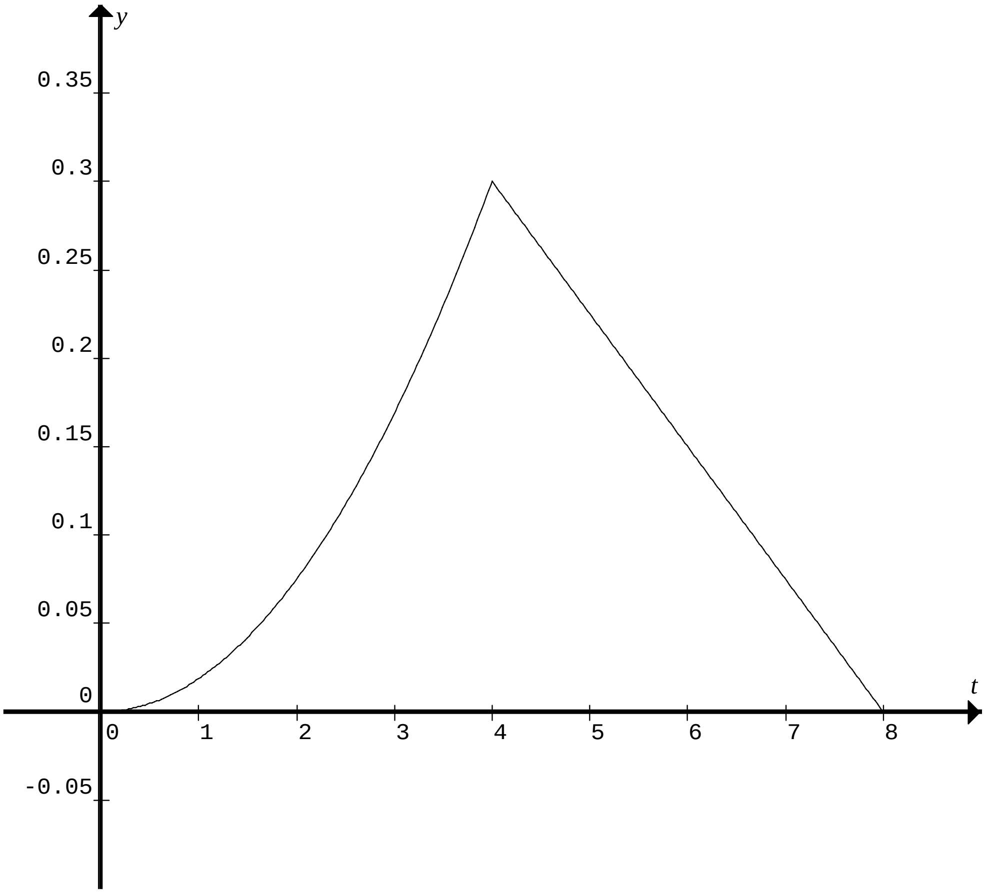
a. the function is continuous $f(4)=16b=4c \quad\Rightarrow\quad c=4b$ A1
the total area under the curve is one.

$b\int_0^4 t^2\,dt+c\int_4^8(8-t)\,dt=1$ A1

$\frac{64b}{3}+8c=1$ and $c=4b$ A1

solving gives $b=\frac{3}{160}$ and $c=\frac{3}{40}$

b. must show point at $(4, 0.3)$ and zero for $t \geq 8$ and $t \leq 0$ G1



c. $\Pr(T > 6) = \frac{3}{40}\int_6^8 (8-t)\,dt$ or the area of a triangle as M1

$\Pr(T > 6) = \frac{1}{2} \text{ x } 2 \text{ x } 2c = \frac{3}{20}$

$\Pr(T > 6) = \frac{3}{20}$ A1

d. $E(T) = \frac{3}{160}\int_0^4 t^3\,dt + \frac{3}{40}\int_4^8 t(8-t)\,dt$ M1

$E(T) = 1.2 + 3.2 = 4.4$ minutes A1

e. Since $\frac{3}{160}\int_0^4 t^2\,dt = 0.4$ the median time m is given by

$\frac{3}{40}\int_4^m (8-t)\,dt = 0.1$ M1

$\frac{-3\left(m^2-16m+48\right)}{80} = 0.1$

solving for m with $4 < m < 8$

$m = 4.35$ minutes A1

f. X is the running time of the movie in minutes

$X \stackrel{d}{=} N\left(\mu = 94, \sigma^2 = 10^2\right)$

$\Pr(X > 109) = \Pr\left(Z > \frac{109-94}{10}\right) = \Pr(Z > 1.5)$

$= 0.0668$ A1

g. $Y \stackrel{d}{=} Bi\left(n = 4, p = 0.0668\right)$

$\Pr(Y \ge 2) = 1 - \left[\Pr(Y=0) + \Pr(Y=1)\right]$

$\Pr(Y \ge 2) = 1 - \left[0.9332^4 + {}^4C_1\,0.0668 \text{x} 0.9332^3\right]$ M1

$\Pr(Y \ge 2) = 0.0244$ A1

h. $\Pr(2 \text{ comedies}) = ACC + CAC + CCA$ M1

$= 0.45 \text{ x } 0.55 \text{ x } 0.65 + 0.55 \text{ x } 0.35 \text{ x } 0.55 + 0.55 \text{ x } 0.65 \text{ x } 0.35$

$= 0.392$ A1

i. $\frac{0.35}{0.35+0.55} = 0.389$

or alternatively
$$\begin{array}{c|cc} & A & C \\ \hline A & 0.45 & 0.35 \\ C & 0.55 & 0.65 \end{array}$$

$$\begin{bmatrix} 0.45 & 0.35 \\ 0.55 & 0.65 \end{bmatrix}^{100} = \begin{bmatrix} 0.389 & 0.389 \\ 0.611 & 0.611 \end{bmatrix}$$

in the long run, the percentage of movies which are actions are 38.9% A1

Question 4

a. $P\left(a, \frac{4}{a^2}\right) \quad O(0,0)$

$s = d(OP) = \sqrt{(a-0)^2 + \left(\frac{4}{a^2} - 0\right)^2}$ M1

$s = \sqrt{a^2 + \frac{16}{a^4}} = \sqrt{\frac{16 + a^6}{a^4}}$ since $a > 0$

$s = \frac{1}{a^2}\sqrt{16 + a^6}$ A1

b.i. $\frac{ds}{da} = \frac{a^6 - 32}{a^3\sqrt{16 + a^6}} = 0$ for minimum distance A1

$a = \sqrt[6]{32} = 2^{\frac{5}{6}}$ A1

if $a > 2^{\frac{5}{6}}$ consider $a = 1.8 \quad \frac{ds}{da} = 0.049 > 0$

and if $a < 2^{\frac{5}{6}}$ consider $a = 1.7 \quad \frac{ds}{da} = -0.25 < 0$

by the sign test it is a minimum. M1

ii. $S_{\min} = \sqrt[3]{2}.\sqrt{3}$ A1

c.i at the point $P\left(a, \frac{4}{a^2}\right) \quad f'(x) = -8x^{-3} \qquad m_T = -\frac{8}{a^3}$ A1

$m_N = \frac{a^3}{8}$

normal $y - \frac{4}{a^2} = \frac{a^3}{8}(x - a)$ or $y = \frac{a^3 x}{8} - \frac{a^4}{8} + \frac{4}{a^2}$ A1

ii. normal passes through origin $(0,0)$ then

$-\frac{a^4}{8} + \frac{4}{a^2} = 0$

$a^6 = 32$

$a = \sqrt[6]{32} = 2^{\frac{5}{6}}$ A1

Question 5

a. $\sqrt{3}\sin(2x)+\cos(2x)=0$

$x=\dfrac{(6k-1)\pi}{12}$ where $k\in Z$ A1

b. since the period of both are π, it follows that $g(x+\pi)=g(x)$

$T=\pi$ A1

c. $f:[0,2\pi]\to R$, $f(x)=\sqrt{3}\sin(2x)+\cos(2x)$

$\sqrt{3}\sin(2x)+\cos(2x)=0$

$\sqrt{3}\sin(2x)=-\cos(2x)$

$\tan(2x)=-\dfrac{1}{\sqrt{3}}$

$x=\dfrac{5\pi}{12}, \dfrac{11\pi}{12}, \dfrac{17\pi}{12}, \dfrac{23\pi}{12}$ A1

d.i. $f'(x)=2\sqrt{3}\cos(2x)-2\sin(2x)=0$ A1

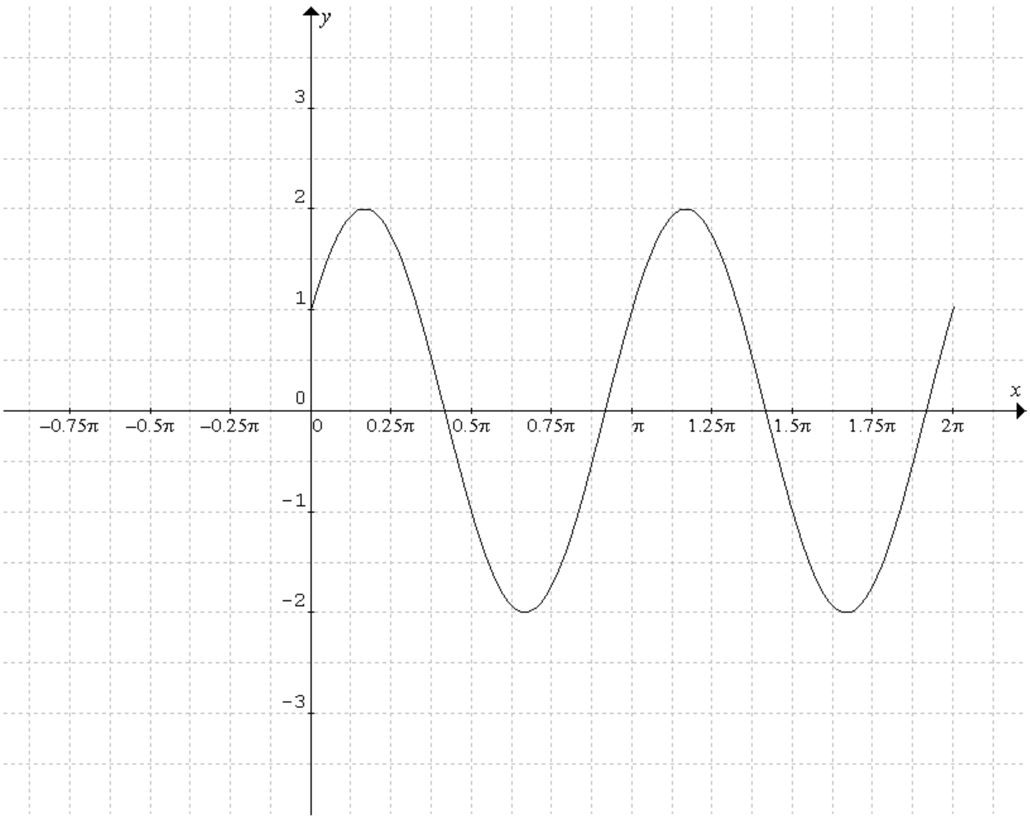
$\sqrt{3}\cos(2x)=\sin(2x)$

$\tan(2x)=\sqrt{3}$

$x=\dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{6}, \dfrac{5\pi}{3}$ A1

ii. max $\left(\dfrac{\pi}{6}, 2\right)$ and $\left(\dfrac{7\pi}{6}, 2\right)$, min $\left(\dfrac{2\pi}{3}, -2\right)$ and $\left(\dfrac{5\pi}{3}, -2\right)$ A1

e. graph on correct domain, correct *x*-intercepts G1
and correct max and min. G1



f. $f(x) = \sqrt{3}\sin(2x) + \cos(2x) = 2\sin\left(2x + \frac{\pi}{6}\right) = 2\sin\left(2\left(x + \frac{\pi}{12}\right)\right)$

translate $2\sin(2x)$, $\frac{\pi}{12}$ to the left parallel to the *x*-axis

$A = 2$ A1

$\alpha = \frac{\pi}{12}$ A1

END OF SECTION 2 SUGGESTED ANSWERS