

Year 2009

VCE

Mathematical Methods
and
Mathematical Methods

(CAS)

Solutions

Trial Examination 1



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Question 1

$y = \frac{\log_e(2x)}{2x^2}$ differentiating using the quotient rule

let $u = \log_e(2x)$ $v = 2x^2$

$\frac{du}{dx} = \frac{1}{x}$ $\frac{dv}{dx} = 4x$ M1

$\frac{dy}{dx} = \frac{2x^2 \times \frac{1}{x} - 4x \log_e(2x)}{4x^4} = \frac{2x - 4x \log_e(2x)}{4x^4}$

$\frac{dy}{dx} = \frac{1}{2x^3} (1 - 2 \log_e(2x))$ A1

Question 2

$\log_6(x+2) + \log_6(2x-2) = 2$

$\log_6((x+2)(2x-2)) = 2$

$2(x+2)(x-1) = 6^2 = 36$

$x^2 + x - 2 = 18$

$x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x = -5 \quad x = 4$

but $x > -2$ so there is only one answer $x = 4$ A1

Question 3

For the function to be differentiable it must be continuous and the gradients must match.

$f(2) = a\sqrt{4} = 2a = 2m + 3$ M1

$f'(x) = \begin{cases} \frac{a}{2\sqrt{x+2}} & x > 2 \\ m & x < 2 \end{cases} \Rightarrow f'(2) = \frac{a}{4} = m$ A1

$a = 4m \Rightarrow 8m = 2m + 3 \quad 6m = 3$

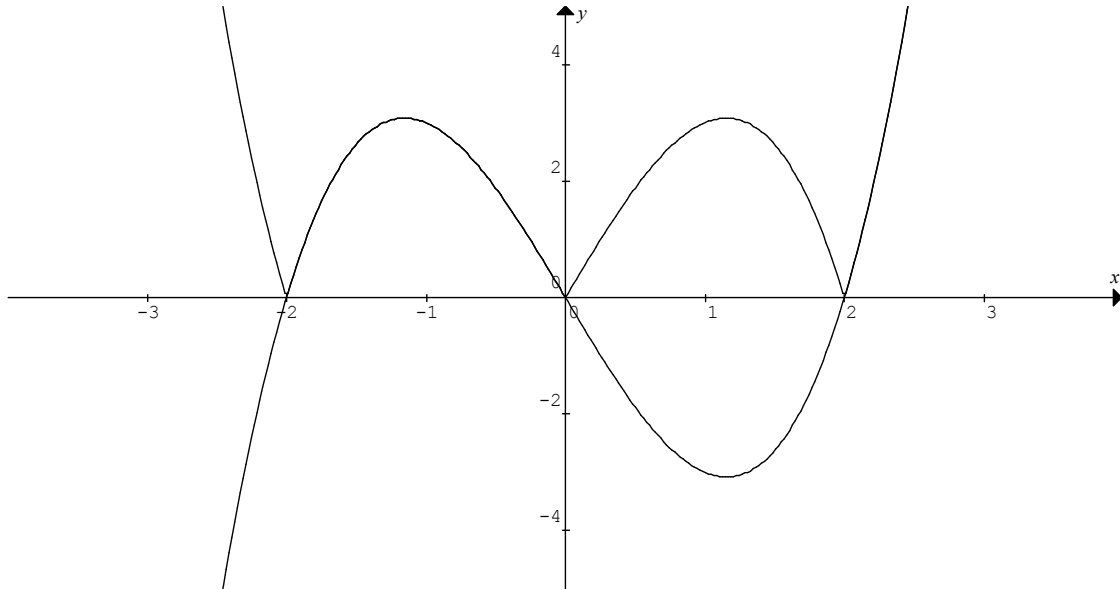
$m = \frac{1}{2}$ A1

$a = 2$ A1

Question 4

a.

G1



b. $A = 2 \int_0^2 (4x - x^3) dx$ by symmetry

$$A = 2 \left[2x^2 - \frac{x^4}{4} \right]_0^2$$

M1

$$A = 2[(8 - 4) - (0)]$$

$$= 8 \text{ units}^2$$

A1

Question 5

f $y = \frac{1}{3}(1 - e^{-2x})$ interchanging y and x

f^{-1} $x = \frac{1}{3}(1 - e^{-2y})$ transposing to make y the subject

M1

f^{-1} $3x = 1 - e^{-2y} \Rightarrow e^{-2y} = 1 - 3x \Rightarrow -2y = \log_e(1 - 3x)$

$$y = f^{-1}(x) = -\frac{1}{2} \log_e(1 - 3x) = \frac{1}{2} \log_e \left(\frac{1}{1 - 3x} \right)$$

A1

the domain of f^{-1} , needs to be stated as we are asked for a function

since $1 - 3x > 0 \Rightarrow x < \frac{1}{3}$ $\text{dom } f^{-1} = \left(-\infty, \frac{1}{3} \right)$

$f^{-1} : \{x : x < \frac{1}{3}\} \rightarrow R$, $f^{-1}(x) = -\frac{1}{2} \log_e(1 - 3x)$

A1

Question 6

- a. $V = \frac{\pi h}{45}(h^2 + 75h) = \frac{\pi}{45}(h^3 + 75h^2)$
 when $h = 1$ $\Delta h = 0.01$ find ΔV
 $\frac{dV}{dh} = \frac{\pi}{45}(3h^2 + 150h) = \frac{\pi}{15}(h^2 + 50h) \approx \frac{\Delta V}{\Delta h}$ M1
 $\Delta V \approx \frac{\pi}{15}(1 + 50) \times 0.01 = \frac{51\pi}{1500} \text{ cm}^3$
 the volume increases by $\frac{17\pi}{500} \text{ cm}^3$ A1
- b. Now given $\frac{dh}{dt} = 10 \text{ cm/min}$
 By the chain rule $\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} = \frac{10\pi}{15}(h^2 + 50h)$ M1
 when $h = 1$ $\left. \frac{dV}{dt} \right|_{h=1} = \frac{2\pi(1+50)}{3} = 34\pi \text{ cm}^3/\text{min}$ A1

Question 7

- a. $\int_0^{\pi} k \sin(3x) dx = 1$
 $-\frac{k}{3} [\cos(3x)]_0^{\pi} = 1$
 $-\frac{k}{3} (\cos(\pi) - \cos(0)) = \frac{2k}{3} = 1$ M1
 $k = \frac{3}{2}$
- b. $\frac{d}{dx}(x \cos(3x)) = \cos(3x) - 3x \sin(3x)$ A1
 $3 \int x \sin(3x) dx = \int \cos(3x) dx - x \cos(3x) = \frac{1}{3} \sin(3x) - x \cos(3x)$ M1
 Now $E(X) = \frac{3}{2} \int_0^{\pi} x \sin(3x) dx$ A1
 $E(X) = \frac{3}{2} \times \frac{1}{3} \left[\frac{1}{3} \sin(3x) - x \cos(3x) \right]_0^{\pi}$ M1
 $E(X) = \frac{1}{2} \left[\left(\frac{1}{3} \sin(\pi) - \frac{\pi}{3} \cos(\pi) \right) - \left(\frac{1}{3} \sin(0) - 0 \right) \right]$
 $E(X) = \frac{\pi}{6}$ A1

Question 8

X	1	2
$\Pr(X = x)$	$\sin(k)$	$2\sin^2(k)$

a. $\sum \Pr(X = x) = \sin(k) + 2\sin^2(k) = 1$
 $2\sin^2(k) + \sin(k) - 1 = 0$ M1
 $(2\sin(k) - 1)(\sin(k) + 1) = 0$
 $\sin(k) = -1$ not possible, since, probabilities must be positive A1
 and $\sin(k) = \frac{1}{2}$
 $k = \frac{\pi}{6}, \frac{5\pi}{6}$ A1

b. $E(X) = \sum x \Pr(X = x) = \sin(k) + 4\sin^2(k)$
 $E(X) = \frac{1}{2} + 4 \times \frac{1}{4}$
 $E(X) = 1.5$ A1

Question 9

a. $g(x) = \tan(x)$ and $h(x) = \frac{\sqrt{x}}{2}$ A1

b. let $f(x) = y = \tan(u)$ $u = \frac{\sqrt{x}}{2} = \frac{1}{2}x^{\frac{1}{2}}$ chain rule M1
 $\frac{dy}{du} = \frac{1}{\cos^2(u)}$ $\frac{du}{dx} = \frac{1}{4}x^{-\frac{1}{2}} = \frac{1}{4\sqrt{x}}$

$f'(x) = \frac{dy}{dx} = \frac{1}{4\sqrt{x} \cos^2\left(\frac{\sqrt{x}}{2}\right)}$ A1

$\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{6}} = f'\left(\frac{\pi^2}{9}\right) = \frac{1}{4 \times \frac{\pi}{3} \cos^2\left(\frac{\pi}{6}\right)} = \frac{3}{4\pi \times \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{3}{4\pi} \times \frac{4}{3}$

$f'\left(\frac{\pi^2}{9}\right) = \frac{1}{\pi}$ A1

Question 10

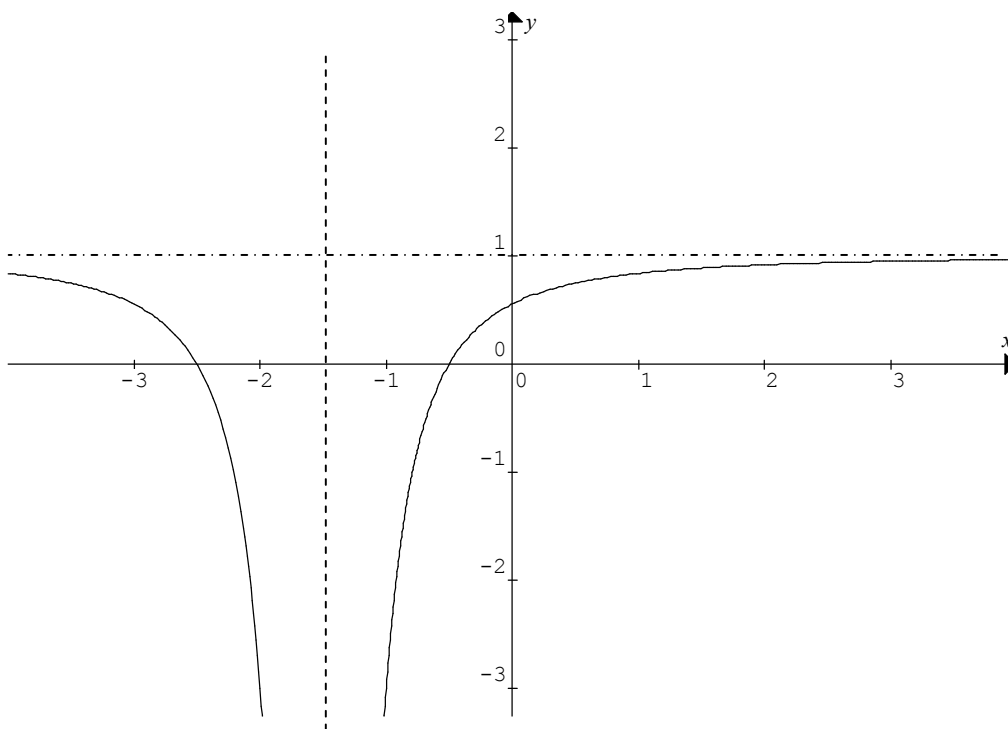
a. $x = -\frac{3}{2}$ is a vertical asymptote, $y = 1$ is a horizontal asymptote

the y-intercept is $y = 1 - \frac{4}{9} = \frac{5}{9}$ $\left(0, \frac{5}{9}\right)$ A1

crosses the x-axis, when $y = 0$, $(2x+3)^2 = 4$ $2x+3 = \pm 2$

$2x = -1, -5$ $x = -\frac{1}{2}, -\frac{5}{2}$ $\left(-\frac{1}{2}, 0\right)$ $\left(-\frac{5}{2}, 0\right)$

correct graph, correct axial intercepts and correct asymptotes G1



b. The area is $A = \int_0^1 \left(1 - \frac{4}{(2x+3)^2}\right) dx$ A1

$$A = \left[x + \frac{2}{2x+3} \right]_0^1 = \left(\left(1 + \frac{2}{5}\right) - \left(0 + \frac{2}{3}\right) \right)$$
 M1

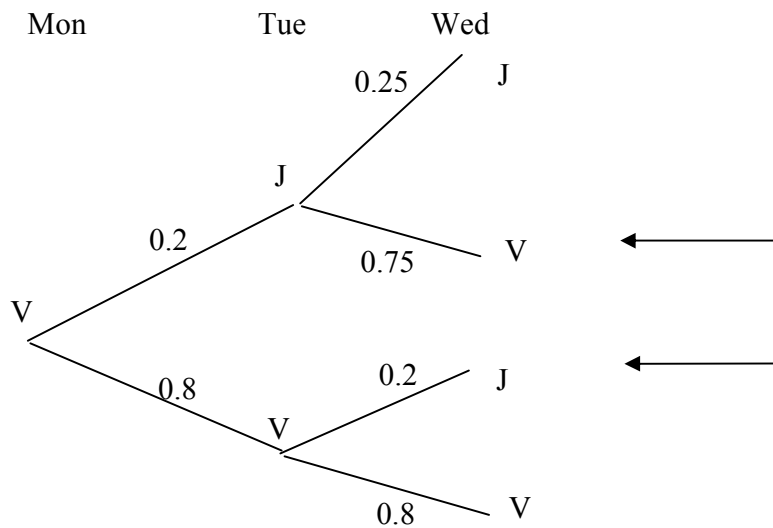
$$A = \frac{11}{15} \text{ units}^2$$
 A1

Question 11

Pr(Vegimite on two days)

M1

= VVJ or VJV



$$= 0.2 \times 0.75 + 0.8 \times 0.2$$

$$= \frac{1}{5} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{5} = \frac{3}{20} + \frac{4}{25} = \frac{15+16}{100}$$

A1

$$= 0.31$$

A1

END OF SUGGESTED SOLUTIONS