2008 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

There were 4106 students who sat the Mathematical Methods (CAS) examination in 2008. Marks ranged from 2 to 79 out of a possible score of 80. Student responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate what they knew.

There is evidence to suggest that the 2008 cohort performed better than the 2007 cohort. The mean for 2008 was 44, compared with 40 last year. The median score for the paper was 45 marks. Of the whole cohort, 12% of students scored 85% or more of the available marks, and 26% scored 75% or more of the available marks.

The mean score for the multiple-choice section was 13 (out of 22). Only five of the multiple-choice questions were distinctive from the Mathematical Methods paper, Questions 1, 3, 6, 9 and 12. Questions 6, 9 and 12 were answered correctly by less than 46% of students.

As stated in the instructions, students must show appropriate working for questions worth more than one mark. This was poorly done by some students, especially in Question 1. Writing out the expression to be evaluated or equation to be solved is considered sufficient working. Students should be encouraged to attempt to write out an expression or equation, even if they think their answer to a previous question is incorrect, because marks can be awarded; for example, in Questions 2aii. and 2cii.

Students must ensure they read questions carefully so that they give the solutions over the required interval. Errors relating to this were made in Questions 2aii., 2bii., 2d., 4b. and 4dii.

Correct mathematical notation should always be used and is expected. Calculator syntax should not be used, as occurred in Questions 1biii., 1cii., 1ciii., 4b., 4di. and 4dii.

In Questions 2 and 4dii., for example, answers could easily have been checked with CAS. Students can solve equations by hand but need to be careful not to make algebraic errors.

Students need to use the variables given in a question. Incorrect variables were used in Question 3g.

In Question 4 some students did not give answers in exact form where this was clearly specified in the question and then did not obtain the corresponding marks. Teachers should draw students’ attention to the April 2009 *VCAA Bulletin* article on the use of exact values. From 2010 it will be assumed that students will provide exact value answers unless specified otherwise.

Students should take more care when drawing graphs and use appropriate scales, make them clearly visible, use the correct domain and use a ruler for linear graphs. They should re-read the question to make sure they have carried out all of the instructions.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	7	3	73	3	13	0	
2	1	2	93	3	1	0	This was the best answered question on the paper.
3	22	7	9	59	2	0	Average value $= \frac{1}{b-a}\int_a^b f(x)dx$ $= \frac{1}{2-0}\int_0^2 \left(\log_e(3x+1)\right)dx$ $= \frac{7\log_e(7)-6}{6}$ 22% of the students found the **average rate of change**, not the **average value**.
4	49	3	34	8	6	0	$\int_1^3 \left(2f(x)-3\right)dx$ $= 2\int_1^3 f(x)dx - \int_1^3 3dx$ $= 2\times 5 - [3x]_1^3$ $= 10 - 9 + 3 = 4$ 34% of students did not find the antiderivative of 3. Option C was $\int_1^3 \left(2f(x)\right)dx - 3 = 7$.
5	7	8	10	6	68	1	
6	12	45	23	8	11	1	For infinitely many solutions the lines must have equivalent equations. $ax + 3y = 0$ $2x + (a+1)y = 0$ Since the *y*-intercept for both equations is the same, i.e. zero, then the values of *a* can be found by letting the determinant of the corresponding matrix equal to zero and solving for *a*, without checking the solutions for parallel lines. $\begin{vmatrix} a & 3 \\ 2 & a+1 \end{vmatrix} = 0$ Hence $a = -3$ or $a = 2$. Alternatively, using multiples of coefficients and equations leads to $a(a+1) = 6$. Hence $a = {}^-3$ or $a = 2$. 23% students found the values *a* for which there was a unique solution.
7	2	3	4	89	2	0	
8	4	2	2	86	6	0	

9	38	34	11	4	12	1	Let (x', y') be the image of (x, y) under the transformation. $x' = 4x + 1$ $x = \frac{x'-1}{4}$ $y' = -2y + 3$ $y = \frac{y'-3}{-2}$ Hence, $\frac{y'-3}{-2} = \left(\frac{x'-1}{4}\right)^3$ $y' = \frac{-(x' - 1)^3}{32} + 3$
10	5	13	45	4	34	0	The period is π. Hence the maximum occurs at the turning point where $x = \frac{\pi}{4}$. $f\left(\frac{\pi}{8}\right) = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$ $f\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{2}\right) = 2$ Hence the range is $\left[\sqrt{2}, 2\right]$. 34% of students assumed the maximum occurred close to the endpoint $\left(\frac{\pi}{3}, \sqrt{3}\right)$, and gave the range as $\left[\sqrt{2}, \sqrt{3}\right)$.
11	4	9	6	76	5	0	
12	15	16	44	16	8	1	$f(2u) = e^{2u} + e^{-2u}$ $\left(f(u)\right)^2 = (e^u + e^{-u})^2 = e^{2u} + 2 + e^{-2u}$ Hence, $f(2u) = (f(u))^2 - 2$.
13	6	17	67	5	6	0	
14	9	8	14	60	9	1	
15	7	62	17	7	7	1	For independent events $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ Let $A = \{1, 2\}$ and $B = \{2, 4, 6\}$ $\Pr(A) = \frac{1}{3}$ and $\Pr(B) = \frac{1}{2}$ $\Pr(A \cap B) = \frac{1}{6}$ $\Pr(A) \times \Pr(B) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ Hence $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ 62% of students chose option B, which contained mutually exclusive events.

16	10	15	52	8	14	1	
17	1	89	4	3	3	0	
18	13	10	31	9	36	0	$f(x) = \sin(4x) + 1$ Reflection in the *x*-axis $h(x) = -\sin(4x) - 1$ Dilation by a factor of 4 from the *y*-axis $g(x) = -\sin(x) - 1$ **Note** the dilation by a factor of 4 from the *y*-axis affects the domain, $\left[0 \times 4, \frac{\pi}{2} \times 4\right] = [0, 2\pi]$
19	25	50	4	3	18	0	
20	18	50	13	8	10	0	
21	3	7	8	75	6	0	
22	9	11	53	16	11	0	

Section 2

Question 1

1ai.

Marks	0	1	Average
%	32	68	0.7

$\Pr(X = 8) = 0.8^8 = 0.1678$, correct to four decimal places

This question was well done. Some students gave the answer correct to only three decimal places and others rounded incorrectly, giving 0.1677 as the answer.

1aii.

Marks	0	1	2	Average
%	24	14	61	1.4

$X \sim \text{Bi}(8, 0.8)$, $\Pr(X = 6) = {}^8C_6\, 0.8^6 0.2^2 = 0.2936$, correct to four decimal places

This question was generally answered well. Some students gave the answer without showing any working. For questions worth more than one mark, appropriate working must be shown.

1bi.

Marks	0	1	Average
%	37	63	0.7

$0.84^7 = 0.2951$, correct to four decimal places

Many students tried to use the transition matrix.

1bii.

Marks	0	1	2	3	Average
%	38	7	9	46	1.7

$\Pr(\text{GGN}) + \Pr(\text{GNG}) + \Pr(\text{NGG}) = 0.84 \times 0.84 \times 0.16 + 0.84 \times 0.16 \times 0.64 + 0.16 \times 0.64 \times 0.84 = 0.2849$, correct to four decimal places

Many students did not know or realise that there were three cases, while others did not show adequate working. Some students used 0.36 as one of the probabilities.

1biii.

Marks	0	1	2	3	Average
%	40	17	26	17	1.3

$\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}^7 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0.80000 \\ 0.199997 \end{bmatrix}$, 0.8000, correct to four decimal places

Many students evaluated $\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}^8 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Some students gave 0.8 as the answer. Most students had the correct transition matrix. Correct mathematical notation must be used and is expected. Students should not use calculator syntax in their responses; for example, [0.84, 0.64; 0.16, 0.36]^8*[1; 0].

1biv.

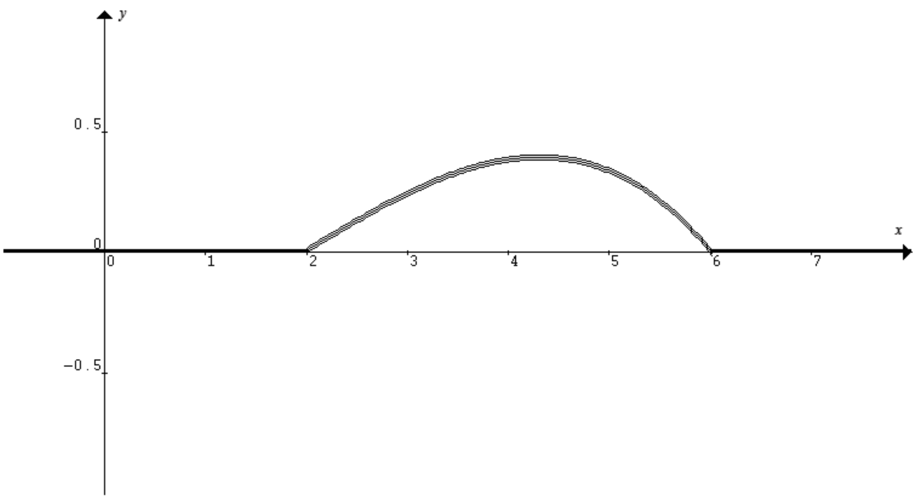
Marks	0	1	Average
%	54	46	0.5

$\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix} \begin{bmatrix} x \\ 1-x \end{bmatrix} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$, $x = 0.8, 80\%$

Some students gave 0.8 as the answer when a percentage was required. As this was a one mark question, working was not required.

1ci.

Marks	0	1	2	Average
%	20	38	42	1.3



Maximum is (4.31, 0.38)

This question was done reasonably well. The local maximum had to be labelled with its coordinates. Some students did not draw the horizontal lines along the *x*-axis corresponding to the ‘elsewhere’ part of the domain of *f*, while others rounded incorrectly or gave their answers correct to only one decimal place when the question asked for two. The *x*-axis must be scaled.

1cii.

Marks	0	1	2	Average
%	23	8	69	1.6

$\int_2^3 f(x)\,dx = 0.1211$, correct to four decimal places

This question was answered well by many students. Some students worked out Pr(X = 3) instead of Pr(X < 3). Others simply gave the answer. Calculator syntax should not be used; for example, $\int(f(x),\ x,\ 2,\ 3)$ is to be written as $\int_2^3 f(x)\,dx$.

1ciii.

Marks	0	1	2	Average
%	37	10	53	1.2

$\int_2^3 xf(x)dx = 4.1333$, correct to four decimal places

Some students worked out the median or the average value of the function. It was pleasing to see that many students had the correct notation, including the *dx*. Some students simply gave the answer without showing any working.

Question 2

2ai.

Marks	0	1	Average
%	41	59	0.6

$$\frac{f(a)-f(1)}{a-1} = \frac{\frac{7}{a}-7}{a-1} = -\frac{7}{a}$$

Some students used $\frac{f(a)-f(1)}{1-a}$ and others left the gradient in the form $\frac{f(a)-f(1)}{a-1}$. Some students made errors when simplifying their answer; this could easily have been checked with CAS.

2aii.

Marks	0	1	2	Average
%	47	17	37	1

$\frac{dy}{dx} = -\frac{7}{x^2} = -\frac{7}{a}$, $x = \sqrt{a}$, as $x > 1$

Many students did not attempt this question. Some students also wrote $x = -\sqrt{a}$ as a solution. Some students made algebraic errors when solving the equation by hand, while others found the antiderivative.

2bi.

Marks	0	1	Average
%	18	82	0.9

$\int_1^e f(x)dx = 7$

This question was answered extremely well. Some students left their answer as $7\log_e(e)$ or 7ln(e).

2bii.

Marks	0	1	2	Average
%	18	18	64	1.5

$\int_b^1 \left(\frac{7}{x}\right)dx = 7,\ b = e^{-1}$

Some students simply wrote down the answer. Others also gave $b = -e^{-1}$ as a solution. The question stated that *b* was positive.

2ci.

Marks	0	1	2	Average
%	63	10	27	0.7

$$Area = A_{\text{trapezium}} = \frac{1}{2}(7+\frac{7}{a})(a-1) = \frac{7a}{2} - \frac{7}{2a} = \frac{7(a^2-1)}{2a}$$

or

$$Area = A_{\text{triangle}} + A_{\text{rectangle}} = \frac{1}{2}(a-1)(7-\frac{7}{a}) + \frac{7}{a}(a-1)$$

or

$$Area = \int_1^a \left(-\frac{7}{a}x + \frac{7}{a} + 7\right)dx = \frac{7(a^2-1)}{2a}$$

This question was done poorly. Very few students used the first two methods. Many students tried unsuccessfully to find the equation of the line segment *CA* and others did not evaluate the integral, as the question asked for the answer in terms of *a*. Some students found the area under the curve of *f*, evaluating $\int_1^a f(x)dx$, while others found the area between the line segment *CA* and the curve of *f*.

2cii.

Marks	0	1	2	Average
%	41	33	26	0.9

$\frac{7(a^2-1)}{2a} = 7,\ a = \sqrt{2}+1$

Many students did not attempt this question. Students should be encouraged to write out the equation, even if they know their answer to the previous question is incorrect.

2ciii.

Marks	0	1	Average
%	89	11	0.1

The area under the curve is less than the area of the trapezium. Hence $\int_1^a f(x)dx < 7$. From **b.i.** $\int_1^e f(x)dx = 7$ but $\int_1^a f(x)dx < 7$, so $a < e$.

This was a difficult question and two statements were required to get one mark. Many students did not attempt to answer this question.

2d.

Marks	0	1	2	Average
%	59	32	9	0.6

$m = e^{\frac{5}{14}}$ and $n = e^{\frac{1}{14}}$ OR $m = -e^{\frac{5}{14}}$ and $n = -e^{\frac{1}{14}}$

Many students tried unsuccessfully to solve the simultaneous equations by hand. Other students gave only one solution, stating that m and n had to be positive but mn and $\frac{m}{n}$ had to be positive. Some students gave the second solution only, possibly because this was the first solution given on the calculator and they did not scroll across to get the second solution.

Question 3

3a.

Marks	0	1	2	Average
%	9	26	66	1.6

$100 = 50\log_e(1+2t)$, $t \approx 3.1945$ h = 192 minutes, to the nearest minute

This question was done quite well. Some students used 99 instead of 100, while others gave 3.19 minutes.

3b.

Marks	0	1	Average
%	49	51	0.6

18 km at 5 km/h = 3.6 h, 3.6 h > 3.1945 h, therefore he will not get the antidote in time.

Other methods were used to answer this question, including working out the concentration. Some students did not give a conclusion.

3c.

Marks	0	1	2	Average
%	40	9	51	1.2

Time(AM) = $\frac{\sqrt{(9+x^2)}}{5}$ = Time(NY); Time(MN) = $\frac{18-2x}{13}$

$$T = 2\left(\frac{\sqrt{(9+x^2)}}{5}\right) + \frac{18-2x}{13} = 2\left(\frac{\sqrt{9+x^2}}{5} + \frac{9-x}{13}\right)$$

Some students appeared to simply unpack the formula and did not relate their discussion to the diagram and/or the context.

3d.

Marks	0	1	2	Average
%	39	10	51	1.2

$\frac{dT}{dx} = 0$, $x = 1.25$ km

Some students simply gave the answer without showing any working, or $x = 1.25$ h, or used $\frac{dy}{dt} = 0$.

3e.

Marks	0	1	Average
%	67	33	**0.4**

When $x = 1.25$, $T \approx 2.492$, 2.492 h < 3.19 h. Therefore, he gets the antidote in time.

Many students did not give a conclusion. Some went further and calculated the concentration.

3f.

Marks	0	1	2	Average
%	23	34	43	**1.3**

$A = (0, 16)$ $C = (1, 24)$

Many students gave the correct coordinates for *A* but then gave the coordinates for *B*, not *C*. Some gave their answers as 16 and 24. Some had $A = (16, 0)$ or $[0, 16]$.

3g.

Marks	0	1	2	Average
%	65	13	22	**0.6**

$$z = \frac{16}{d} + 8$$

Some students used *y* and *x*, instead of *z* and *d* or wrote $CD = \frac{16}{d} + 8$ or just $\frac{16}{d} + 8$. Students should use the variables as given in the question. Some found the equation of the straight line *CD*. Many students did not attempt this question.

3h.

Marks	0	1	Average
%	84	16	**0.2**

6 days

Many students did not attempt this question. Five and seven days were very popular answers.

Question 4
4ai.

Marks	0	1	Average
%	25	75	**0.8**

$$f'\left(\frac{\pi}{2}\right) = 1$$

This question was answered quite well.

4aii.

Marks	0	1	2	Average
%	37	10	52	**1.2**

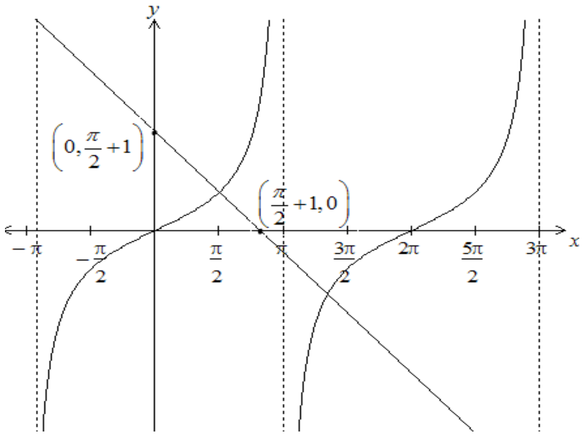
$$m = -1, \left(\frac{\pi}{2}, 1\right), \quad y = -x + \frac{\pi}{2} + 1$$

This question was generally well done. Some students did not give exact answers and some worked out the equation of the tangent rather than the normal.

4aiii.

Marks	0	1	2	3	Average
%	35	14	11	39	**1.6**

x-intercept is $1+\frac{\pi}{2}$, y-intercept is $1+\frac{\pi}{2}$



Once again, some students did not give the exact answers for the intercepts. Some students were not careful about where the normal should be drawn.

4b.

Marks	0	1	2	Average
%	58	12	30	**0.8**

$-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

This question was not done well. Many students gave a general solution to $\frac{d}{dx}\left(\tan\left(\frac{x}{2}\right)\right)=1$ using calculator syntax; for example, $x=\frac{(8en16+1)\pi}{2}$. Students should be familiar with representing general solutions using mathematical notation, involving a suitable parameter and the substitution of relevant values for the parameter to determine solutions within the required interval.

4c.

Marks	0	1	2	Average
%	50	13	37	**0.9**

$\tan\left(\frac{1-a}{2}\right)=1, \quad a=1-\frac{\pi}{2}$

Many students tried to solve $\tan\left(\frac{1}{2}-a\right)=1$.

4di.

Marks	0	1	Average
%	29	71	**0.8**

$h'(x)=\frac{1}{2}\cos\left(\frac{x}{2}\right)+\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

This question was done quite well. Some students wrote $\frac{\pi}{360}$ instead of $\frac{1}{2}$ and others wrote $h'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right) + \frac{1}{2}\sec\left(\frac{x}{2}\right)^2$.

4dii.

Marks	0	1	2	Average
%	45	14	40	1

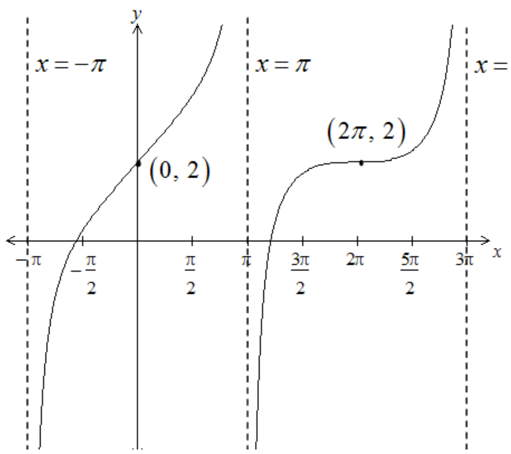
$h'(x) = 0, x = 2\pi$

Some students gave the general solution using calculator syntax.

4e.

Marks	0	1	2	Average
%	53	21	26	0.8

Stationary point at (2π, 2), correct shape, equations of the asymptotes and correct y-intercept



It appeared that some students did not take enough care when drawing this graph or possibly ran out of time. Many did not show the stationary point. Some students did not give the equations of the asymptotes or they wrote y = '…' instead of x = '…'. Some sketched the graph of the derivative. The asymptotic behaviour was shown better this year than in 2007.