Student Name: _____

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2

2008 Trial Examination

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*	*Suggested times (minutes)*
1	22	22	22	33
2	5	5	58	87
			Total 80	

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) or, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 23 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and on the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 – Multiple-choice questions

Instructions for Section 1

Answer **all** questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

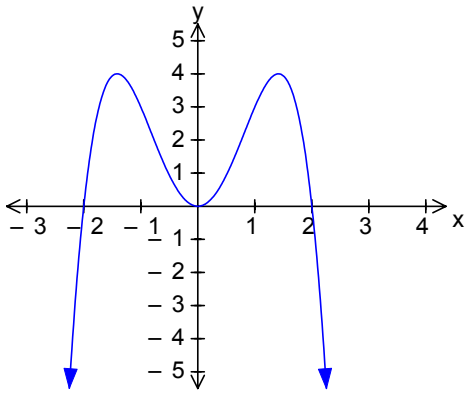
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The graph shown is given by the equation:



A. $y = x^2(x-2)(x+2)$

B. $y = x(x-2)(x+2)$

C. $y = x^4 - 4x^2$

D. $y = 4x^2 - x^4$

E. $y = -x\ (x-2)(x+2)$

Question 2

The range of the function $y = (x-3)^2(x+4), x \in (1,5]$ is given by:

A. (0, 50.81]

B. [0, 36]

C. R

D. [0,18)

E. (20, 36]

Question 3

Consider the equation $2\cos\left(\sqrt{3}x\right)+1=2$, $x \geq 0$. The seventh solution is given by:

A. $\frac{17\pi}{3}$

B. $\frac{19\sqrt{3}\pi}{9}$

C. $\frac{37\sqrt{3}\pi}{9}$

D. $\frac{19\pi}{3}$

E. $\frac{41\sqrt{3}\pi}{9}$

SECTION 1 – continued

Question 4

Consider this system of simultaneous linear equations:

$$\begin{aligned} 2a + \ b - 3c + \tfrac{1}{2}d - \tfrac{1}{5}e &= -14.24 \\ a - \ b + \ c - \ d + \ e &= \ 9.3 \\ a - 3b + 2c - \tfrac{3}{10}d + \tfrac{1}{10}e &= 12.39 \\ \tfrac{3}{10}a - 2b + 7c - \ d + \tfrac{1}{10}e &= 32.3 \\ a - 3b - 4c - 5d - 2e &= -3.3 \end{aligned}$$

The solution yields which of the following relationships between the unknowns:

A. $a = 12b$

B. $b = 2d$

C. $c = 2e$

D. $c = 39a$

E. $d = -0.5e$

Question 5

If $f\left(\sqrt{3}g(x-1)\right) = 81x^4 - 310.5x^3 + 333.32x^2 - 68.54x - 0.64$, then $f(x)$ and $g(x)$, are:

A. $f(x) = 27x^2 + 7x - 0.5$ and $g(x) = 3x^2 - 0.5x + \sqrt{2}$

B. $f(x) = 3x^2 - 7x - 0.5$ and $g(x) = 3x^2 + 0.25x - \sqrt{2}$

C. $f(x) = 3x^2 - 0.25x - \sqrt{2}$ and $g(x) = 27x^2 + 7x - 0.5$

D. $f(x) = 3x^2 - 7x - 0.5$ and $g(x) = 3x^2 - 0.25x + \sqrt{2}$

E. $f(x) = 3x^2 - 0.25x + \sqrt{2}$ and $g(x) = 3x^2 - 7x - 0.5$

Question 6

The graph of $y = e^{2x-4} - 2$ is obtained from the graph of $y = e^x$ by:

A. dilation by a factor of $\frac{1}{2}$ from the y axis, translation of 2 units right and 2 units down.

B. dilation by a factor of $\frac{1}{2}$ from the x axis, translation of 4 units right and 2 units down.

C. dilation by a factor of 2 from the x axis, translation of 2 units left and 2 units down.

D. dilation by a factor of 2 from the y axis, translation of 4 units right and 2 units down.

E. translation of 2 units right and 2 units down.

Question 7

The **exact** x intercept and y intercept of the graph with equation $y = \log_e(x+3) - 1$ are respectively:

A. $(-0.28, 0)$ & $(0, 0.99)$

B. $(e-3, 0)$ & $(0, \log_e(3)-1)$

C. $(-1, 0)$ & $(0, \log_e(3)-1)$

D. $(-2, 0)$ & $(0, -1)$

E. $(\log_e(3)-1, 0)$ & $(e-3, 0)$

Question 8

The function $f(x) = \dfrac{1}{(x-5)^2}$ will have an inverse function if:

A. $x \in R \setminus \{5\}$

B. $x \in R$

C. $x \in (-\infty, 5)$

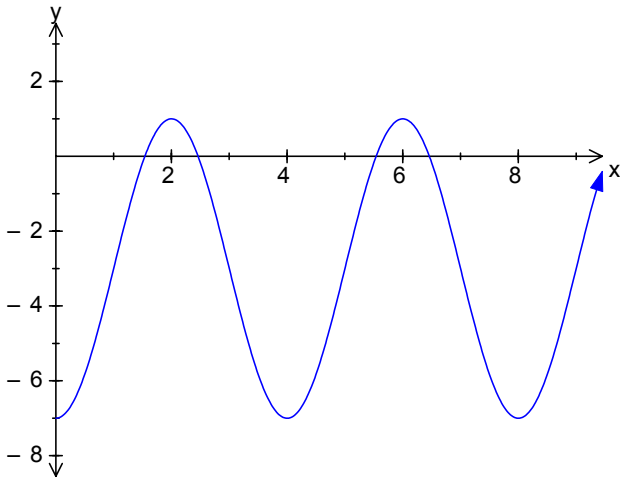
D. $x \in (-5, \infty)$

E. $x \in [0, \infty)$

Question 9

The diagram shows a graph of the form $y = a\cos(nx) + c$.

The values of a, n and c are respectively:



A. $8, \dfrac{\pi}{2}, -4$

B. $4, \dfrac{\pi}{2}, -3$

C. $8, \dfrac{2}{\pi}, -3$

D. $-4, \dfrac{\pi}{2}, -3$

E. $-4, \dfrac{2}{\pi}, -4$

SECTION 1 - continued

Question 10

The derivative of $f(x) = \frac{e^{4hx}}{3}\sqrt{x^3}$ is:

A. $\frac{e^{4hx}}{3}\left(4h\sqrt{3}x^2 + 2\sqrt{3}x\right)$

B. $\frac{e^{4x}}{3}\left(4\sqrt{3}x^2 + 2\sqrt{3}x\right)$

C. $\frac{e^{4x}x^{2\sqrt{3}}}{3}\left(\frac{2\sqrt{3}}{x} + 4\right)$

D. $e^{4h}\sqrt{3}x^2$

E. $\frac{e^{4hx}x^{\frac{1}{2}}}{3}\left(\frac{3}{2} + 4hx\right)$

Question 11

If $\frac{dy}{dx} = x^3 - 7x^2 + xe^x - 2$, then the average value of y over $\left[\frac{\pi}{2},\ \sqrt{7}\right]$ is closest to:

A. -4.906

B. -1.220

C. -5.143

D. -4.784

E. -3.721

Question 12

Let $f : R \to R$ be a differentiable function. If $f'(2) = 0$ and $f'(x) < 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$ and $f(2) = 6$, then which of the following is true?

A. (0, 6) is a point of inflection

B. (2, 0) is a local minimum

C. (2, 0) is a local maximum

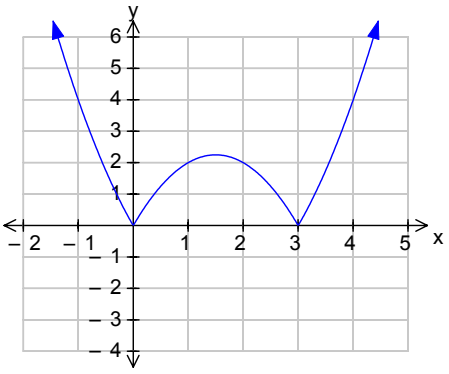
D. (2, 6) is a local maximum

E. (2, 6) is a local minimum

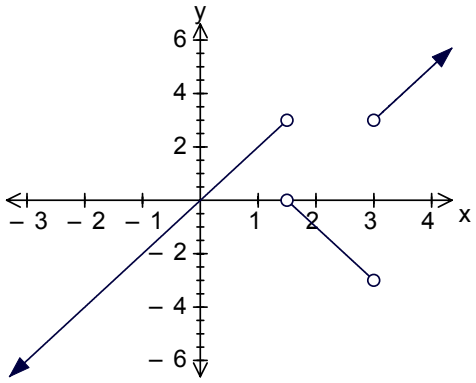
Question 13

The graph of $y = f(x)$ is shown.

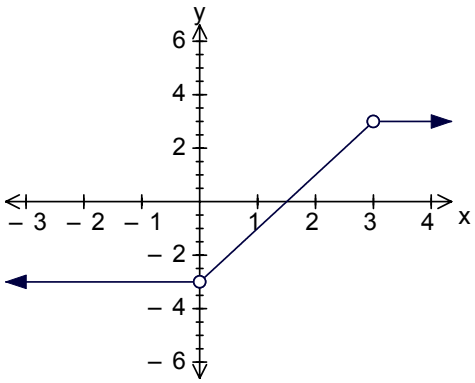
The graph of the gradient function $y = f'(x)$ is best represented by:



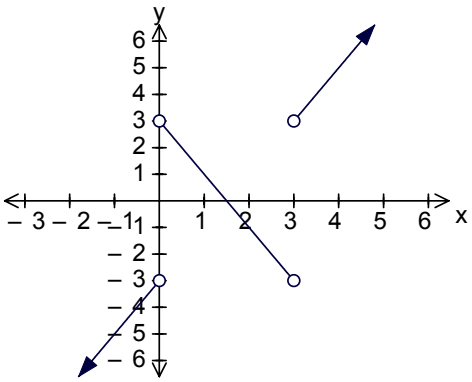
A.



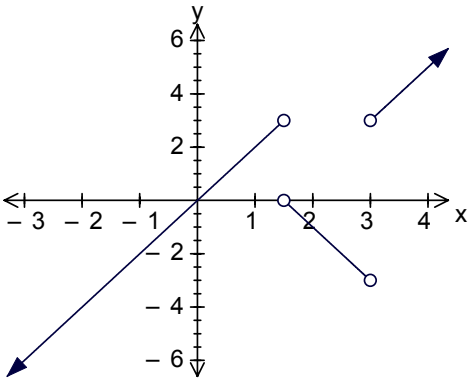
B.



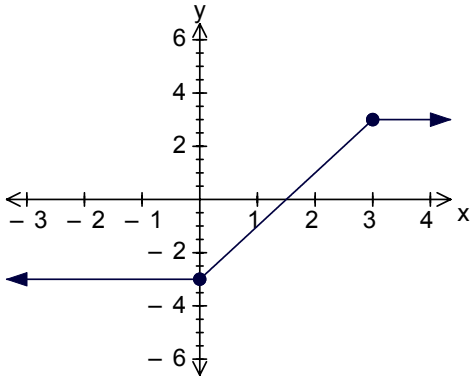
C.



D.



E.



SECTION 1 - continued

Question 14

If $f'(x) = \dfrac{6x+1}{2x-3}$ then $f(x)$ is equal to:

A. $3x + 5\log_e|2x-3| + c$

B. $3x + \dfrac{10}{(2x-3)^2} + c$

C. $-\dfrac{4}{3}x + c$

D. $\dfrac{3x^2 - x}{x^2 - 3x} + c$

E. $3x - 4\log_e|2x-3| + c$

Question 15

If $\dfrac{dV}{dt} = -3$ and $V = \dfrac{4}{3}\pi r^3$, then an expression for $\dfrac{dr}{dt}$ would be given by:

A. $-\dfrac{4\pi r^2}{3}$

B. $-12\pi r^2$

C. $-\dfrac{9}{4\pi r^3}$

D. $-4\pi r^3$

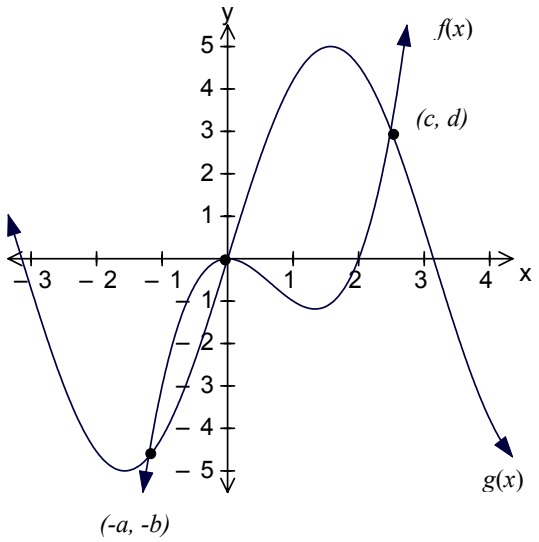
E. $-\dfrac{3}{4\pi r^2}$

SECTION 1 - continued
TURN OVER

Question 16

The graphs shown intersect at the points (*-a, -b*), (0, 0) and (*c, d*).

The total area enclosed by the graphs is given by:



A. $\int_{-b}^{0}[f(x)-g(x)]dx+\int_{0}^{d}[g(x)-f(x)]dx$

B. $\int_{-a}^{0}[f(x)-g(x)]dx+\int_{0}^{c}[g(x)-f(x)]dx$

C. $\int_{-a}^{c}f(x)dx-\int_{-a}^{c}g(x)dx$

D. $\int_{-b}^{0}[f(x)-g(x)]dx+\int_{0}^{d}[f(x)-g(x)]dx$

E. $\int_{-a}^{0}[f(x)-g(x)]dx+\int_{0}^{c}[f(x)-g(x)]dx$

Question 17

If Pr(A) = 0.6, Pr(A $\cup$ B) = 0.9 and Pr(B) = 0.65, then the value of Pr($A'\cap B'$) is equal to:

A. 0.35

B. 0.30

C. 0.40

D. 0.10

E. 0.25

Question 18

A binomial random variable has *E(X)* = 12 and *Var(X)* = 4. The values of *n* and *p*, respectively are:

A. 6, $\frac{1}{3}$

B. 18, $\frac{2}{3}$

C. 18, $\frac{1}{3}$

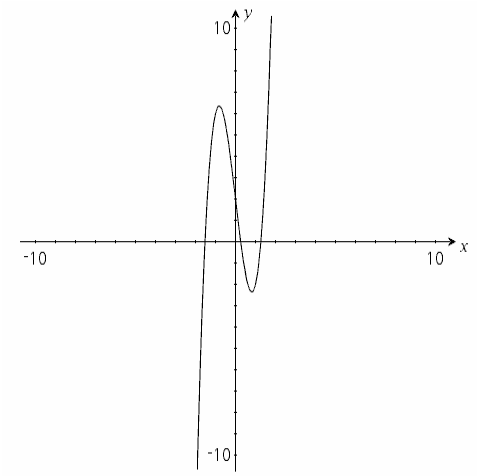
D. 4, $\frac{1}{3}$

E. 6, $\frac{2}{3}$

SECTION 1 - continued

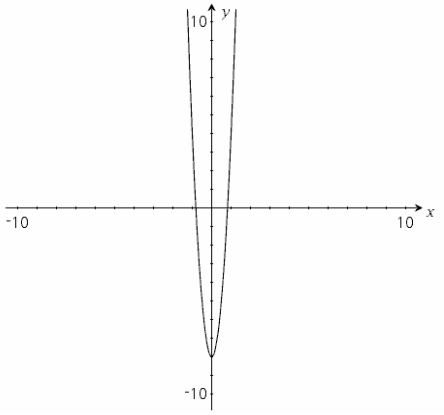
Question 19

Consider the following graph of $f'(x)$:

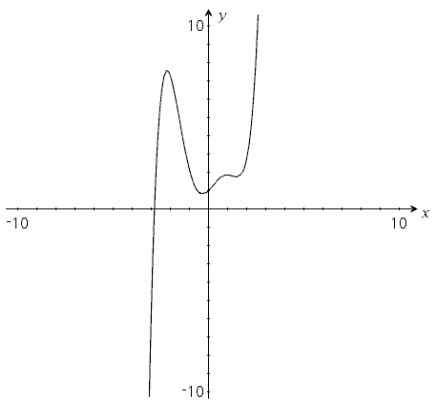


The graph of $f(x)$ is closest to:

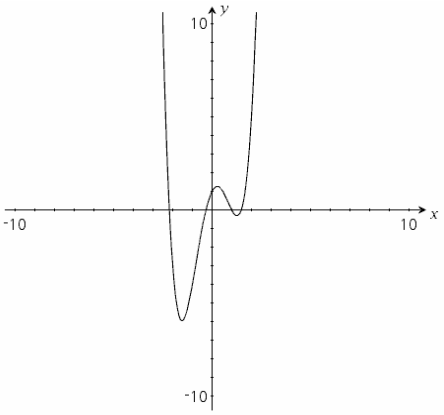
A.



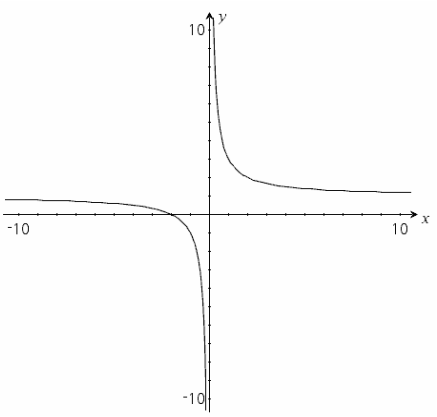
B.



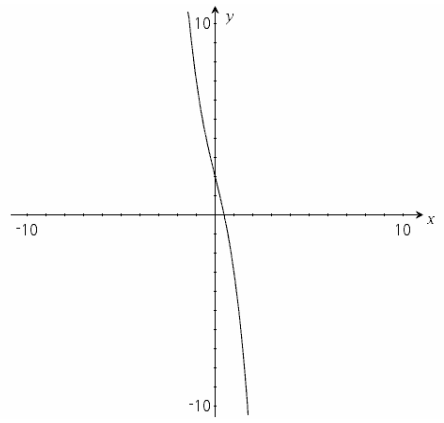
C.



D.



E.



SECTION 1 - continued

TURN OVER

Question 20

The probability of a defective toy is *k.* If 150 toys are tested, then the probability that 10% are defective is:

A. ${}^{150}C_{15}(k)^{15}$

B. ${}^{150}C_{10}(1-k)^{10}(k)^{140}$

C. ${}^{150}C_{15}(k)^{15}(1-k)^{135}$

D. ${}^{150}C_{15}(1-k)^{15}(k)^{135}$

E. ${}^{150}C_{10}(k)^{10}(1-k)^{140}$

Question 21

The probability distribution for a continuous random variable X is defined by the probability density function $f(x) = \begin{cases} \sqrt{2}\cos(x) & ,0 \le x \le \frac{\pi}{4} \\ 0 & ,elsewhere \end{cases}$

The variance of the distribution is closest to:

A. 0.0499

B. 0.1876

C. 0.1836

D. 0.1327

E. 0.3719

Question 22

The mass of new born Belgian Blue calves follow a normal distribution, with a mean of 52*kg* and a standard deviation of 2*kg*. The percentage of calves born over 55*kg* is closest to:

A. 6 %

B. 7 %

C. 93 %

D. 94 %

E. 8 %

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A certain National Park is completely bounded by two roads. The path of one of the roads is given by $g(x) = e^{0.1(x+10)} - 5$. The other road has axial intercepts (-5.550, 0), (0, 20), (3.097,0) and (17.453, 0). Assume North is pointing to the top of the page.

a. Determine the equation of $f(x)$ if $f(x)$ is in the form $y = ax^3 + bx^2 + cx + d$.

_____

_____

_____

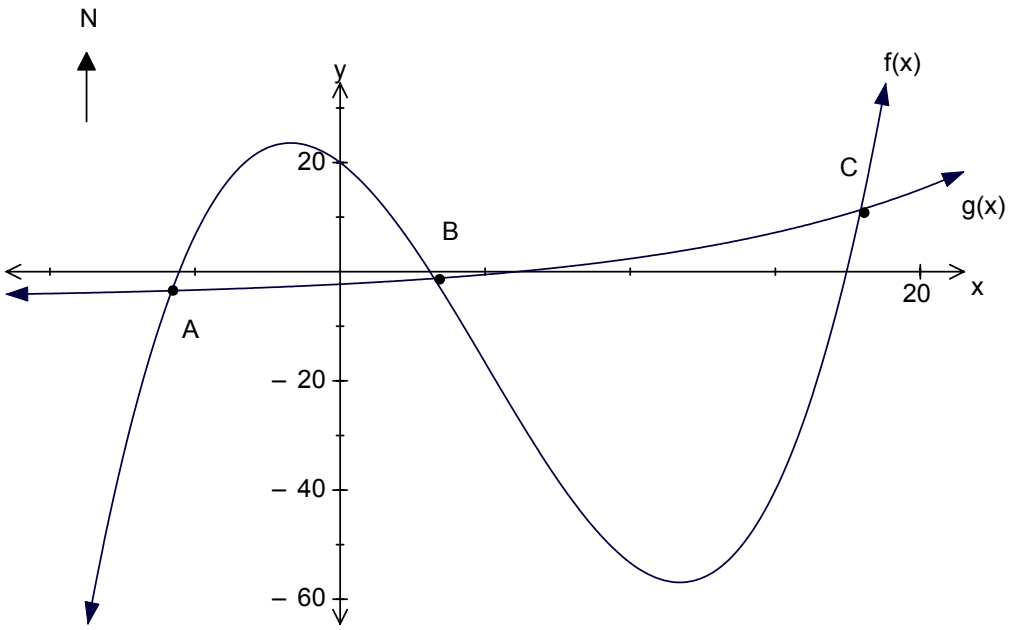
_____

_____

_____

3 marks

b. The graphs of $f(x)$ and $g(x)$ are shown below. Find the coordinates of A, B and C, the points of intersection between the graphs (correct to 3 decimal places).



3 marks

c. Use **calculus** to find the total area of the National Park correct to 3 decimal places. Assume all measurements are in kilometres.

3 marks

SECTION 2 – Question 1 – continued

d. Due to recent shortages of water, the government has decided to turn the area between points A and B into a large reservoir. If the reservoir is to have an average depth of 25 m, find the capacity of the reservoir to the nearest **megalitre**. ($1km^3 = 10^6 ML$)

_____

_____

_____

1 mark

e. The Bureau of Meteorology is predicting a very hot summer, so the National Parks ranger wants a fire break (road) built in the most heavily treed area. If the road is to follow a path approximated by the normal to $f(x)$ at $x = 5$, find the equation that models the path of the road.

_____

_____

_____

_____

_____

_____

3 marks

f. If the firebreak only runs between the two points where it connects the road modelled by the equation $f(x)$, find the length of the road correct to 2 decimal places. (Assume the road is flat).

_____

_____

_____

_____

_____

2 marks

Total 15 marks

SECTION 2 – continued

TURN OVER

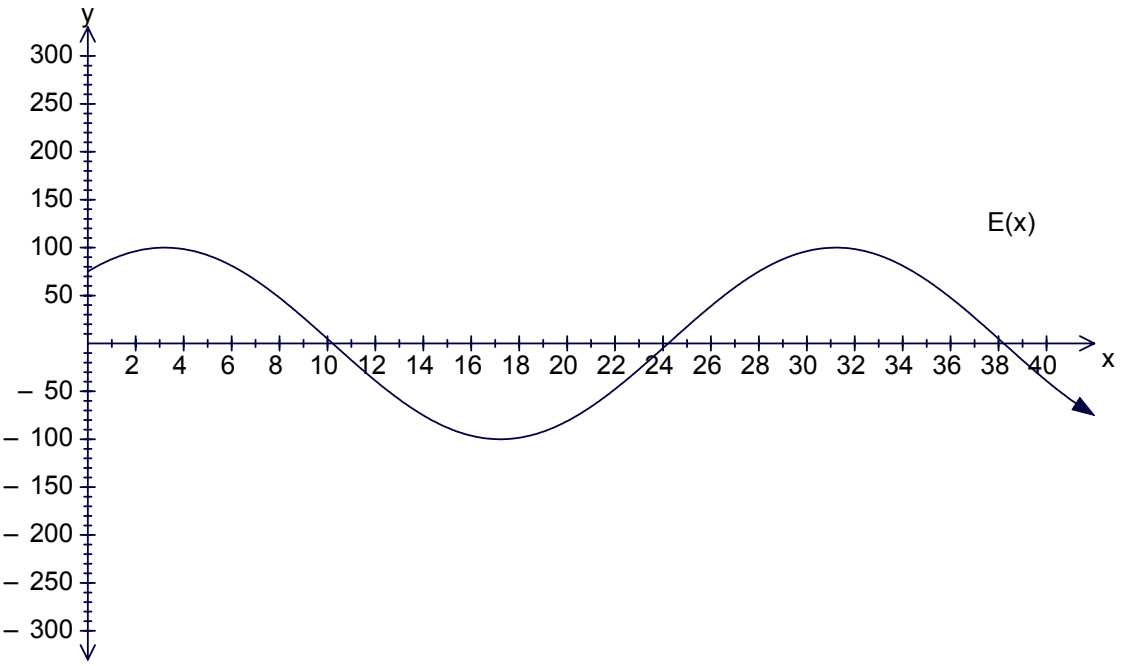
Question 2

Astrologists and numerologists believe we go through cycles called Biorhythms. There are three kinds of Biorhythms: Physical, Emotional and Intellectual.

High days (+100) are associated with doing well, but low days (−100) are associated with poor performance. Zero days are critical days and caution is needed.

Nancy has worked out her Biorhythms for the month of October. She wants to go for a job interview and decides to look at her Emotional and Intellectual cycles. Her Emotional cycle $E(x) = 100\sin\left(\frac{\pi x}{14} - 18\right)$ is shown below, where x is the number of days after September the 30^{th}.

a. If Nancy's Intellectual Biorhythm is given by $I(x) = 100\sin\left(\frac{2\pi x}{33} - 32\right)$ sketch this graph on the same set of axes with E(x).



2 marks

SECTION 2 – Question 2 – continued

b. Using Nancy's Intellectual Biorhythm, calculate the dates of her critical days, high days and her low days for October.

_____

_____

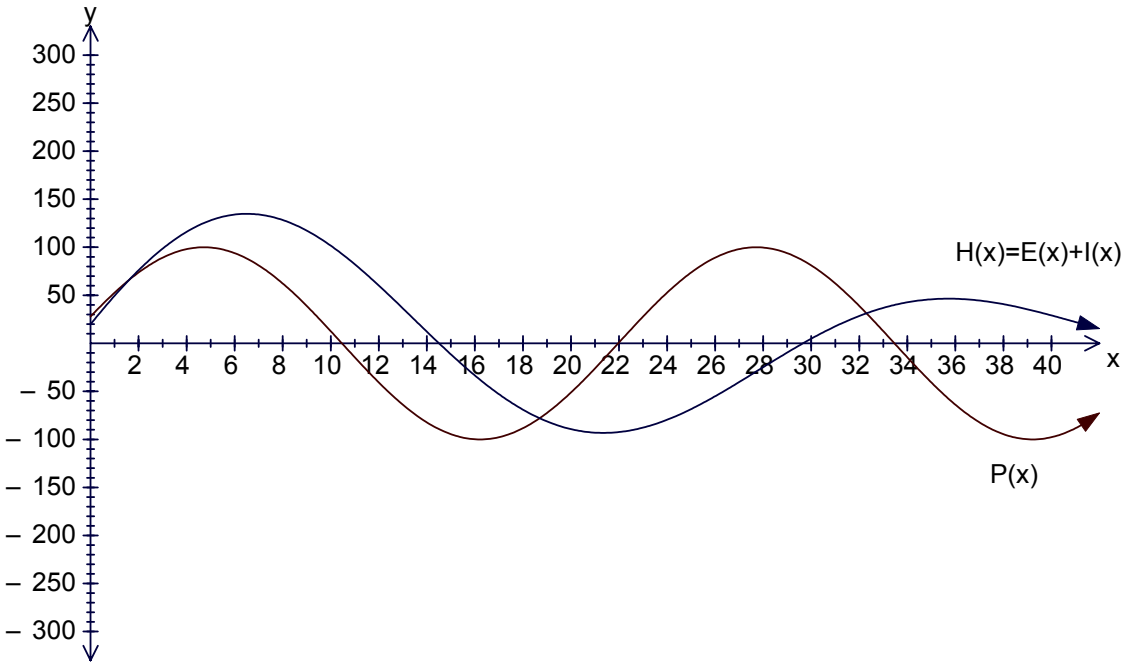
_____

_____

3 marks

c. Nancy decides that if she wants to have the best possible chance for her job interview, she needs to consider all three of her biorhythms. She decides to add all three graphs together to find her super high and super low days. Nancy's Emotional and Intellectual biorhythms have already been added and are shown below by H*(x)*.

Her Physical biorhythm $P(x) = 100\sin\left(\frac{2\pi x}{23} - 6\right)$ is also shown. Sketch her Super Biorhythm $S(x) = H(x) + P(x)$ on the same set of axes using addition of ordinates.



2 marks

SECTION 2 - Question 2– continued

TURN OVER

d. Using Nancy's new graph $S(x)$ what would be the best date and the worst date for a job interview?

_____

_____

_____

2 marks

e. Nancy decides that any day that is at least 105 on $S(x)$ would be suitable for a job interview. Between which dates does this occur?

_____

_____

_____

2 marks

Total 11 marks

SECTION 2 - Question 3– continued

Question 3

A certain radioactive substance called Raybon, decays according to the rule $R(t) = Ae^{-kt}$, where R is the amount of Raybon in grams and *t* is the time in days.

a. If there was 9.2500g of Raybon initially and 5.9724g of Raybon after 9 days, find the values of A and *k* correct to four decimal places.

3 marks

b. How much of the Raybon decays in the first hour (in grams to four decimal places)?

2 marks

c. The half-life of a substance is the time it takes for half of the compound to decay. Find the half-life of Raybon (in days to 4 decimal places).

2 marks

SECTION 2 – Question 3 - continued

TURN OVER

A chemist has 5.2700g of another substance called Decabon, that decays according to the rule $D(t) = D_0 e^{-0.0309t}$, where D is the amount of Decabon in grams, D_0 is the amount of Decabon present initially and t is the time in days. He gets the Decabon the same day that he gets the Raybon.

d.

i. When will the chemist have the same mass of Raybon and Decabon to the nearest hour?

_____

_____

_____

ii. How much of each will he have (in grams correct to 4 decimal places)?

_____

_____

1 + 1 = 2 marks
Total 9 marks

SECTION 2 - continued

Question 4

A new Confectionary company is running a competition with its best selling chocolate bar called Chocofull. They claim that one in eight Chocofull bars has a winning wrapper. The local milk bar buys Chocofull bars in boxes of 50.

a. Find the probability (correct to 4 decimal places) that the first three bars the milk bar sells from the box are not winners, but the next two bars are winners.

_____

_____

_____

2 marks

b. Find the probability (correct to 4 decimal places) that there is a least one winner in the box.

_____

_____

_____

_____

2 marks

c. Find the probability that there are no more than 10 winners in the box, given that there is at least one winner in the box.

_____

_____

_____

_____

3 marks

SECTION 2 – Question 4 - continued

TURN OVER

d. The weights of Chocofull bars are normally distributed with a standard deviation of 0.15g. If 75% of the bars have weights above 40g, show that the mean weight of the bars (correct to two decimal places) is 40.10g

_____

_____

_____

_____

_____

_____

3 marks

e. Find the probability that a Chocofull bar weighs less than 40g.

_____

_____

_____

_____

2 marks

Total 12 marks

SECTION 2 - continued

Question 5

If X is a continuous random variable with a probability density function given by

$$f(x) = \begin{cases} k(x^2+1) & , \quad 0 \le x \le 2 \\ 0 & , \quad \textit{elsewhere} \end{cases}$$

a. Use calculus to find the value of k.

_____

_____

_____

_____

2 marks

b. Find the median of X (correct to four decimal places)**.**

_____

_____

_____

_____

_____

3 marks

c. Find the mode of X.

_____

_____

_____

1 mark

SECTION 2 – Question 5 – continued
TURN OVER

d. Find the mean of X (correct to 4 decimal places).

2 marks

e. Find the probability that X is smaller than the mean (correct to 4 decimal places).

2 marks

f. Find the variance of X.

2 marks

SECTION 2 – Question 5 - continued

g. Find the standard deviation of X.

_____

_____

_____

_____

1 mark

Total 11 marks

END OF QUESTION AND ANSWER BOOK

