

# MATHEMATICAL METHODS (CAS)

## Units 3 & 4 – Written examination 1



### 2008 Trial Examination

### SOLUTIONS

#### Question 1

- a. Reflection in  $x$  axis, dilation from  $x$  axis by factor 4, translation right 3 units, translation down 2 units

A1

- b. Show asymptotes, intercepts, correct shape  
Asymptotes  $y = -2$ ,  $x = 3$ .

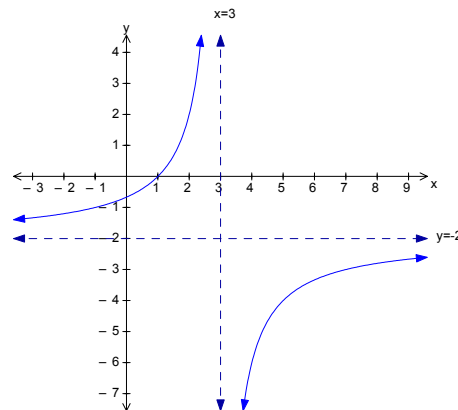
$$\text{Let } x = 0 \quad y = \frac{4}{3} - 2 = -\frac{2}{3}$$

$$2 = -\frac{4}{x-3}$$

$$\text{Let } y = 0, \quad 2x - 6 = -4$$

$$2x = 2$$

$$x = 1$$



A3

#### Question 2

$$(2^x)^2 - 2^x 2^3 + 2^4 = 0$$

$$\text{let } \dots 2^x = a$$

$$a^2 - 8a + 16 = 0$$

$$(a - 4)^2 = 0$$

$$a = 4$$

$$2^x = 4 \therefore x = 2$$

M2 + A1

**Question 3**

a.

$$x = 1 - 2e^{y-1}$$

$$x - 1 = -2e^{y-1}$$

$$1 - x = 2e^{y-1}$$

$$\frac{1-x}{2} = e^{y-1}$$

$$\log_e \left| \frac{1-x}{2} \right| = y - 1$$

$$\log_e \left| \frac{1-x}{2} \right| + 1 = y \dots \therefore f^{-1}(x) = \log_e \left| \frac{1-x}{2} \right| + 1$$

M3

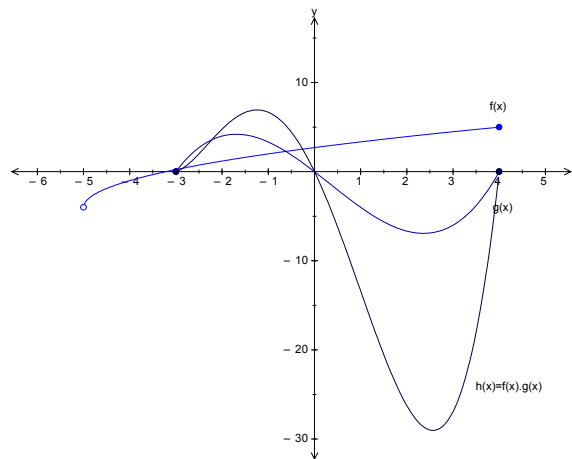
b. Range  $\mathbb{R}$ , domain  $(-\infty, 1)$

A1

**Question 4**

a. Show shape

$x$ -intercepts  $x = -3, 0, 4$



M1 + A1

b. Domain  $[-3, 4]$

A1

**Question 5**

a. 
$$f'(x) = \frac{24x(3x^2 - 2)^3 \cos(x) + \sin(x)(3x^2 - 2)^4}{\cos^2(x)}$$

This can be simplified considerably and re-expressed algebraically, but for one mark this answer is sufficient

A1

b. 
$$\frac{dy}{dx} = 2xe^{2x} + 2x^2e^{2x}$$

A1

c.  $\int (\frac{1}{2} f'(x) + 1) dx = \frac{1}{2} f(x) + x + c = \frac{1}{2} x^2 e^{2x} + x + c$

A1

**Question 6**

a.  $\frac{\sqrt{3} \sin(3x)}{\cos(3x)} = 1$  and  $-3\pi \leq x \leq 3\pi$

$\tan(3x)$  is positive in 1<sup>st</sup> and 3<sup>rd</sup> quadrants

$\tan(3x) = \frac{1}{\sqrt{3}}$

$3x = -\frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{17\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$

$x = -\frac{5\pi}{18}, -\frac{11\pi}{18}, -\frac{17\pi}{18}, \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}$

M2 + A1

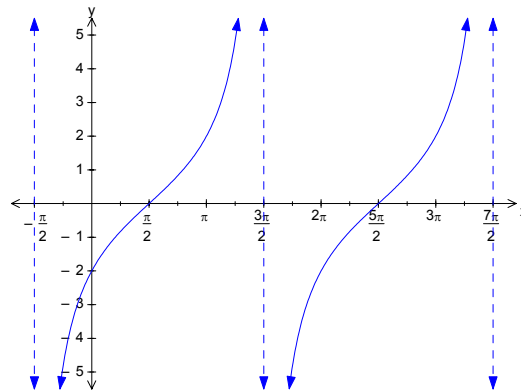
**Question 7**

a. period =  $\pi \div \frac{1}{2} = 2\pi$ , translated  $\frac{\pi}{2}$  right

asymptotes:  $x = \frac{\pi}{2} \pm \pi = -\frac{\pi}{2}, \frac{3\pi}{2}$

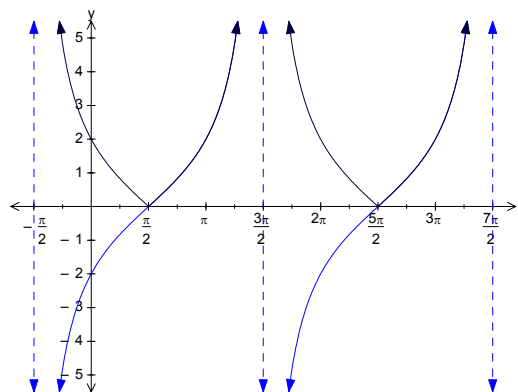
and..  $\frac{3\pi}{2} + 2\pi = \frac{7\pi}{2}$

let  $x = 0, y = 2 \tan(-\frac{\pi}{4}) = -2$



M3

b. correct shape and asymptotes



A1

**Question 8**

a.  $\frac{dy}{dx} = \frac{3(2)}{2x-3}$ , gradient at  $x = 3$   $m = \frac{6}{6-3} = 2$   
*sub.*  $x = 3, y = 3 \ln 3$   
 $y - 3 \ln 3 = 2(x - 3)$   
 $y = 2x + 3 \ln 3 - 6$

M2 + A1

**Question 9**

- a. Area can be calculated a few ways. Best method shown.  
 Area = area bounded by coordinate axes and  $x = 2$  and  $y = 9 - (x+1)^2$  - area bounded by curve,  $x$  axis and  $x = 0$  and  $x = 2$

$$\begin{aligned} A &= 18 - \int_0^2 (x+1)^2 dx \\ &= 18 - \int_0^2 x^2 + 2x + 1 dx \\ &= 18 - \left[ \frac{x^3}{3} + x^2 + x \right]_0^2 \\ &= 18 - \left[ \frac{8}{3} + 4 + 2 \right] \\ &= 9\frac{1}{3} \text{ sq. units} \end{aligned}$$

M2 + A1

**Question 10**

a.  
 $E(X) = 0 \times 0.2 + a + 2b + 3 \times 0.2 + 4 \times 0.2$   
 $2.1 = 1.4 + a + 2b$   
 $0.7 = a + 2b$   
 $a = 0.7 - 2b \dots \dots \dots (1)$   
 $0.2 + a + b + 0.2 + 0.2 = 1$   
 $a + b = 0.4 \dots \dots \dots (2)$   
*sub.* (1) *in* (2),  $0.7 - 2b + b = 0.4$   
 $\dots \dots \dots -b = -0.3$   
*sub.*  $b = 0.3$  *in* (1)  $a = 0.7 - 0.6$   
 $\therefore a = 0.1, b = 0.3$

M1 + A1

b.

$$E(2X + 1) = 2E(X) + 1$$

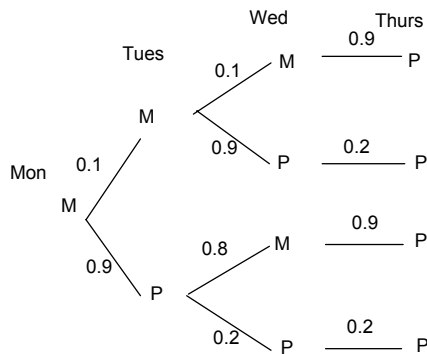
$$\dots\dots\dots = 2(2.1) + 1$$

$$\dots\dots\dots = 5.2$$

A1

**Question 11**

a.



$$\begin{aligned} \Pr(\text{Pellets.on.Thurs} | \text{Milk.on.Mon}) &= \frac{1 \times 1 \times 9}{1000} + \frac{1 \times 9 \times 2}{1000} + \frac{9 \times 8 \times 9}{1000} + \frac{9 \times 2 \times 2}{1000} \\ &= \frac{9 + 18 + 648 + 36}{1000} \\ &= \frac{711}{1000} \end{aligned}$$

M3 + A1

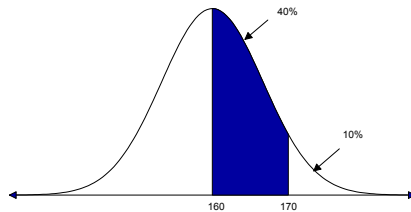
**Question 12**

a. For 95% data lies within  $\mu \pm 2\sigma$

$$\therefore 144 \leq x \leq 176 \text{cm}$$

A1

b.



$$\begin{aligned} \Pr(H < 170 | H > 160) &= \frac{\Pr(H < 170 \cap H > 160)}{\Pr(H > 160)} = \frac{\Pr(160 < H < 170)}{\Pr(H > 160)} \\ &= \frac{0.4}{0.5} = \frac{4}{5} \end{aligned}$$

M1 + A1