



Trial Examination 2008

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

$$\text{Let } u = \sin(x^2) \quad \therefore y = u^2$$

For use of chain rule M1

$$\frac{du}{dx} = ? \quad \frac{dy}{du} = 2u$$

$$\text{let } v = x^2 \quad \therefore u = \sin v$$

$$\frac{dv}{dx} = 2x \quad \frac{du}{dv} = \cos v$$

$$\begin{aligned} \therefore \frac{du}{dx} &= \frac{du}{dv} \times \frac{dv}{dx} \\ &= 2x \cos(x^2) \end{aligned}$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 2x \cos(x^2) \times 2 \sin(x^2) \\ &= 4x \cos(x^2) \sin(x^2) \end{aligned}$$

A1

Question 2

$$\begin{aligned} \text{a. } f(x) &= (x+3)^2(x-2) \\ &= (x^2 + 6x + 9)(x-2) \\ &= x^3 + 4x^2 - 3x - 18 \\ \therefore a &= -4 \end{aligned}$$

A1

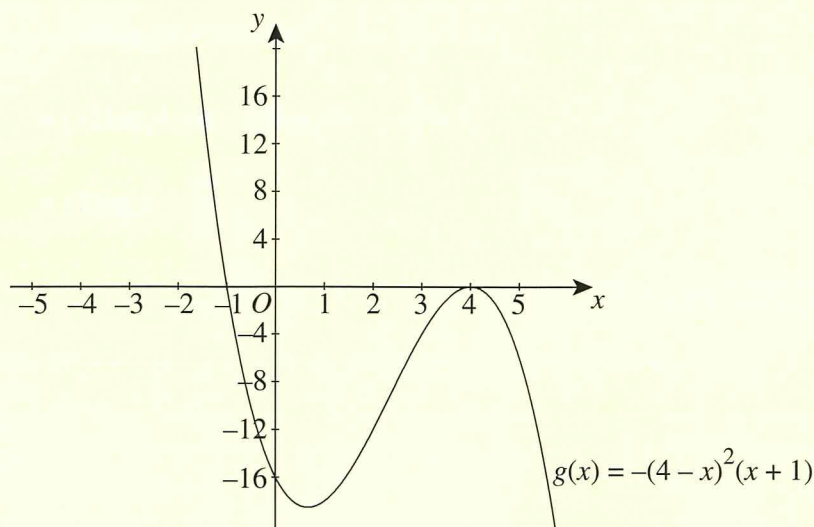
$$\text{Alternatively, } f'(x) = 3x^2 - 2ax - 3$$

$$= 0 \quad \text{if } x = -3$$

$$f'(-3) = 27 + 6a - 3 = 24 + 6a$$

$$= 0 \quad \text{if } a = -4$$

- b. The graph of $y = f(1-x) = f[-(x-1)]$ is obtained by reflecting $y = f(x)$ in the x -axis followed by a translation of 1 unit to the right.



M1

$$\therefore \{x: f(1-x) \geq 0\} = (-\infty, -1] \cup \{4\}$$

A1

Question 3

$$e^{-2x} - 6e^{-x} + 8 = 0$$

$$(e^{-x} - 2)(e^{-x} - 4) = 0$$

M1

$$e^{-x} = 2 \text{ or } e^{-x} = 4$$

$$\Rightarrow x = -\log_e 2 \text{ or } x = -\log_e 4$$

A1

Hence, the sum of the roots is $-\log_e(2) - \log_e(4) = -\log_e(8) = \log_e\left(\frac{1}{8}\right)$.

A1

Question 4

$$f'(x) = \frac{3\pi}{5} \cos\left(\frac{3\pi x}{5}\right)$$

$$f'(x)_{min} = -\frac{3\pi}{5} \text{ as minimum value of } \cos(\theta) \text{ is } -1$$

A1

$$\cos\left(\frac{3\pi x}{5}\right) = -1$$

$$\frac{3\pi x}{5} = \pi$$

$$x = \frac{5}{3}$$

A1

Question 5

a. $g(x) = -2\left[x^2 + 4x + \frac{11}{2}\right]$

$$= -2\left[(x^2 + 4x + 4) - 4 + \frac{11}{2}\right]$$

M1

$$= -2\left[(x+2)^2 + \frac{3}{2}\right]$$

$$= -2(x+2)^2 - 3$$

This is a parabola with a turning point at $(-2, -3)$.

A1

Alternatively, $g'(x) = -4x - 8$

$$= 0 \text{ if } x = -2$$

$$g(-2) = -8 + 16 - 11$$

$$= -3$$

so turning point at $(-2, -3)$

b. Dilate $f(x) = x^2$ by a factor of 2 parallel to the y-axis to get $f_1(x) = 2x^2$.

A1

Reflect $f_1(x)$ in the x-axis to get $f_2(x) = -2x^2$.

Translate $f_2(x)$ 2 units to the left and 3 units down to get $g(x)$.

A1

Note that there are other possible correct solutions.

Question 6

$$\begin{aligned} \text{a. } f\left(g\left(\frac{\pi}{3}\right)\right) &= \tan\left(2\left(\frac{\pi}{3} + \frac{\pi}{2}\right)\right) \\ &= \tan\left(\frac{5\pi}{3}\right) \\ &= -\tan\left(\frac{\pi}{3}\right) \\ &= -\sqrt{3} \end{aligned}$$

A1

$$\text{b. } f(g(x)) = -1 \Rightarrow \tan\left(2\left(x + \frac{\pi}{2}\right)\right) = -1$$

$$2x + \pi = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 3\pi - \frac{\pi}{4}, 4\pi - \frac{\pi}{4}$$

$$2x = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$$

M1

$$x = -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}$$

$$\text{As } x \in [0, 2\pi], x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$$

A1

Question 7

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

M1

$$V = 2\pi^2(30r^2 + r^3)$$

$$\frac{dV}{dr} = 2\pi^2(60r + 3r^2)$$

$$\therefore \frac{dr}{dt} = \frac{1}{2\pi^2(60r + 3r^2)} \times 20$$

A1

$$= \frac{1}{2\pi^2(60(2) + 3(2)^2)} \times 20, \text{ when } r = 2$$

$$= \frac{5}{66\pi^2} \text{ cm/s}$$

A1

Question 8

$$\int 2x \log_e(2x) + x \, dx = x^2 \log_e(2x)$$

$$\int 2x \log_e(2x) \, dx = x^2 \log_e(2x) - \int x \, dx$$

M1

$$\int x \log_e(2x) \, dx = \frac{1}{2} \left(x^2 \log_e(2x) - \frac{1}{2} x^2 \right)$$

A1

Question 9

- a. By appropriate substitution we obtain the probability values:

$$\Pr(X=0) = -4a \quad \Pr(X=1) = -9a \quad \Pr(X=2) = -10a \quad \Pr(X=3) = -7a \quad \text{M1}$$

As $\sum \Pr(X=x) = 1$ we have $-4a - 9a - 10a - 7a = 1$

$$a = -\frac{1}{30} \quad \text{A1}$$

- b. As $E(X) = \sum \Pr(X=x)$ we have

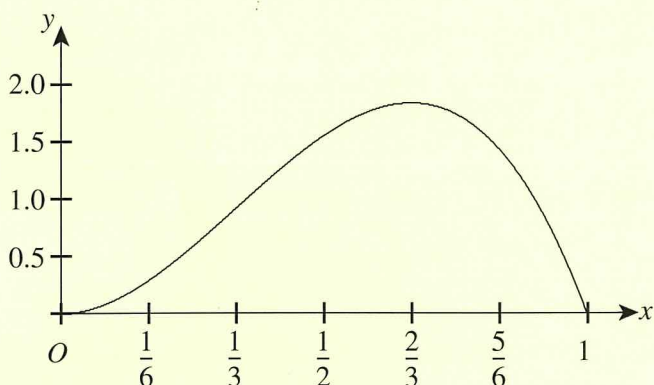
$$E(X) = 0 \times -4a + 1 \times -9a + 2 \times -10a + 3 \times -7a = -50a = -50 \times -\frac{1}{30} = \frac{5}{3} \quad \text{M1, A1}$$

So we require $\Pr\left(X > \frac{5}{3}\right)$

$$= \Pr(X \geq 2) = -17a = \frac{17}{30} \quad \text{A1}$$

Question 10

- a.



A1

- b. $\Pr\left(\frac{1}{2} \leq X \leq 1\right)$

$$= \int_{\frac{1}{2}}^1 (12x^2 - 12x^3) dx \quad \text{M1}$$

$$= [4x^3 - 3x^4]_{\frac{1}{2}}^1$$

$$= (4 - 3) - \left(\frac{4}{8} - \frac{3}{16}\right)$$

$$= 1 - \frac{5}{16} \quad \text{A1}$$

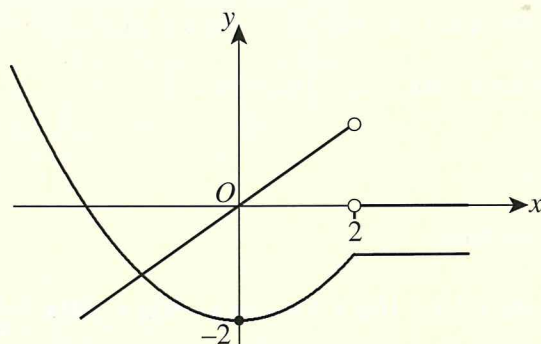
$$= \frac{11}{16}$$

- c. The median is the value of X such that $\Pr(X \leq m) = \frac{1}{2}$

$$\text{Thus } \int_0^m 12x^2(1-x) dx = \frac{1}{2} \quad \text{A1}$$

Question 11

a.



Parabolic with turning point $(0, -2)$ A1
 Right-hand end of parabola could end above x -axis

Continuous at $x = 2$ A1

Horizontal line for $x > 2$ A1

b. range of $f(x)$; $y \in [-2, \infty)$

A1

Question 12

$$\begin{aligned} \text{a. } f(0) &= \frac{12}{(0-2)} - 3 \\ &= 3 \end{aligned}$$

$$f'(x) = -12(x-2)^{-2}$$

M1

$$\begin{aligned} f'(0) &= -\frac{12}{(0-2)^2} \\ &= -3 \end{aligned}$$

\therefore The equation of the tangent is $y = -3x + 3$

A1

$$\text{b. Area} = \int_0^2 \left(\frac{12}{x+2} - 3x \right) dx - \int_0^1 (-3x + 3) dx$$

Integration of two regions M1

$$= [12 \log_e(x+2) - 3x]_0^2 - \left[-\frac{3x^2}{2} + 3x \right]_0^1$$

M1

$$= (12 \log_e 4 - 6 - 12 \log_e 2) - \left(-\frac{3}{2} + 3 \right)$$

$$= 12 \log_e 2 - \frac{15}{2}$$

A1