

**Year 2008**

**VCE**

**Mathematical Methods**  
**and**  
**Mathematical Methods**  
**( CAS )**

**Trial Examination 1**



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**Instructions**

Answer **all** questions in the spaces provided.  
A decimal approximation will not be accepted if an **exact** answer is required to a question.  
In questions where more than one mark is available, appropriate working must be shown.  
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

Let  $f(x) = \frac{\sin(3x)}{3x^2}$ . Find  $f'(x)$ .

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2 marks

**Question 2**

Solve the equation  $\tan^2(x) + (1 - \sqrt{3})\tan(x) = \sqrt{3}$  for  $x \in [0, 2\pi]$ , giving exact values in terms of  $\pi$ .

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3 marks

**Question 3**

Given the function  $f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ \cos(x) & \text{for } 0 \leq x < \pi \\ \pi - x & \text{for } x \geq \pi \end{cases}$

- a. State the gradient function,  $f'(x)$  clearly indicating its domain.

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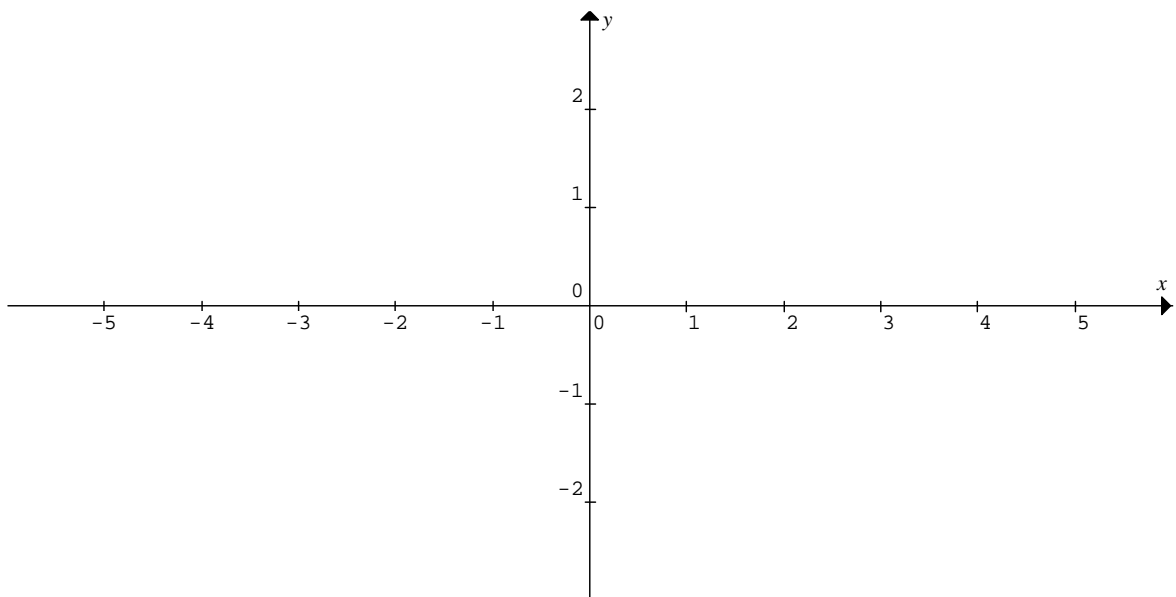
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2 marks

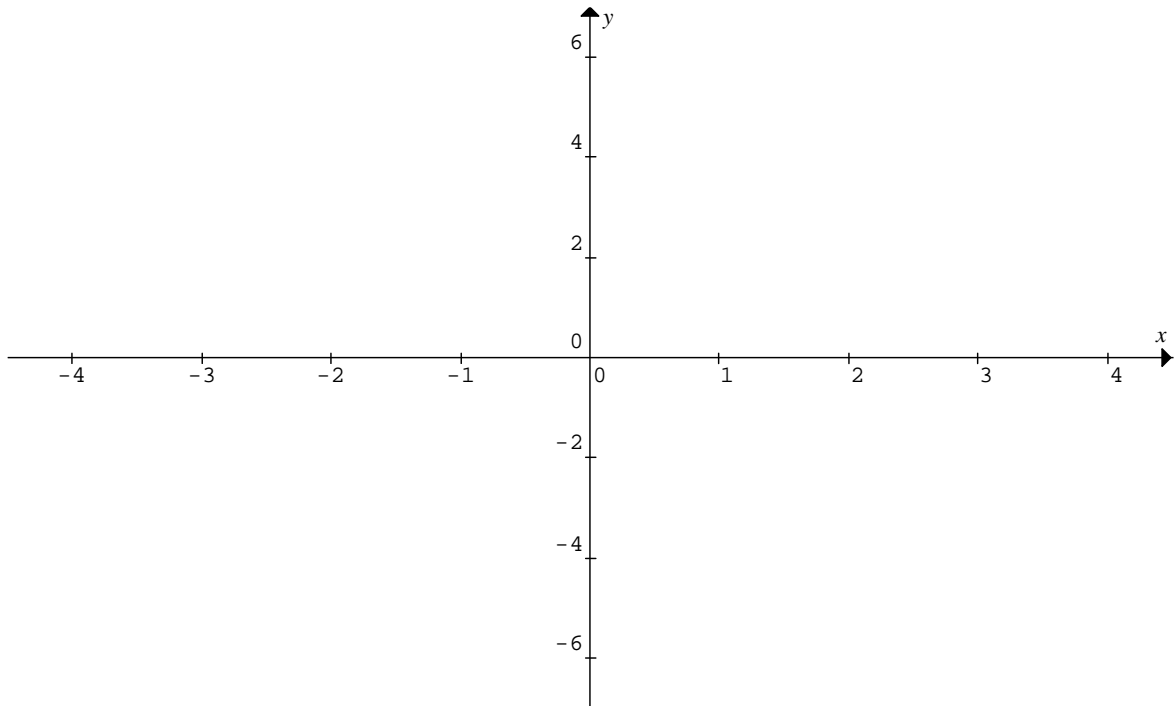
- b. Sketch the graph of the gradient function, on the axes below.



2 marks

**Question 4**

Sketch the graph of the function  $y = |x^2 - 4| - 5$ , on the axes below, clearly showing the coordinates of all axial intercepts.



2 marks

**Question 5**

For the function  $f : \{x : x < a\} \rightarrow R$ ,  $f(x) = x^2 - 6x + 5$ .

- a.** Find the largest value of the real number  $a$ , for which the function  $f$ , is a one-one decreasing function.

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1 mark

- b.** Find the inverse function  $f^{-1}$ .

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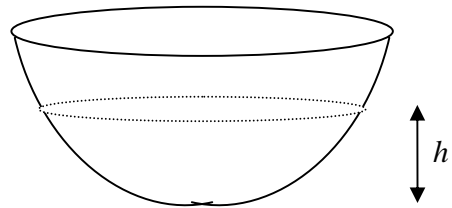
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3 marks

**Question 6**

When a hemispherical bowl contains water to a depth of  $h$  cm, the volume  $V$  cm<sup>3</sup> of water in the bowl is given by  $V = \frac{\pi h^2}{3}(120 - h)$



- a. If the bowl is being filled with water at a rate of 200 cm<sup>3</sup> per minute, find the rate (in cm per minute) at which the depth of the water is increasing, when the depth is 10 cm.

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2 marks

- b. Find, using calculus, the approximate change in the volume of the water in the bowl, when the depth of the water in the bowl increases from 10 to 10.01 cm.

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2 marks



**Question 7**

**a.** The probability density function of a continuous random variable  $X$  is given by

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

**i.** Show that  $k = \frac{3}{8}$ .

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1 mark

**ii.** Find  $E(X)$ , giving your answer correct to one decimal place.

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1 mark

**b.** A discrete random variable  $X$  has a probability distribution given by

$$p(x) = cx^2, \quad x = 0, 1, 2$$

**i.** Show that  $c = \frac{1}{5}$ .

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1 mark

**ii.** Find  $E(X)$ , giving your answer correct to one decimal place.

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1 mark



**Question 9**

Let  $f(x) = \cos\left(\frac{1}{x}\right)$ . Evaluate  $f'\left(\frac{6}{\pi}\right)$ .

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3 marks

**Question 10**

The line  $6y - x + d = 0$  is a normal to the curve  $y = x^4 + px$  at the point  $x = -1$ . Find the values of  $p$  and  $d$ .

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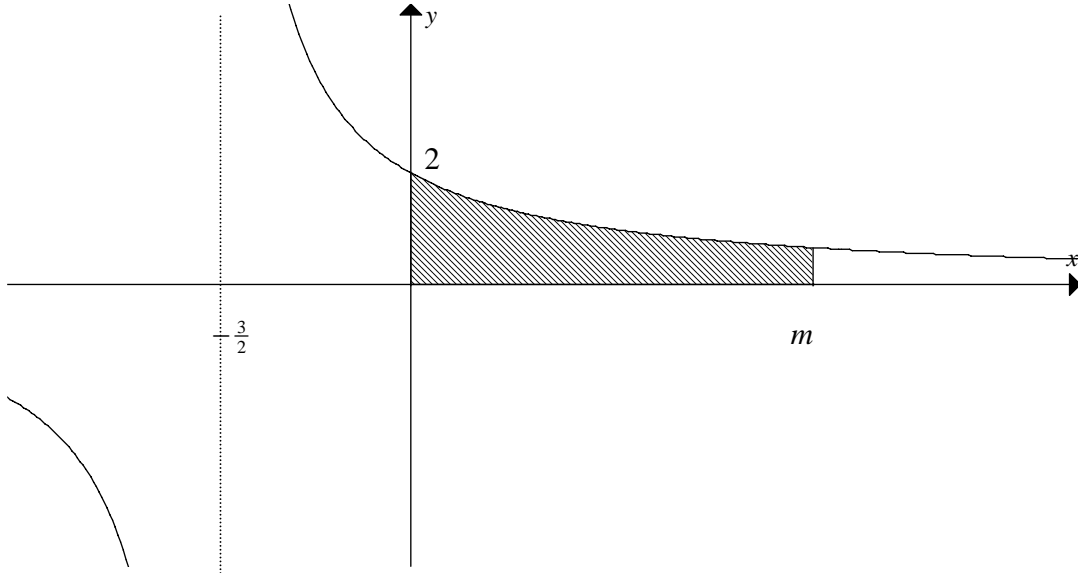
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4 marks

**Question 11**

Consider the graph of the function  $f : \mathbb{R} \setminus \{-\frac{3}{2}\} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{b}{2x+a}$ . The graph has a vertical asymptote at  $x = -\frac{3}{2}$  and crosses the  $y$ -axis at  $y = 2$ , as shown below.



The shaded area is the area bounded by the graph of  $y = \frac{b}{2x+a}$ , the coordinate axes and the line  $x = m$ . If the shaded area is equal to  $\log_e(27)$  square units, find the values of  $a$ ,  $b$  and  $m$ .

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4 marks



# **MATHEMATICAL METHODS**

## **Written examination 1**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods and CAS Formulas

### Mensuration

|  |   |
|--|---|
| area of a trapezium: $\frac{1}{2}(a+b)h$     | volume of a pyramid: $\frac{1}{3}Ah$      |
| curved surface area of a cylinder: $2\pi rh$ | volume of a sphere: $\frac{4}{3}\pi r^3$  |
| volume of a cylinder: $\pi r^2 h$            | area of triangle: $\frac{1}{2}bc \sin(A)$ |
| volume of a cone: $\frac{1}{3}\pi r^2 h$     |   |

### Calculus

|  |   |
|--|---|
| $\frac{d}{dx}(x^n) = nx^{n-1}$                                 | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$ |
| $\frac{d}{dx}(e^{ax}) = ae^{ax}$                               | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$            |
| $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$                        | $\int \frac{1}{x} dx = \log_e x  + c$               |
| $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$                          | $\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$       |
| $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$                         | $\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$        |
| $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$ |   |

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation:  $f(x+h) \approx f(x) + h f'(x)$

### Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Mean:  $\mu = E(X)$

variance:  $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

| probability distribution |                                     | mean                                      | variance   |
|--------------------------|-------------------------------------|---|--|
| discrete                 | $\Pr(X = x) = p(x)$                 | $\mu = E(X)$                              | $\sigma^2 = \sum (x - \mu)^2 p(x)$                       |
| continuous               | $\Pr(a < X < b) = \int_a^b f(x) dx$ | $\mu = \int_{-\infty}^{\infty} x f(x) dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ |