Year 2008

VCE

Mathematical Methods and Mathematical Methods (CAS)

Trial Examination 1



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Victorian Certificate of Education 2008

STUDENT NUMBER

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Figures							
Words							

MATHEMATICAL METHODS AND MATHEMATICAL METHOD (CAS)

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
12	12	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

•	4		
In	stri	ıctı	ons

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working must be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

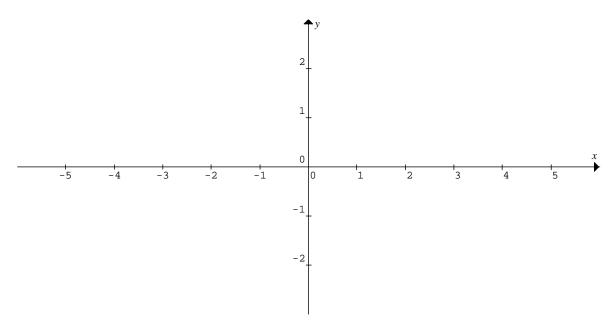
Question 1	
Let $f(x) = \frac{\sin(3x)}{3x^2}$. Find $f'(x)$.	
Question 2	2 marks
Solve the equation $\tan^2(x) + (1 - \sqrt{3})\tan(x) = \sqrt{3}$ terms of π .	for $x \in [0, 2\pi]$, giving exact values in

Given the function
$$f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ \cos(x) & \text{for } 0 \le x < \pi \\ \pi - x & \text{for } x \ge \pi \end{cases}$$

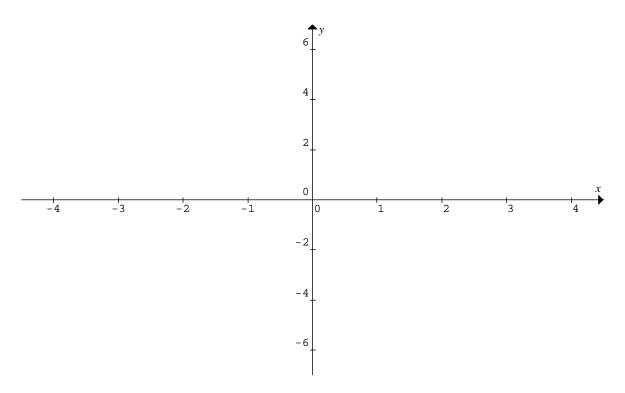
a. State the gradient function, f'(x) clearly indicating its domain.

2 marks

b. Sketch the graph of the gradient function, on the axes below.



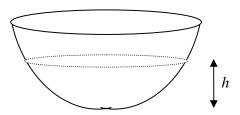
Sketch the graph of the function $y = |x^2 - 4| - 5$, on the axes below, clearly showing the coordinates of all axial intercepts.



Question 5 For the function $f:\{x:x < a\} \to R$, $f(x) = x^2 - 6x + 5$. **a.** Find the largest value of the real number a, for which the function f, is a one-one decreasing function. 1 mark **b.** Find the inverse function f^{-1} .

When a hemispherical bowl contains water to a depth of h cm, the volume $V \text{ cm}^3$ of water in the

bowl is given by $V = \frac{\pi h^2}{3} (120 - h)$



a.	If the bowl is being filled with water at a rate of 200 cm ³ per minute, find the rate (in cm per minute) at which the depth of the water is increasing, when the depth is 10 cm.
	2 mark
b.	Find, using calculus, the approximate change in the volume of the water in the bowl, when the depth of the water in the bowl increases from 10 to 10.01 cm.

a. The probability density function of a continuous random variable *X* is given by

 $f(x) = \begin{cases} kx^2 & 0 \le x \le 2\\ 0 & \text{otherwise} \end{cases}$

i. Show that $k = \frac{3}{8}$.

.

1 mark

ii. Find E(X), giving your answer correct to one decimal place.

1 mark

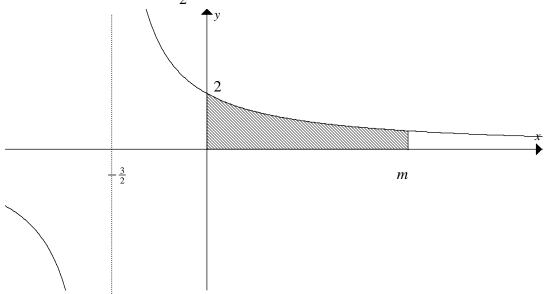
b.	A discrete random variable X has a probability distribution given by	
	$p(x) = cx^2$, $x = 0,1,2$	
i.	Show that $c = \frac{1}{5}$.	
	1 ma	- rk
ii.	Find $E(X)$, giving your answer correct to one decimal place.	

1 mark

Question 8	
Differentiate $x e^{-3x}$ and, hence, find $\int x e^{-3x} dx$.	

Question 9	
Let $f(x) = \cos\left(\frac{1}{x}\right)$. Evaluate $f'\left(\frac{6}{\pi}\right)$.	
	3 marks
Question 10	
Question 10 The line $6y - x + d = 0$ is a normal to the curve $y = x^4 + px$ at the point $x = -1$. Find the values of p and d .	
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Consider the graph of the function $f: R \setminus \{-\frac{3}{2}\} \to R$, $f(x) = \frac{b}{2x+a}$. The graph has a vertical asymptote at $x = -\frac{3}{2}$ and crosses the y-axis at y = 2, as shown below.



The shaded area is the area bounded by the graph of $y = \frac{b}{2x+a}$, the coordinate axes and the line x = m. If the shaded area is equal to $\log_e(27)$ square units, find the values of a, b and m.

Ashley has either cereal or toast for breakfast every morning. If he has toast one morning, the probability he has cereal the next morning is 0.25. If he has cereal one morning, the probability he has toast the next morning is 0.4. Suppose he has toast on a Monday morning. What is the probability that from Monday to Wednesday inclusive he has toast for breakfast exactly twice?

3 marks

END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

- 1 area of a trapezium: $\frac{1}{2}(a+b)h$ volume of a pyramid: $\frac{1}{3}Ah$
 - curved surface area of a cylinder: $2\pi rh$ volume of a sphere: $\frac{4}{3}\pi r^3$
- volume of a cylinder: $\pi r^2 h$ area of triangle: $\frac{1}{2}bc\sin(A)$
- volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^{n}) = nx^{n-1}$$

$$\int x^{n} dx = \frac{1}{n+1}x^{n+1} + c , n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_{e}|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

- product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} u\frac{dv}{dx}}{v^2}$
- Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Mean:
$$\mu = E(X)$$
 variance: $\operatorname{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probabi	lity distribution	mean	variance
discrete	$\Pr(X=x) = p(x)$	$\mu = E(X)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_{a}^{b} f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$