Year 2008

VCE

Mathematical Methods
and
Mathematical Methods
(CAS)
Solutions
Trial Examination 1

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Question 1

$y = \dfrac{\sin(3x)}{3x^2}$ differentiating using the quotient rule

let $u = \sin(3x)$ $\quad v = 3x^2$

$\dfrac{du}{dx} = 3\cos(3x) \qquad \dfrac{dv}{dx} = 6x$ M1

$\dfrac{dy}{dx} = \dfrac{9x^2\cos(3x) - 6x\sin(3x)}{9x^4} = \dfrac{3x(3x\cos(3x) - 2\sin(3x))}{9x^4}$

$\dfrac{dy}{dx} = \dfrac{1}{3x^3}(3x\cos(3x) - 2\sin(3x))$ A1

Question 2

$\tan^2(x) + (1-\sqrt{3})\tan(x) - \sqrt{3} = 0$

$(\tan(x) - \sqrt{3})(\tan(x) + 1) = 0$

$\tan(x) = \sqrt{3}$ or $\tan(x) = -1$ A1

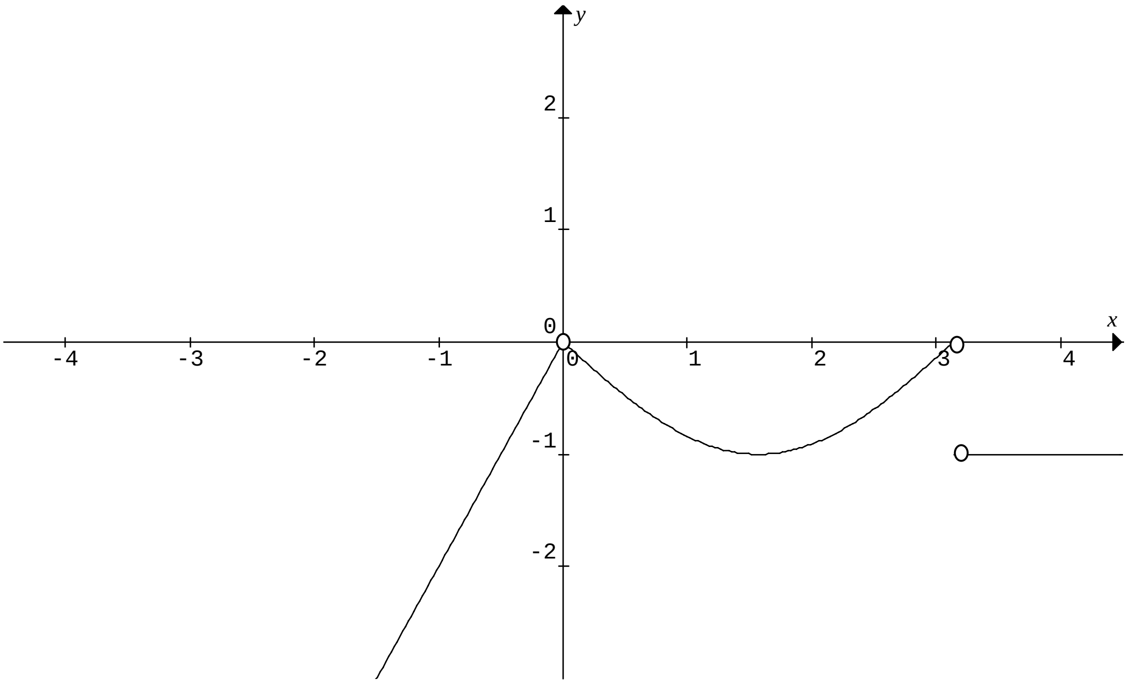
$x = \dfrac{\pi}{3},\ \pi + \dfrac{\pi}{3}$ or $x = \dfrac{3\pi}{4},\ \pi + \dfrac{3\pi}{4}$ M1

$x = \dfrac{\pi}{3},\ \dfrac{3\pi}{4},\ \dfrac{4\pi}{3},\ \dfrac{7\pi}{4}$ A1

Question 3

a. $f'(x) = \begin{cases} 2x & \text{for } x < 0 \\ -\sin(x) & \text{for } 0 < x < \pi \\ -1 & \text{for } x > \pi \end{cases}$ A2

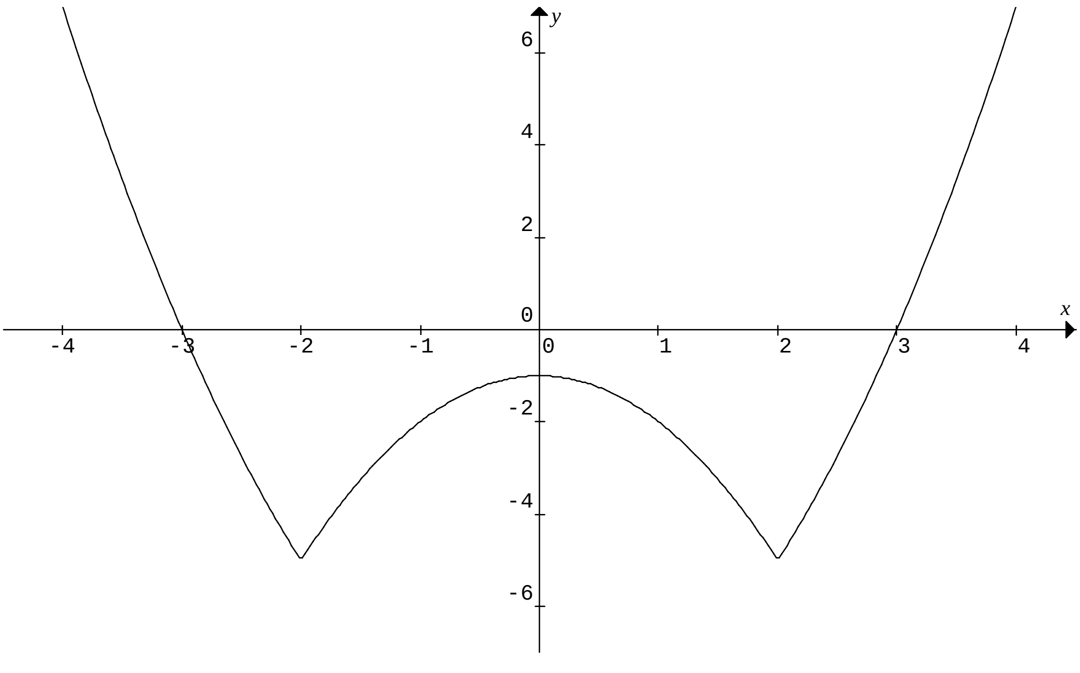
b. note that the graph must have open circles at $x = 0$ and $x = \pi$
the gradient function is not defined at $x = 0$ or $x = \pi$ A2



Question 4

$y = \left|x^2 - 4\right| - 5$, graph must pass through the y-intercept $(0,-1)$

and x-intercepts $(-3,0)\ (3,0)$ and $(-2,-5)$, $(2,-5)$ A2



Question 5

a. $f(x) = x^2 - 6x + 5$

$f(x) = (x^2 - 6x + 9) + 5 - 9$

$f(x) = (x-3)^2 - 4$

for f to be a one-one decreasing function $a = 3$ A1

b. $f \quad y = (x-3)^2 - 4$ interchanging y and x

$f^{-1} \quad x = (y-3)^2 - 4$ transposing to make y the subject M1

$f^{-1} \quad x + 4 = (y-3)^2$

$f^{-1} \quad y - 3 = \pm\sqrt{x+4}$

the domain of f^{-1}, needs to be stated as we are asked for a function

dom $f^{-1} = (-4, \infty)$ A1

since ran $f^{-1} =$ dom $f = (-\infty, 3)$, we need to take the negative in the square root

$f^{-1}(x) = 3 - \sqrt{x+4}$ A1

Question 6

a. Given $\frac{dV}{dt} = 200$ cm^3/min and $V = \frac{\pi}{3}\left(120h^2 - h^3\right)$

$\frac{dV}{dh} = \frac{\pi}{3}\left(240h - 3h^2\right) = \pi\left(80h - h^2\right)$ A1

By the chain rule $\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt} = \frac{200}{\pi\left(80h - h^2\right)}$

when $h = 10$ $\left.\frac{dh}{dt}\right|_{h=10} = \frac{200}{\pi(800-100)} = \frac{2}{7\pi}$ cm/min A1

b. when $h = 10$ $\Delta h = 0.01$ find ΔV

$\frac{dV}{dh} = \pi\left(80h - h^2\right) \approx \frac{\Delta V}{\Delta h}$ M1

$\Delta V \approx \pi\left(80h - h^2\right)\Delta h = \pi(800-100) \times 0.01 = 7\pi$ cm^3

the volume increases by 7π cm^3 A1

Question 7

a.i. $f(x)=\begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

$\int_0^2 kx^2\,dx = 1$ $\qquad k\left[\dfrac{x^3}{3}\right]_0^2 = 1$

M1

$k\left[\dfrac{2^3}{3} - 0\right] = 1$

$\dfrac{8k}{3} = 1$

$k = \dfrac{3}{8}$

ii. $E(X) = \int_a^b x\,f(x)\,dx = \int_0^2 kx^3\,dx$

$E(X) = \dfrac{3}{8}\left[\dfrac{x^4}{4}\right]_0^2 = \dfrac{3}{32}(16-0)$

$E(X) = 1.5$ A1

b.i. $p(x) = cx^2 \quad x = 0, 1, 2$

X	0	1	2
$\Pr(X = x)$	0	c	$4c$

Since $\sum \Pr(X = x) = 5c = 1$

$c = \dfrac{1}{5}$ A1

ii. $E(X) = \sum x \Pr(X = x) = 0 + c + 8c = 9c$

$E(X) = \dfrac{9}{5} = 1.8$ A1

Question 8

$\frac{d}{dx}\left(xe^{-3x}\right) = e^{-3x} - 3xe^{-3x}$ product rule A1

hence $\int\left(e^{-3x} - 3xe^{-3x}\right)dx = xe^{-3x}$ M1

$$\int e^{-3x}dx - 3\int xe^{-3x}dx = xe^{-3x}$$

$$-3\int xe^{-3x}dx = xe^{-3x} - \int e^{-3x}\,dx = xe^{-3x} + \frac{1}{3}e^{-3x}$$

$\int xe^{-3x}dx = -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C = -\frac{e^{-3x}}{9}(3x+1) + C$ A1

Question 9

let $f(x) = y = \cos\left(\frac{1}{x}\right) = \cos(u)$ $u = \frac{1}{x} = x^{-1}$ chain rule

$\frac{dy}{du} = -\sin(u)$ $\frac{du}{dx} = -x^{-2} = -\frac{1}{x^2}$

$\frac{dy}{dx} = \frac{1}{x^2}\sin\left(\frac{1}{x}\right)$ A1

$\frac{dy}{dx}$ when $x = \frac{6}{\pi}$ $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = f'\left(\frac{6}{\pi}\right) = \frac{1}{\left(\frac{6}{\pi}\right)^2}\sin\left(\frac{\pi}{6}\right) = \frac{\pi^2}{36} \text{ x } \frac{1}{2}$ M1

$f'\left(\frac{6}{\pi}\right) = \frac{\pi^2}{72}$ A1

Question 10

The line $6y-x+d=0 \quad 6y=x-d \quad y=\frac{x}{6}-\frac{d}{6}$

has a gradient of $\frac{1}{6}$ so $m_N=\frac{1}{6}$ so the tangent has a gradient of $m_T=-6$ A1

$y=x^4+px \quad \Rightarrow \quad \frac{dy}{dx}=4x^3+p=-6 \quad \text{at} \quad x=-1$ M1

$4(-1)^3+p=-4+p=-6 \quad \text{so that} \quad p=-2$

The curve is $y=x^4-2x$ at the point $x=-1$

$y(-1)=(-1)^4-p=1+2=3$ M1

the point $P(-1, 3)$ is also on the line

$6y-x+d=0 \qquad 18+1+d=0$

so $d=-19$ A1

Question 11

since $x=-\frac{3}{2}$ is a vertical asymptote, the denominator is $2x+3$, so that $a=3$

since the y-intercept is $y=2=\frac{b}{a}$, it follows that $b=6$ A1

The area is $\int_0^m \frac{6}{2x+3}\,dx=\log_e(27)$ A1

$=3\left[\log_e(2x+3)\right]_0^m=3\left(\log_e(2m+3)-\log_e(3)\right)$

$=3\log_e\left(\frac{2m+3}{3}\right)=\log_e(27)=3\log_e(3)$ M1

$\frac{2m+3}{3}=3 \quad \Rightarrow 2m+3=9 \quad \Rightarrow \quad 2m=6$

$m=3$ A1

Question 12

$\Pr(\text{Toast on two days})$ M1

$=T\ T\ C \text{ or } T\ C\ T$

$=0.75 \times 0.25+0.25 \times 0.4$

$=\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{2}{5}=\frac{3}{16}+\frac{1}{10}=\frac{15+8}{80}$ A1

$=\frac{23}{80}$ A1

END OF SUGGESTED SOLUTIONS