2008 VCAA Mathematical Methods Exam 1 Solutions
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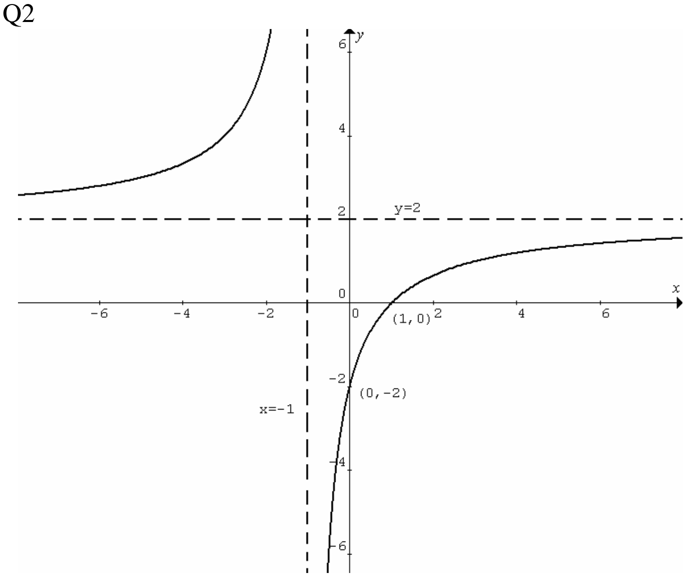
Q1a The chain rule: $\frac{dy}{dx}=5\left(3x^2-5x\right)^4(6x-5)$

Q1b The product rule: $f'(x)=3xe^{3x}+e^{3x}=e^{3x}(3x+1)$,
$f'(0)=e^0(1)=1$

Q2



Q3 $\cos\left(\frac{3x}{2}\right)=\frac{1}{2}$, $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$, $\therefore -\frac{3\pi}{4}\le\frac{3x}{2}\le\frac{3\pi}{4}$.
$\therefore \frac{3x}{2}=-\frac{\pi}{3},\frac{\pi}{3}$, $\therefore x=-\frac{2\pi}{9},\frac{2\pi}{9}$.

Q4a $\int_{-\infty}^{\infty}f(x)dx=1$, $\therefore \int_0^1 k\sin(\pi x)dx=1$, $\left[\frac{-k\cos(\pi x)}{\pi}\right]_0^1=1$,
$\therefore \frac{-k\cos\pi}{\pi}+\frac{k\cos 0}{\pi}=1$, $\frac{k}{\pi}+\frac{k}{\pi}=1$, $\frac{2k}{\pi}=1$, $\therefore k=\frac{\pi}{2}$.

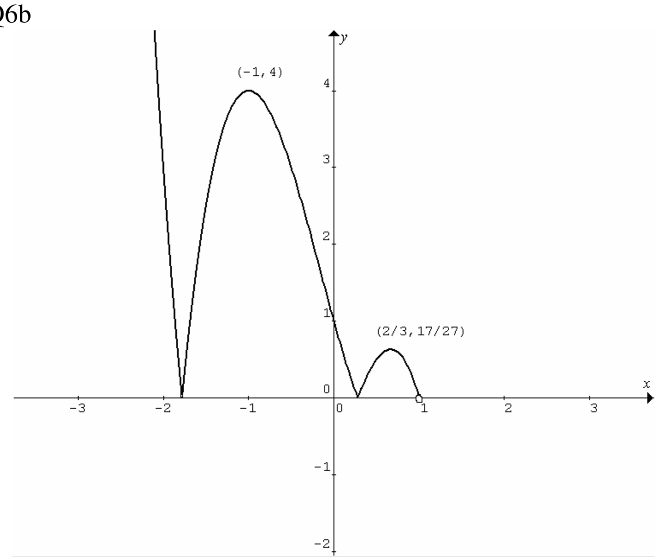
Q4b $\Pr\left(X\le\frac{1}{4}\mid X\le\frac{1}{2}\right)=\frac{\Pr\left(X\le\frac{1}{4}\cap X\le\frac{1}{2}\right)}{\Pr\left(X\le\frac{1}{2}\right)}=\frac{\Pr\left(X\le\frac{1}{4}\right)}{\Pr\left(X\le\frac{1}{2}\right)}$

$=\frac{\int_0^{\frac{1}{4}}\frac{\pi}{2}\sin(\pi x)dx}{\int_0^{\frac{1}{2}}\frac{\pi}{2}\sin(\pi x)dx}=1-\frac{\sqrt{2}}{2}$.

Q5 $\int_0^C e^{2x}dx=\frac{5}{2}$, $\left[\frac{e^{2x}}{2}\right]_0^C=\frac{5}{2}$, $\frac{e^{2C}}{2}-\frac{1}{2}=\frac{5}{2}$, $\therefore e^{2C}=6$,
$C=\frac{1}{2}\log_e 6$ or $\log_e\sqrt{6}$.

Q6a Domain of f' is $(-\infty,1)\cup(1,2)\cup(2,\infty)$.

Q6b



Q7a The mode of X is 3.

Q7b $\Pr=0.1^2+0.2^2+0.3^2+0.4^2=0.3$

Q8 $\Pr(DCC)+\Pr(CDC)+\Pr(CCD)$
$=0.6\times0.5\times0.4+0.4\times0.6\times0.5+0.4\times0.4\times0.6=0.336$

Q9a Area of equilateral triangle $=\frac{1}{2}x^2\sin 60°=\frac{\sqrt{3}}{4}x^2$.
$\therefore V=\frac{\sqrt{3}}{4}x^2y=1000$. Hence $y=\frac{4000}{x^2\sqrt{3}}$.

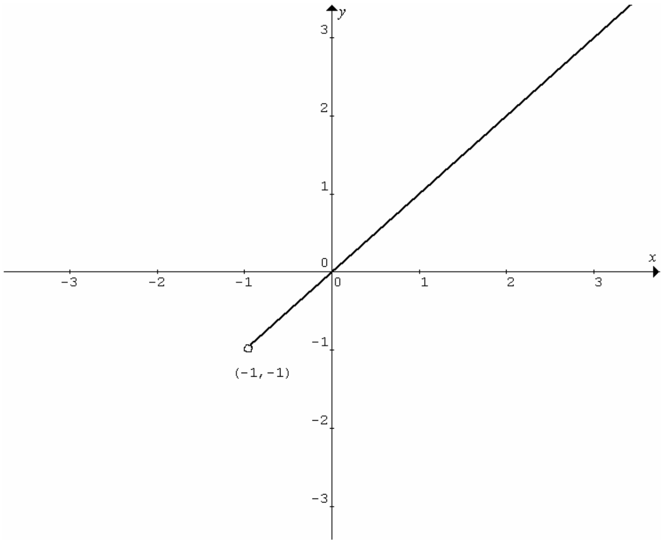
Q9b Total surface area $A=3xy+2\left(\frac{\sqrt{3}}{4}x^2\right)=\frac{4000\sqrt{3}}{x}+\frac{\sqrt{3}x^2}{2}$

Q9c Let $\frac{dA}{dx}=-\frac{4000\sqrt{3}}{x^2}+\sqrt{3}x=0$,
$\therefore x^3=4000$, $x=10\times\sqrt[3]{4}$ or $10\times 2^{\frac{2}{3}}$.

Q10a The range of *f* is $(-1,\infty)$, it becomes the domain of f^{-1}.
The equation of *f* is $y=e^{2x}-1$, the equation of f^{-1} is
$x=e^{2y}-1$. $\therefore e^{2y}=x+1$, $y=\frac{1}{2}\log_e(x+1)$.

Hence $f^{-1}(x)=\frac{1}{2}\log_e(x+1)$, $x\in(-1,\infty)$.

Q10b $y = f\left(f^{-1}(x)\right) = x$, $x \in (-1, \infty)$.



Q10c $f^{-1}(x) = \frac{1}{2}\log_e(x+1)$, $-f^{-1}(2x) = -\frac{1}{2}\log_e(2x+1)$,

$f(x) = e^{2x} - 1$, $f\left(-f^{-1}(2x)\right) = e^{2\left(-\frac{1}{2}\log_e(2x+1)\right)} - 1$

$= e^{-\log_e(2x+1)} - 1 = (2x+1)^{-1} - 1 = \frac{1}{2x+1} - 1 = \frac{-2x}{2x+1}$.

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