



Trial Examination 2007

# **VCE Mathematical Methods Units 3 & 4**

Written Examination 1

**Suggested Solutions**

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**Question 1**

$$g(f(x)) = g((x-1)^2)$$

$$= \sqrt{(x-1)^2} + 1 \text{ or } |x-1| + 1$$

A1

Note:  $|x-1| + 1 \neq x$ **Question 2**

- a. The turning point of the graph of  $y = f(x)$  is  $(-1, -1)$ .

Therefore the maximal domain for  $f$  to be a one-to-one function is  $(-\infty, -1)$ .

Therefore  $a = -1$ .

A1

- b. Originally, we have  $y = x^2 + 2x$ .

For the inverse,  $x = y^2 + 2y$ .

M1

$$x = y^2 + 2y + 1 - 1$$

$$= (y+1)^2 - 1$$

M1

$$x + 1 = (y+1)^2$$

$$y = -1 \pm \sqrt{x+1}$$

However,  $\text{ran } f^{-1} = \text{dom } f = (-\infty, -1)$

$$f^{-1}(x) = -1 - \sqrt{x+1}$$

A1

**Question 3**

- a. i. period =  $\frac{2\pi}{\frac{\pi}{6}} = 12$

A1

- ii. range :  $[-1, 3]$

A1

- b.  $g(x) = 1 - 2\sin\left(\frac{\pi}{6}(x-2)\right)$

$g(x)$  has a minimum where  $\sin\left(\frac{\pi}{6}(x-2)\right) = 1$

M1

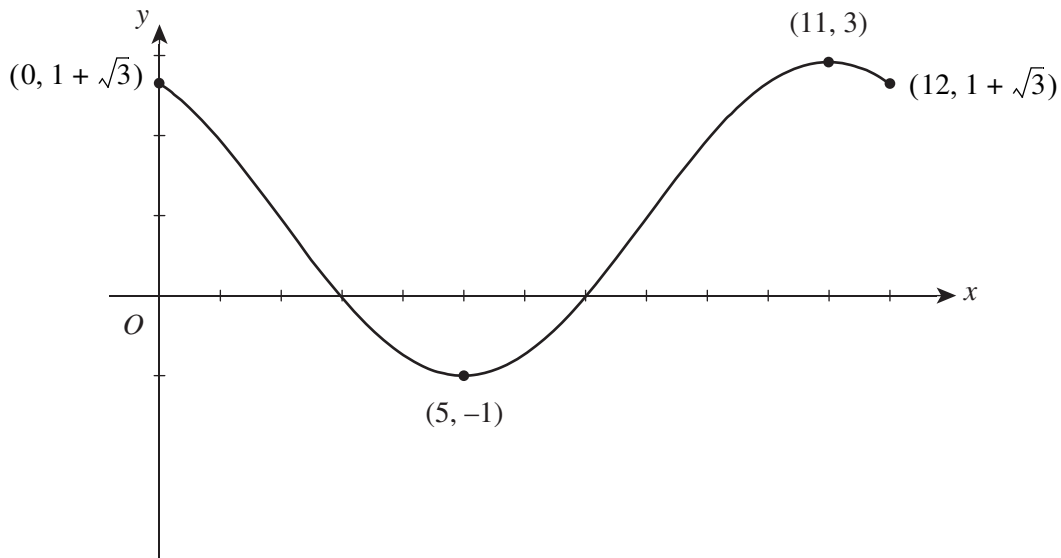
$$\therefore \frac{\pi}{6}(x-2) = \frac{\pi}{2}$$

$$x = 5$$

So the minimum occurs at  $(5, -1)$ .

A1

c.



Correct shape A1  
 Correct endpoints A1  
 Correct maximum and minimum A1

**Question 4**

$$\begin{aligned}
 f(x) &= \log_e\left(\frac{x}{x-1}\right) \\
 &= \log_e\left(\frac{-x}{1-x}\right) \\
 &= \log_e(-x) - \log_e(1-x) \text{ (so that } f\left(-\frac{1}{2}\right) \text{ exists)}
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= \frac{-1}{-x} - \frac{-1}{1-x} \\
 &= \frac{1}{x} - \frac{1}{x-1} \\
 &= \frac{x-1-x}{x(x-1)} \\
 &= \frac{-1}{x(x-1)} && \text{M1} \\
 &= \frac{1}{x(1-x)}
 \end{aligned}$$

$$\begin{aligned}
 f'\left(-\frac{1}{2}\right) &= \frac{1}{-\frac{1}{2}\left(1+\frac{1}{2}\right)} && \text{M1} \\
 &= -\frac{4}{3} && \text{A1}
 \end{aligned}$$

Thus the gradient of the tangent is  $-\frac{4}{3}$ , so the gradient of the normal is  $\frac{3}{4}$ . A1

**Question 5**

$$\int_5^1 2(h(x) - 1)dx = \int_5^1 (2h(x) - 2)dx$$

$$= -2 \int_1^5 h(x)dx - \int_5^1 2dx$$

M1

$$= -2[4] - [2x]_5^1$$

$$= -8 - (2 - 10)$$

$$= 0$$

A1

**Question 6**

a. As  $f(x)$  is a probability density function,  $\int_1^6 \frac{k}{x^2} dx = 1$ .

M1

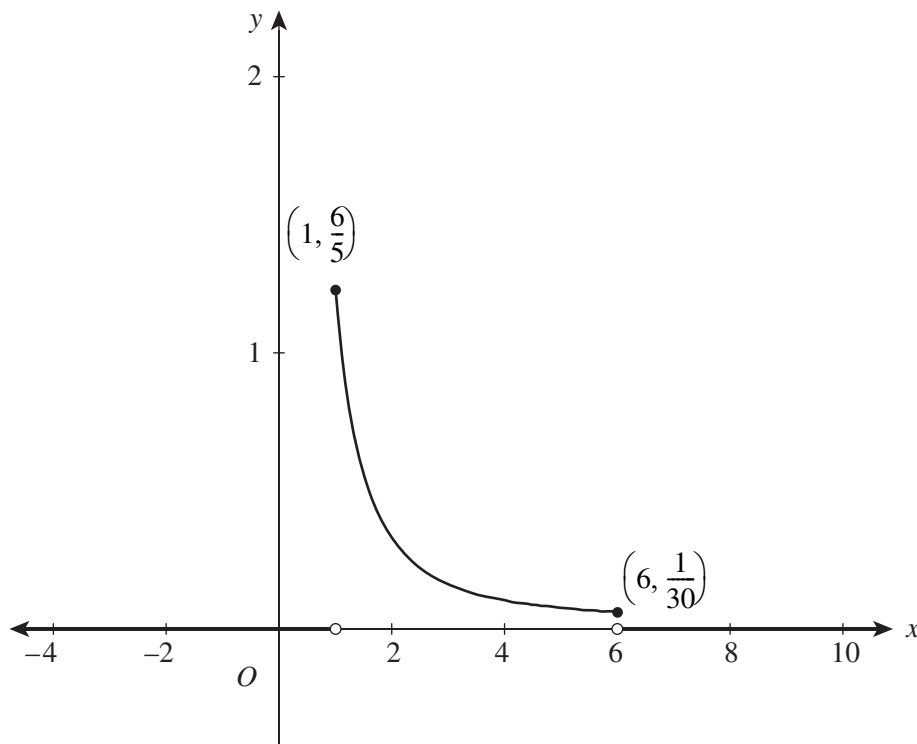
$$\left[ -\frac{k}{x} \right]_1^6 = 1$$

$$-\frac{k}{6} + \frac{k}{1} = 1$$

$$k = \frac{6}{5}$$

A1

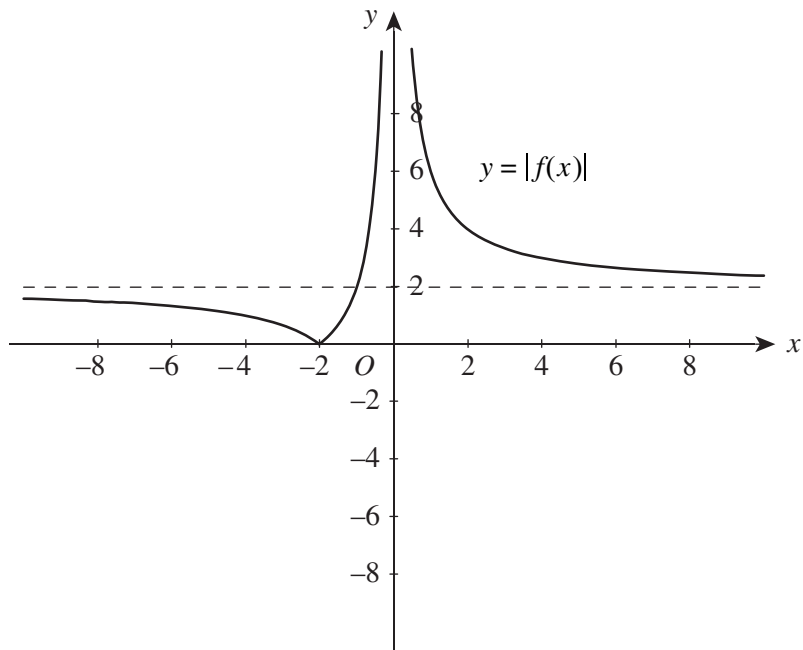
b.



Correct endpoints A1  
Correct graph on  $(-\infty, \infty)$  A1

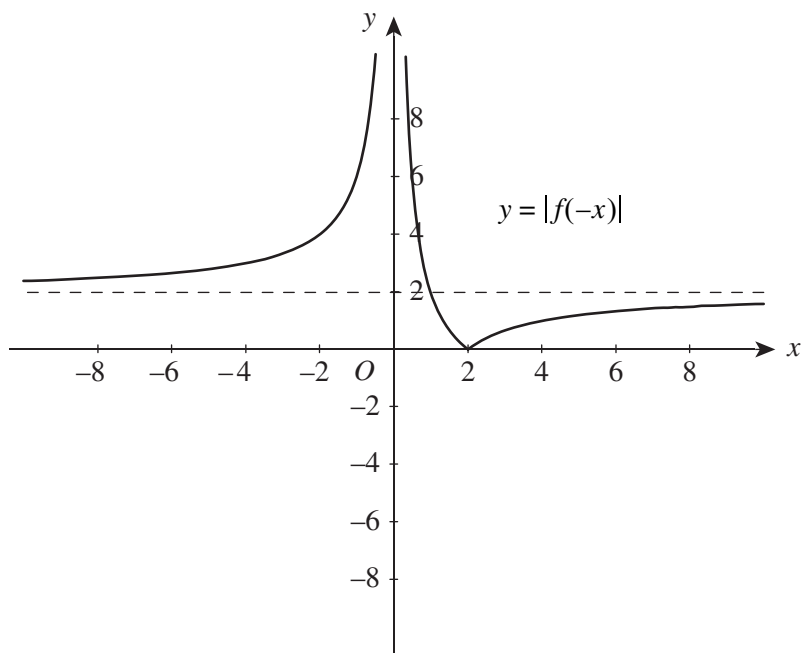
**Question 7**

**a.**



Correct graph of  $|f(x)|$  A1

**b.**



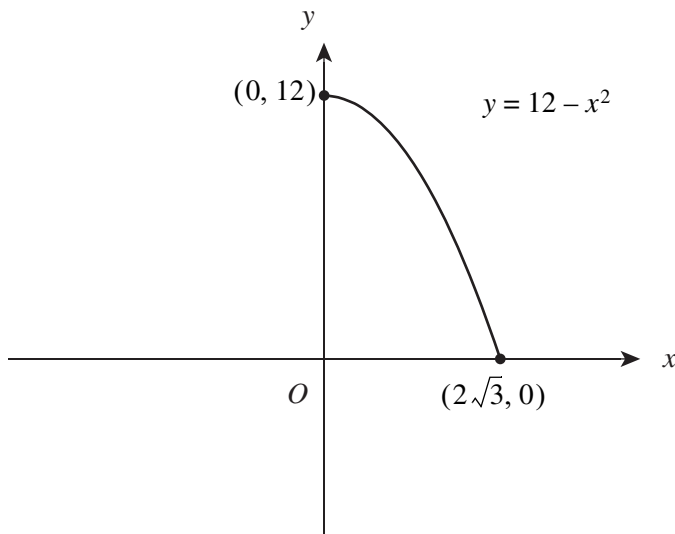
Correct graph of  $|f(-x)|$  A1

**c.** range :  $\mathbb{R}^+ \cup \{0\}$

A1

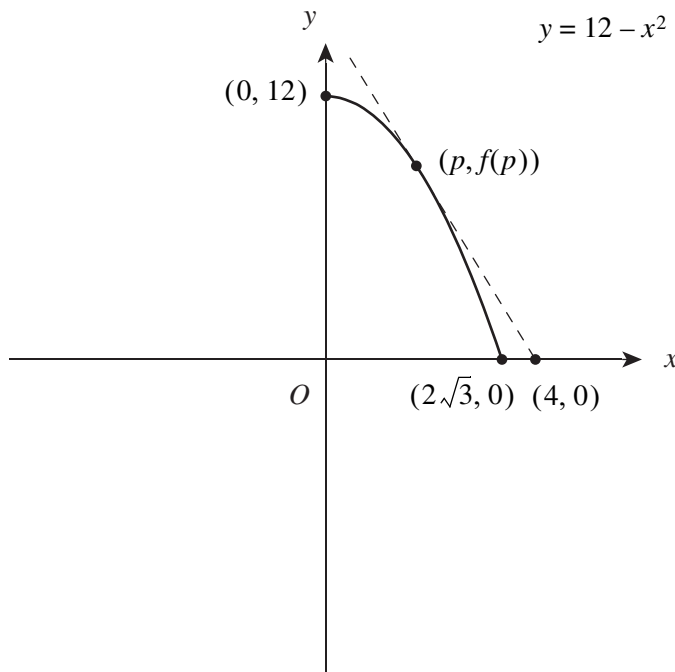
**Question 8**

a.



Correct graph A1

b.



The gradient of the tangent at  $x = p$  is given by  $f'(p)$ .

$$f'(x) = -2x \text{ so } f'(p) = -2p$$

$$\text{Equation of the tangent: } y - y_1 = m(x - x_1)$$

$$\therefore y - (12 - p^2) = -2p(x - p)$$

$$y = -2px + p^2 + 12$$

M1 A1

As the tangent passes through the point (4, 0)

$$0 = -8p + p^2 + 12$$

$$0 = (p - 6)(p - 2)$$

M1

$$p = 6 \text{ or } p = 2$$

Clearly  $0 < p < 2\sqrt{3}$  so  $p = 2$

A1

**Question 9**

a.  $\frac{dr}{dt} = \frac{1}{25}$ ,  $V = \frac{4}{3}\pi r^3$ ,  $\frac{dV}{dr} = 4\pi r^2$

Using the chain rule,

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \times \frac{1}{25} = \frac{4\pi r^2}{25}$$

M1

At  $r = 10$ ,

$$\frac{dV}{dt} = \frac{4\pi \times 100}{25} = 16\pi \text{ cm}^3/\text{s}$$

A1

b. The area of a cross-section through the centre of the sphere is  $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{6\pi}{25}$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

M1

$$\frac{6\pi}{25} = 2\pi r \times \frac{1}{25}$$

$$r = 3$$

At  $r = 3$ ,

$$V = \frac{4}{3}\pi \times 3^3 = 36\pi \text{ cm}^3$$

A1

**Question 10**

Point of intersection:

$$\sin(x) = \cos(x)$$

$$\tan(x) = 1$$

$$x = \frac{\pi}{4}$$

A1

$$\text{area} = \int_0^{\frac{\pi}{4}} (\cos(x) - \sin(x)) dx$$

$$= [\sin(x) + \cos(x)]_0^{\frac{\pi}{4}}$$

M1

$$= \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) - (0 + 1)$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

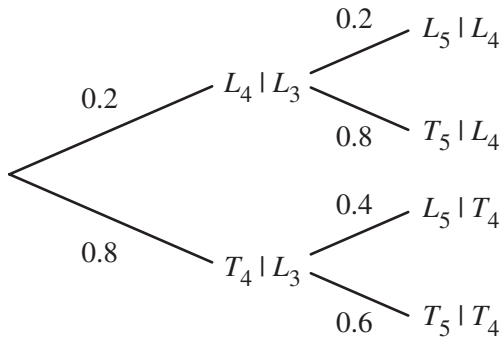
$$= \frac{2 - \sqrt{2}}{\sqrt{2}}$$

$$= \sqrt{2} - 1$$

A1

**Question 11**

Let  $T_i$  be the event 'on time for school on the  $i$ th day of the week', and let  $L_i$  be the event 'late for school on the  $i$ th day of the week'.



Tree diagram or table M1  
Correct probabilities A1

$$\begin{aligned}\Pr(L_5) &= 0.2 \times 0.2 + 0.8 \times 0.4 \\ &= 0.36\end{aligned}$$

A1