2007 VCAA Mathematical Methods Exam 1 Solutions
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Q1 $f(x)=\frac{x^3}{\sin x}$,

$f'(x)=\frac{(\sin x)(3x^2)-x^3\cos x}{\sin^2 x}=\frac{x^2(3\sin x-x\cos x)}{\sin^2 x}$.

Q2a $\log_e(3x+5)+\log_e 2=2$,

$\log_e 2(3x+5)=2$, $\log_e(6x+10)=2$,

$6x+10=e^2$, $x=\frac{1}{6}\left(e^2-10\right)$

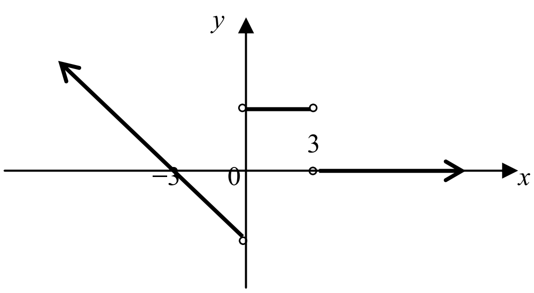
Q2b Let $u=\tan x$.

$g'(x)=\frac{d}{dx}\log_e(\tan x)=\frac{d}{du}\log_e u\times\frac{du}{dx}$

$=\frac{1}{u}\sec^2 x=\frac{\sec^2 x}{\tan x}$

$\therefore g'\left(\frac{\pi}{4}\right)=\frac{\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)}=\frac{\left(\sqrt{2}\right)^2}{1}=2$.

Q3a



Q3b $R\setminus\{0,3\}$

Q4 Given $\frac{dV}{dt}={}^{+}8\ \text{cm}^3\text{s}^{-1}$, $V=4x^{\frac{3}{2}}$.

$\frac{dV}{dx}=6x^{\frac{1}{2}}=12$ when $x=4$.

$\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$, $\therefore {}^{+}8=12\frac{dx}{dt}$, $\frac{dx}{dt}={}^{+}\frac{2}{3}$ cms^{-1}.

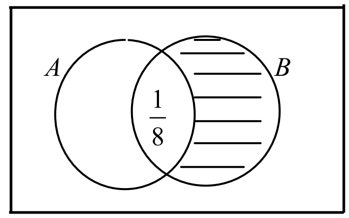
The rate of increase is $\frac{2}{3}$ cms^{-1}.

Q5 Binomial, $n=4$, $p=\frac{1}{2}$, $\therefore q=\frac{1}{2}$.

$\Pr(X>2)=\Pr(X=3)+\Pr(X=4)$

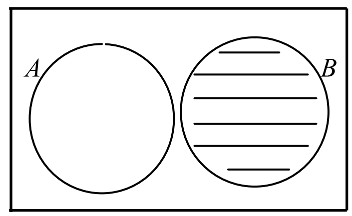
$={}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1+{}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0=4\times\frac{1}{16}+1\times\frac{1}{16}=\frac{5}{16}$.

Q6a



$\Pr(A'\cap B)=\Pr(B)-\Pr(A\cap B)=\frac{1}{3}-\frac{1}{8}=\frac{5}{24}$.

Q6b



$\Pr(A'\cap B)=\Pr(B)-\Pr(A\cap B)=\frac{1}{3}-0=\frac{1}{3}$.

Q7 $f'(x)=\cos 3x-3x\sin 3x$

$\int f'(x)dx=\int\cos 3x\,dx-\int 3x\sin 3x\,dx$

$f(x)=\frac{1}{3}\sin 3x-3\int x\sin 3x\,dx$

$x\cos 3x=\frac{1}{3}\sin 3x-3\int x\sin 3x\,dx$

$3\int x\sin 3x\,dx=\frac{1}{3}\sin 3x-x\cos 3x$

$\int x\sin 3x\,dx=\frac{1}{9}\sin 3x-\frac{1}{3}x\cos 3x$.

Q8a $\sin\frac{2\pi x}{3}=-\frac{\sqrt{3}}{2}$, $0\le x\le 3$, i.e. $0\le\frac{2\pi x}{3}\le 2\pi$.

$\frac{2\pi x}{3}=\frac{4\pi}{3}$ or $\frac{5\pi}{3}$, $\therefore x=2$ or $\frac{5}{2}$.

Q8b Since $\sin\frac{2\pi x}{3}=1$, $0\le x\le 3$, $x=\frac{3}{4}$.

The maximum of $f(x)=\sin\frac{2\pi x}{3}$ is at $x=\frac{3}{4}$.

Let $h(x)=3f(x)+2$. The transformations on $f(x)$ do not change the x-coordinate of the maximum point, i.e. $x=\frac{3}{4}$.

$\therefore$ for $g(x)=3f(x-1)+2$, the maximum point is translated to the right by 1 unit. Hence $x=\frac{3}{4}+1=\frac{7}{4}$.

Q9a $f(x)=e^{\frac{x}{2}}+1, x \in R$. When $x=0$, $y=f(0)=e^0+1=2$.

$f'(x)=\frac{1}{2}e^{\frac{x}{2}}$. At $x=0$, $f'(0)=\frac{1}{2}e^0=\frac{1}{2}$, which is the gradient of the tangent m_T at $(0,2)$.

$\therefore$ the gradient of the normal at $(0,2)$ is $m_N=-\frac{1}{m_T}=-2$.

Equation the normal at $(0,2)$:
$y-2=-2(x-0)$, i.e. $y=-2x+2$.

Q9b x-intercept of the normal: $y=0$, $x=1$.

Area of the shaded region $=\int_0^1\left[\left(e^{\frac{x}{2}}+1\right)-(-2x+2)\right]dx$

$=\int_0^1\left(e^{\frac{x}{2}}+2x-1\right)dx=\left[2e^{\frac{x}{2}}+x^2-x\right]_0^1$

$=2\left(e^{\frac{1}{2}}-1\right)$ square units.

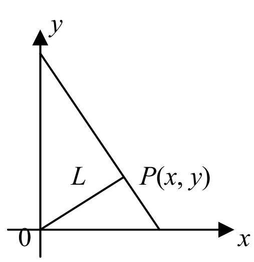
Q10 $\int_0^9 kx^{\frac{1}{2}}dx=27$, $k\int_0^9 x^{\frac{1}{2}}dx=27$, $k\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^9=27$.

$\therefore 18k=27$, $k=\frac{3}{2}$.

Q11a Let *T* be 'on time' and *F* be 'fine'.
Given $\Pr(T \mid F)=0.8$, $\Pr(T \mid F')=0.6$ and $\Pr(F)=0.4$.
$\therefore \Pr(F')=0.6$, $\Pr(T \cap F)=\Pr(T \mid F)\Pr(F)=0.8\times 0.4=0.32$
and $\Pr(T \cap F')=\Pr(T \mid F')\Pr(F')=0.6\times 0.6=0.36$.
$\therefore \Pr(T)=\Pr(T\cap F)+\Pr(T \cap F')=0.32+0.36=0.68$

Q11b $\Pr(F \mid T)=\frac{\Pr(F\cap T)}{\Pr(T)}=\frac{0.32}{0.68}=\frac{8}{17}$.

Q12



Let *L* be the the length of *OP*.
$\therefore L=\sqrt{x^2+y^2}$
Since $y=-2x+10$,
$\therefore L=\sqrt{x^2+4x^2-40x+100}=\sqrt{5x^2-40x+100}$.
$\frac{dL}{dx}=\frac{10x-40}{2\sqrt{5x^2-40x+100}}=\frac{5x-20}{\sqrt{5x^2-40x+100}}$.
Let $\frac{dL}{dx}=0$, $\therefore 5x-20=0$, $x=4$ and $y=2$. $\therefore P(4,2)$.
$L_{\min}=\sqrt{4^2+2^2}=2\sqrt{5}$ units.

Instead of using calculus, other methods can be used.

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