Mathematical Methods

2007

Trial Examination 2

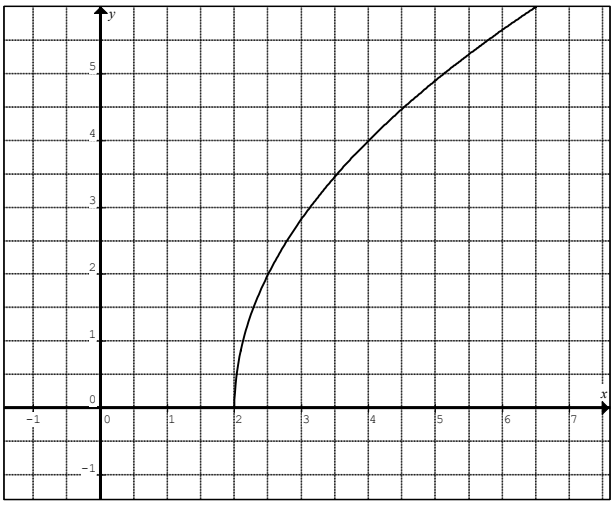
SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The graph of $y = \sqrt{ax+b}$ is shown below.



The values of a and b are

A. $a = 4,\ b = 16$

B. $a = 8,\ b = 16$

C. $a = 4,\ b = -16$

D. $a = 8,\ b = -16$

E. $a = 2,\ b = -8$

Question 2

The value of $\log_5 10$ is closest to

A. 1.43 B. 1.430 C. 1.431 D. 2 E. 100000

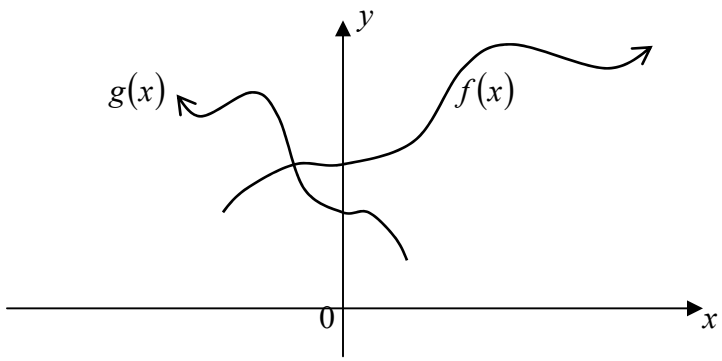
Question 3

The number of solutions of $|\cos(3x)| = 0.15$ for $-\pi \le x \le \pi$ is

A. 2 B. 4 C. 6 D. 12 E. 24

Question 4

The graphs of function $f(x)$ and its transformation $g(x)$ are shown below.



Which one of the following statements is possibly true?

A. $g(x)=f(x)-1$

B. $g(x)=f(-x)+1$

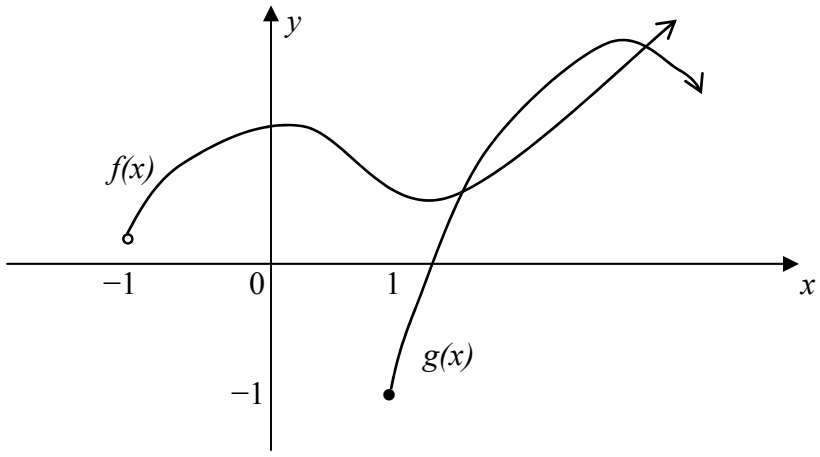
C. $g(x)=-f(x)-1$

D. $g(x)=-f(2x)+1$

E. $g(x)=f(-2x)-1$

Question 5

The graphs of functions $f(x)$ and $g(x)$ are shown below.



The domain of $f[g(x)]$ is

A. $(1,\infty)$ B. $[1,\infty)$ C. $(-\infty,1)$ D. $(-\infty,1]$ E. $(-1,\infty)$

Question 6

The solution(s) of $\frac{x^2e^x}{2\pi x}=1$ is/are

A. 0, 1.46 B. 1.46 C. 0, 0.68 D. 0.68 E. 0.69

Question 7

Given $f(x)=-x(2x+a)^3(x-2b)^2$ and $a,b\in R^-$, some of the x-intercepts of the graph of $y=f'(x)$ are

A. 0, $\frac{a}{2}$, $-2b$

B. $-\frac{a}{2}$, $2b$

C. 0, $-\frac{a}{2}$, $2b$

D. $\frac{a}{2}$, $-2b$

E. a, $-b$

Question 8

The inverse of $f:[-2,-1)\cup(-1,\infty)\to R$, $f(x)=\frac{1}{x+1}+2$ is

A. $f^{-1}:(-\infty,2)\cup(2,\infty)\to R$, $f^{-1}(x)=\frac{1}{x-2}-1$

B. $f^{-1}:(-\infty,1]\cup(2,\infty)\to R$, $f^{-1}(x)=\frac{1}{x-2}-1$

C. $f^{-1}:(-\infty,2)\cup(2,\infty)\to R$, $f^{-1}(x)=\frac{1}{x+2}+1$

D. $f^{-1}:(-\infty,1]\cup(2,\infty)\to R$, $f^{-1}(x)=\frac{1}{x+2}+1$

E. $f^{-1}:(-\infty,2)\cup(2,\infty)\to R$, $f^{-1}(x)=\frac{1}{x-2}+1$

Question 9

For $2x+1\overline{)\ 32x^5+8x^3+2x+1}$

the remainder is

A. -2 B. 2 C. -4 D. 4 E. 0

Question 10

The tangent to the curve $y=\sin\left(\frac{\pi x}{2}\right)-\frac{\pi x}{2}$ for $-2\pi \le x \le 2\pi$ is parallel to the x-axis at $x=$

A. $-4, -2, 0, 2, 4$

B. $-4, -2, 2, 4$

C. $-2\pi,\ 0,\ 2\pi$

D. $-4, 0, 4$

E. $-2\pi,\ -\pi,\ 0,\ \pi,\ 2\pi$

Question 11

The derivative of $e^{\sqrt{1+x^2}}$ is

A. $\frac{e^{\sqrt{1+x^2}}}{2x}$

B. $2xe^{\sqrt{1+x^2}}$

C. $\frac{2xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$

D. $\frac{e^{\sqrt{1+x^2}}}{2\sqrt{1+x^2}}$

E. $\frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$

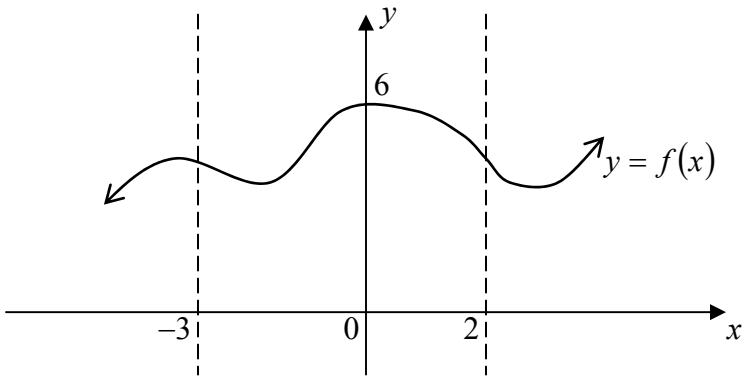
Question 12

The rate of **decrease** of $\dfrac{\log_e\left|x^3+a\right|}{x^3+a}$ with respect to x at $x=-2$ when $a=3$ is closest to

A. -0.29 B. 0.29 C. -0.32 D. 0.32 E. undefined

Question 13

Consider the graph of $f(x)$ shown below.



The estimated value of $\int_{-3}^{2} f(x)dx$ is closest to

A. 15 B. 20 C. 25 D. 30 E. 31

Question 14

The exact value of $\int_{0}^{2} \dfrac{1}{2x-5}dx$ is

A. $-\log_e\sqrt{5}$

B. $-\dfrac{1}{2}(1-\log_e 5)$

C. $\dfrac{1}{2}(1-\log_e 5)$

D. $-\dfrac{1}{2}(1+\log_e 5)$

E. $\dfrac{1}{2}(1+\log_e 5)$

Question 15

Given $F(x)=\int f(x)dx$, $F(2)=5$ and $F(1)=1$, the value of $\int_1^2 [2f(x)-3]dx$ is

A. -2 B. -3 C. 4 D. 5 E. 6

Question 16

Given $\frac{d}{dx}\left[\sin^2\left(x^2\right)\right]=2x\sin\left(2x^2\right)$, the value of $\int_0^{\frac{\sqrt{\pi}}{2}} x\sin\left(2x^2\right)dx$ is

A. 4 B. 1 C. $\frac{1}{2}$ D. $\frac{1}{4}$ E. $\frac{\sqrt{2}}{4}$

Question 17

If $f(x)=\frac{\sqrt{2-(x-p)^2}}{2}$, the value of $f'(p+1)$ is

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{1}{2p}$ D. $-\frac{1}{4}$ E. $-\frac{1}{2}$

Question 18

A die is marked with numbers from 1 to 6. The result of a roll of the die is the number that appears on the uppermost face of the die.
Which one of the followings could be a random variable of the probability experiment – roll a die twice?

A. The difference of the results in the two rolls.

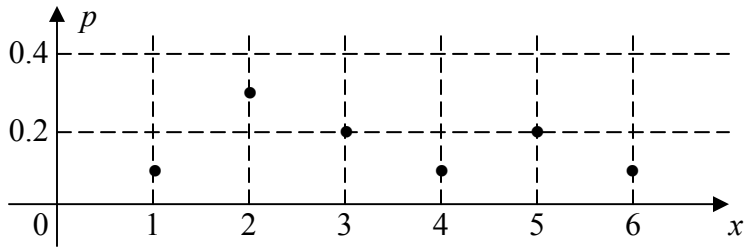
B. The result in the first roll is 5.

C. The difference of the results in the two rolls is 5.

D. The result in the first roll is greater than the result in the second roll.

E. The results in the two rolls are both even numbers.

Question 19
The graph below shows the probability distribution of random variable X.



Which one of the following statements in relation to the above probability distribution of X is correct?

A. Mode > median > mean

B. Median > mode > mean

C. Mean > mode > median

D. Mean > median > mode

E. Median > mean > mode

Question 20
X is a random variable with a binomial distribution, and $\Pr(X < 2) = (0.9)^6 + 6(0.1)(0.9)^5$. The values of n (number of trials) and p (probability for success) are

A. $n = 5,\ p = 0.9$

B. $n = 5,\ p = 0.1$

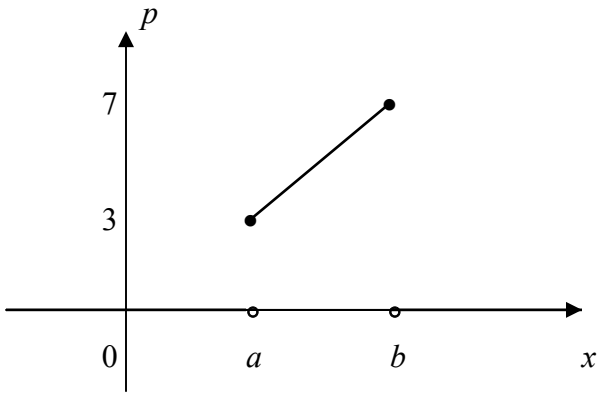
C. $n = 6,\ p = 0.1$

D. $n = 6,\ p = 0.5$

E. $n = 6,\ p = 0.9$

Question 21

$p(x)$ is a probability density function as shown in the graph below.



The value of $b - a$ is

A. 0.01 B. 0.1 C. 0.2 D. 0.3 E. 0.5

Question 22

X is a random variable with a standard normal distribution. If $\Pr(a \leq X < 2) = 0.8$, the value of a is closest to

A. 0.92 B. -0.92 C. -0.93 D. 0.93 E. 0

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Function $f(x) = x^4 - 4x^3 - 12x^2 + 32x$ has stationary points (–2, –64), (1, 17) and (4, –64).

a. Sketch the graph of $f(x)$ showing the stationary points and intercepts, correct to two decimal places if necessary.

3 marks

b. Function $g(x) = x^4 - 4x^3 - 12x^2 + 32x + p$ has exactly two x-intercepts. Find the value(s) of p.

2 marks

c. Function $h(x) = \frac{1}{4}\left[f(x) - x^4 + 4x^3\right]$ and function $k(x) = -h(1-x) + 2$.

Express $k(x)$ as a polynomial in x.

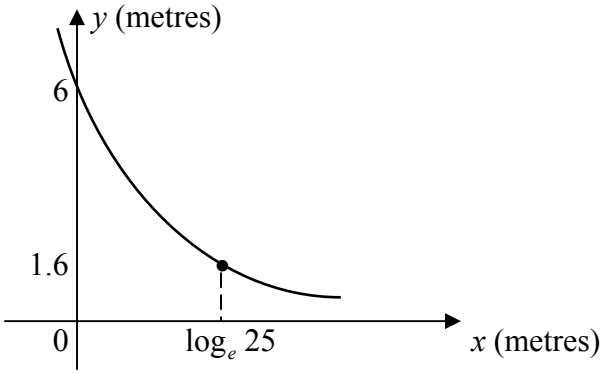
3 marks

d. i. Find the area (correct to 2 decimal places) bounded by $y = f(x)$ and the x-axis.

ii. Hence find the area (correct to 2 decimal places) bounded by $y = \frac{1}{2} f\left(1 - \frac{x}{2}\right)$ and the x-axis.

2 + 1 = 3 marks

Question 2



The curve above has equation in the form $y = pe^{\frac{-x}{r}} + q$, where $r = 2$.
It passes through the point ($\log_e 25$, 1.6) and cuts the y-axis at 6.

a. Write two equations involving p and q.

2 marks

b. Show that $p = 5.5$ and $q = 0.5$.

2 marks

c. Write down the equation of the asymptote.

1 mark

A waterslide has a profile given by $y = 5.5e^{\frac{-x}{2}} + 0.5$, $0 \le x \le 5$.

d. Find the average gradient (correct to 2 decimal places) of the waterslide.

2 marks

A child slides down the waterslide. Let (x, y) be the position of the child at time t seconds, where $y = 5.5e^{\frac{-x}{2}} + 0.5$, $0 \le x \le 5$.

e. Write down an equation that relates the vertical velocity $\frac{dy}{dt}$ to the horizontal velocity $\frac{dx}{dt}$ of the child at time t when $0 \le x \le 5$.

2 marks

At a particular time, $\frac{dy}{dt} = -1.1$ ms^{-1} and $\frac{dx}{dt} = 0.8$ ms^{-1}.

f. i. Find the exact gradient of the waterslide at where the child is at that particular time.

ii. Find the exact coordinates of the child at that particular time.

2 + 3 = 5 marks

The waterslide with profile $y = pe^{\frac{-x}{r}} + q$ for $0 \le x \le 5$, where $r = 2$, $p = 5.5$ and $q = 0.5$, is to be redesigned to make the magnitude of the average gradient of the waterslide **smaller**.

g. i. Which one of the parameters, p, q or r, would need to be changed without changing the maximum height of the water slide?

ii. Do you increase or decrease the value of the chosen parameter?

1 + 1 = 2 marks

Question 3

A parcel of land is bounded by a stream $y = x + \frac{3\pi}{4} + \frac{\pi}{\sqrt{2}}\cos\left(x + \frac{3\pi}{4}\right)$ and three wire fences $x = 0$, $y = 0$ and $x = 2\pi$. Length is measured in 100 metres. ***The y-axis points to the north***.

a. Find the exact coordinates of the four corners of the parcel of land.

3 marks

b. Sketch the parcel of land. Label the boundaries with equations, and the corners with coordinates.

2 marks

c. i. Write down a definite integral for the area of the parcel of land.

ii. Find the exact area (in m^2) of the parcel of land.

1 + 4 = 5 marks

A square building is to be erected on the parcel of land as far to the west as possible. Building regulations require it to be at least 15 metres from all boundaries.

d. Determine the floor area (correct to the nearest m^2) of the largest building possible.

4 marks

Question 4

The bonding strength X of superglue **A** is normally distributed. The mean is 50 kg and the standard deviation is 4kg.

The bonding strength X of superglue **B** has the following probability density function:

$$f(x) = \begin{cases} ke^{-kx} & x \geq 0 \\ 0 & elsewhere \end{cases}, \text{ where } k \in R^+ .$$

A broken rod is repaired with a drop of superglue **A** and then subjected to a test load of 49 kg.

a. What is the probability (correct to 3 decimal places) that the bonding will fail?

2 marks

Consider superglue B in questions **b i**, **b ii** and **b iii**.

b. i. Show that $\int_0^a f(x)dx \to 1$ as $a \to \infty$.

ii. Given $\Pr(0 \leq X \leq 50) = 0.5$, show that $k = 0.0139$.

iii. What is the value of the median of X?

2 + 3 + 1 = 6 marks

An identical broken rod is repaired with a drop of superglue **B** and then subjected to a test load of 49 kg.

c. What is the probability (correct to 3 decimal places) that the bonding will fail?

2 marks

Ten identical broken rods are repaired, four with superglue **A** and six with superglue **B**. Five of these rods are selected randomly one at a time and returned before the next one is selected.

d. i. What is the probability (correct to 2 decimal places) that more rods are repaired with superglue **A** than with superglue **B**?

ii. Given that in the random selection less rods or none are repaired with superglue **A**, what is the probability (correct to 2 decimal places) that two rods are repaired with superglue **A**?

3 + 2 = 5 marks

One of these ten repaired rods is selected randomly and subjected to a test load of 49 kg.

e. What is the probability (correct to 2 decimal places) that the bonding will fail?

2 marks

End of exam 2