Mathematical Methods 3/4 Trial Exam 2 Solutions 2007
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SECTION 1

1	2	3	4	5	6	7	8	9	10	11
D	C	D	E	A	B	B	B	D	E	E

12	13	14	15	16	17	18	19	20	21	22
B	C	A	D	D	E	A	D	C	C	C

Q1 $(2,0)$, $0=\sqrt{2a+b}$, $\therefore 2a+b=0$ ……(1)
$(4,4)$, $4=\sqrt{4a+b}$, $\therefore 4a+b=16$ ……(2)
Solve (1) and (2) simultaneously, $a=8$ and $b=-16$.

Q2 $\log_5 10=\dfrac{\log_{10}10}{\log_{10}5}=\dfrac{1}{\log_{10}5}=1.431$.

Q3 Use graphics calc to sketch $y=|\cos(3x)|$ and $y=0.15$. The number of intersections in $-\pi\le x\le\pi$ is 12.

Q4 $g(x)$ is the result of $f(x)$ undergoing reflection in the y-axis, horizontal dilation by a factor of $\frac{1}{2}$ and downward translation.

Q5 Domain of $f(x)$ is $(-1,\infty)$. For $f[g(x)]$ to be defined, $g(x)\in(-1,\infty)$ and hence $x\in(1,\infty)$.

Q6 Use graphics calc to sketch $y=\dfrac{x^2e^x}{(2\pi x)}$ and $y=1$. The x-coordinate of the intersection is 1.46

Q7 The repeated factors $(2x+a)^3$ and $(x-2b)^2$ indicate that $f(x)$ has a stationary inflection point on the x-axis at $x=-\dfrac{a}{2}$ and a turning point on the x-axis at $x=2b$. $\therefore f'(x)=0$ at $x=-\dfrac{a}{2}$ and $x=2b$.

Q8 The range of f is $(-\infty,1]\cup(2,\infty)$. This becomes the domain of f^{-1} .
Equation of f is $y=\dfrac{1}{x+1}+2$, $\therefore$equation of f^{-1} is $x=\dfrac{1}{y+1}+2$. Express y as the subject, $y=\dfrac{1}{x-2}-1$,
$\therefore f^{-1}(x)=\dfrac{1}{x-2}-1$.

Q9 Remainder theorem:
$R=P\left(-\frac{1}{2}\right)=32\left(-\frac{1}{2}\right)^5+8\left(-\frac{1}{2}\right)^3+2\left(-\frac{1}{2}\right)+1=-2$

Q10 $y=\sin\left(\dfrac{\pi x}{2}\right)-\dfrac{\pi x}{2}$, $-2\pi\le x\le 2\pi$. The tangent is parallel to the x-axis when $\dfrac{dy}{dx}=\dfrac{\pi}{2}\cos\left(\dfrac{\pi x}{2}\right)-\dfrac{\pi}{2}=0$, i.e. $\cos\left(\dfrac{\pi x}{2}\right)=1$.
Hence $\dfrac{\pi x}{2}=-2\pi,0,2\pi$, $\therefore x=-4,0,4$.

Q11 Let $y=e^{\sqrt{1+x^2}}$ and $u=\sqrt{1+x^2}$.
$\dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}=e^{\sqrt{1+x^2}}\times\dfrac{x}{\sqrt{1+x^2}}=\dfrac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$.

Q12 Rate of change = gradient of $y=\dfrac{\log_e|x^3+3|}{x^3+3}$.
Use graphics calc to find $\dfrac{dy}{dx}$ at $x=-2$.
$\dfrac{dy}{dx}=-0.29$, $\therefore$rate of decrease = 0.29 .

Q13 Use rectangles (left or right) to estimate.

Q14 $\displaystyle\int_0^2\frac{1}{2x-5}dx=\left[\frac{\log_e|2x-5|}{2}\right]_0^2$
$=\dfrac{\log_e|-1|}{2}-\dfrac{\log_e|-5|}{2}=-\dfrac{1}{2}\log_e 5=-\log_e\sqrt{5}$.

Q15 $\displaystyle\int_1^2[2f(x)-3]dx=2\int_1^2 f(x)dx-\int_1^2 3dx$
$=2[F(x)]_1^2-[3x]_1^2=2(F(2)-F(1))-(6-3)=2(4)-3=5$.

Q16
$\displaystyle\int_0^{\frac{\sqrt{\pi}}{2}}x\sin(2x^2)dx=\left[\frac{1}{2}\sin^2(x^2)\right]_0^{\frac{\sqrt{\pi}}{2}}=\frac{1}{2}\sin^2\left(\frac{\pi}{4}\right)=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2=\frac{1}{4}$.

Q17 $f'(x)=-\dfrac{x-p}{2\sqrt{2-(x-p)^2}}$, $f'(p+1)=-\dfrac{1}{2}$.

Q18 The difference of the results can be 0, 1, 2, 3, 4 or 5. It is a random variable.

Q19 Mode = 2,
mean $=1\times0.1+2\times0.3+3\times0.2+4\times0.1+5\times0.2+6\times0.1=3.3$,
median = 3.

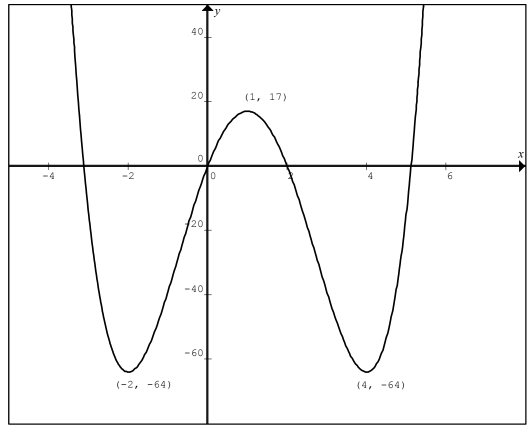
Q20 $X < 2 \Rightarrow X = 0,1$, i.e. one success or none.
$\therefore\ p = 0.1$ and $n = 1 + 5 = 6$.

Q21 Area under $p(x) = 1$, $\therefore \frac{1}{2}(3+7)(b-a) = 1$,
$\therefore b - a = 0.2$.

Q22 $\Pr(a \le X < 2) = 0.8$, $\therefore \Pr(X < 2) - \Pr(X < a) = 0.8$,
$\therefore \Pr(X < a) = \Pr(X < 2) - 0.8 = 0.977 - 0.8 = 0.177$
$\therefore a = -0.93$.

SECTION 2

Q1a Use graphics calc, find x-intercepts at x = -3.12, 0, 2, 5.12



Q1b $g(x) = f(x) + p$ has exactly two x-intercepts when $p = 64$ or $p < -17$, i.e. $f(x)$ is translated upwards by 64 units or downwards by more than 17 units.

Q1c $h(x) = \frac{1}{4}\left[f(x) - x^4 + 4x^3\right] = \frac{1}{4}\left(-12x^2 + 32x\right) = -3x^2 + 8x$
$k(x) = -h(1-x) + 2 = -\left[-3(1-x)^2 + 8(1-x)\right] + 2 = 3x^2 + 2x - 3$.

Q1di Use graphics calc to find the area of the three regions.
Area $= 123.3485 \times 2 + 22.4 = 269.10$ unit^2.

Q1dii The regions are dilated vertically by a factor of $\frac{1}{2}$ and horizontally by a factor of 2. The area remains the same, i.e. 269.10 unit^2. Reflection and horizontal translation do not change the area.

Q2a $(0,6)$ gives $p + q = 6$.
$(\log_e 25, 1.6)$ gives $pe^{\frac{-\log_e 25}{2}} + q = 1.6$.

Q2b $e^{\frac{-\log_e 25}{2}} = \left(e^{\log_e 25}\right)^{\frac{-1}{2}} = (25)^{\frac{-1}{2}} = \frac{1}{\sqrt{25}} = 0.2$.
Solve $p + q = 6$ and $0.2p + q = 1.6$ simultaneously to obtain $p = 5.5$ and $q = 0.5$

Q2c As $x \to \infty$, $e^{\frac{-x}{2}} \to 0$, $\therefore y \to 0.5$, $\therefore$asymptote is $y = 0.5$

Q2d When $x = 0$, $y = 6$. When $x = 5$, $y = 5.5e^{-2.5} + 0.5$.
Average gradient $= \frac{5.5e^{-2.5} + 0.5 - 6}{5 - 0} = -1.01$

Q2e $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$, $\frac{dy}{dt} = -2.75e^{\frac{-x}{2}} \times \frac{dx}{dt}$.

Q2fi $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-1.1}{0.8}=-1.375$

Q2fii $\frac{dy}{dx}=-2.75e^{\frac{-x}{2}}$, $-1.375=-2.75e^{\frac{-x}{2}}$, $\therefore e^{\frac{-x}{2}}=\frac{1}{2}$,

$e^{\frac{x}{2}}=2$, $\frac{x}{2}=\log_e 2$, $x=2\log_e 2$.

$y=5.5e^{\frac{-x}{2}}+0.5=5.5\times\frac{1}{2}+0.5=3.25$

Coordinates ($2\log_e 2$, 3.25).

Q2gi Horizontal dilation is required, so change parameter *r*.

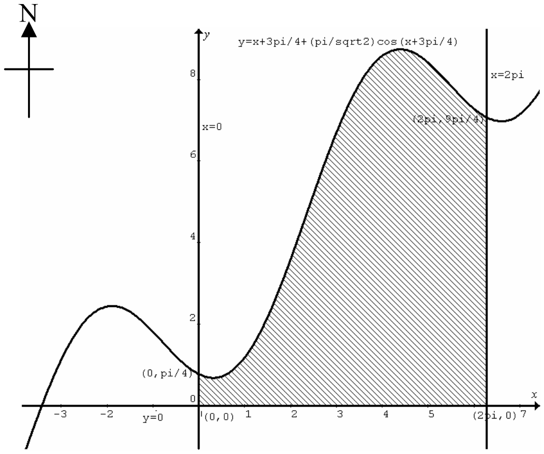
Q2gii Increase the dilation factor *r*.

Q3a $x=0$, $y=\frac{3\pi}{4}+\frac{\pi}{\sqrt{2}}\cos\left(\frac{3\pi}{4}\right)=\frac{3\pi}{4}-\frac{\pi}{2}=\frac{\pi}{4}$. $\left(0,\frac{\pi}{4}\right)$

$x=2\pi$, $y=2\pi+\frac{3\pi}{4}+\frac{\pi}{\sqrt{2}}\cos\left(2\pi+\frac{3\pi}{4}\right)=\frac{9\pi}{4}$. $\left(2\pi,\frac{9\pi}{4}\right)$

$(0,0)$, $(2\pi,0)$.

Q3b



Q3ci $\int_0^{2\pi}\left[x+\frac{3\pi}{4}+\frac{\pi}{\sqrt{2}}\cos\left(x+\frac{3\pi}{4}\right)\right]dx$

Q3cii $=\left[\frac{x^2}{2}+\frac{3\pi x}{4}+\frac{\pi}{\sqrt{2}}\sin\left(x+\frac{3\pi}{4}\right)\right]_0^{2\pi}$

$=\left[2\pi^2+\frac{3\pi^2}{2}+\frac{\pi}{2}\right]-\left[\frac{\pi}{2}\right]=\frac{7\pi^2}{2}$.

Land area $=\frac{7\pi^2}{2}\times 100^2$ m^2 $=35000\pi^2$ m^2.

Q3d Using graphics calc, the local minimum in $[0,2\pi]$ is (0.3185, 0.6910).

$\therefore$ the shortest distance between the north and the south boundaries is $0.6910\times 100=69.10$ m. Take off 15 m clearance from each boundary. The floor area $=(69.10-15\times 2)^2=1529$ m^2.

Q4a $\Pr(fail)=\Pr(X\le 49)=normalcdf(-E99,49,50,4)=0.401$

Q4bi $\int_0^a ke^{-kx}dx=\left[-e^{-kx}\right]_0^a=-e^{-ka}+1$.

As $a\to\infty$, $e^{-ka}\to 0$, $\therefore\int_0^a ke^{-kx}dx\to 1$.

Q4bii $\Pr(0\le X\le 50)=\int_0^{50}ke^{-kx}dx=\left[-e^{-kx}\right]_0^{50}=-e^{-k50}+1=0.5$,

$e^{-50k}=0.5$, $-50k=\log_e 0.5$, $\therefore k=0.0139$.

Q4biii Since $\Pr(0\le X\le 50)=0.5$, the median of X is 50.

Q4c $\Pr(fail)=\Pr(0\le X\le 49)=\int_0^{49}0.0139e^{-0.0139x}dx=0.494$.

Q4di Let *X* be the random variable – number of broken rods repaired with superglue A. $p=\frac{4}{10}=0.4$, $\therefore q=0.6$.

Pr (more with A than with B) $=\Pr(X=3)+\Pr(X=4)$

$={}^5C_3(0.4)^3(0.6)^2+{}^5C_4(0.4)^4(0.6)^1=0.31$.

Or $=binompdf(5,0.4,3)+binompdf(5,0.4,4)=0.31$.

Q4dii $\Pr(X=2|\,X=0,\text{ 1 or }2)=\frac{\Pr(X=2\cap X=0,1,2)}{\Pr(X=0,1,2)}$

$=\frac{\Pr(X=2)}{\Pr(X=0,1,2)}=\frac{binompdf(5,0.4,2)}{binomcdf(5,0.4,2)}=0.51$.

Q4e $\Pr(fail)=0.401\times 0.4+0.494\times 0.6=0.46$.

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