Insight Publications

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Trial Exam Paper

2007

MATHEMATICAL METHODS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided
- The question and answer book of 12 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions
- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = 2x - 5$ and $g(x) = \cos x$. Write down the rule of $f(g(x))$.

_____

_____

1 mark

Question 2

For the function $f : (2, \infty) \to R,\ f(x) = 2\log_e(x-1)$,

a. find the rule for the inverse function f^{-1}.

_____

_____

_____

2 marks

b. find the domain of the inverse function f^{-1}.

_____

_____

1 mark

Question 3

For the function $f:[-\pi,\pi] \to R, f(x) = -2\sin(3(x - \frac{\pi}{4}))$

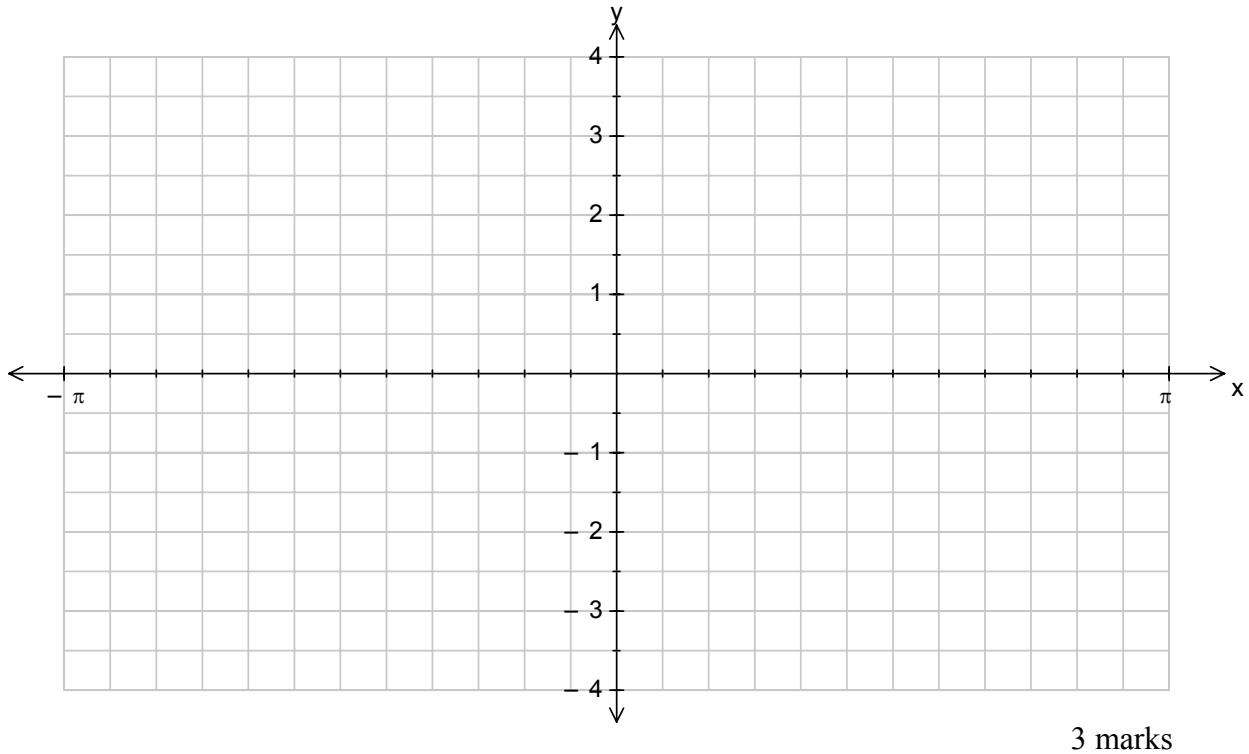
a. write down the amplitude and period of the function.

_____

_____

2 marks

b. on the set of axes below, sketch the graph of the function *f*. Label the axis intercepts with their coordinates. Label the end-points of the graph with their coordinates.



3 marks

Question 4

a. Let $f(x) = \log_e(\sin(x))$. Find $f'(x)$.

_____

_____

1 mark

b. Let $y = x^2 \cos(x)$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$.

_____

_____

_____

_____

_____

2 marks

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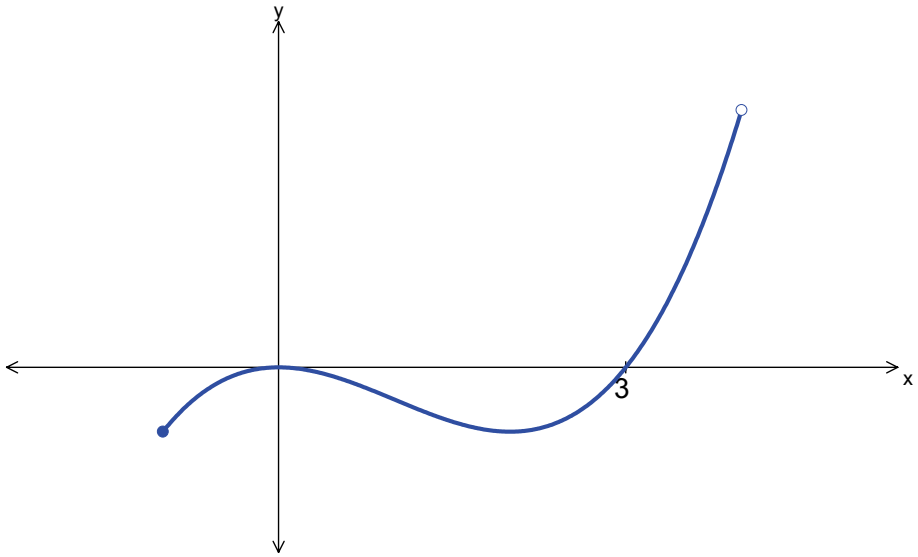
Question 5

The graph of $f:[-1,4] \to R$ where $f(x) = x^3 - 3x^2$ is shown below.



a. Let $g(x) = |f(x)|$. On the same set of axes, sketch the graph of g.

2 marks

b. State the domain of the derivative function g'.

_____

_____

1 mark

Question 6

Solve the equation $\sin(2x) - \sqrt{3}\cos(2x) = 0$ for $x \in [0, 2\pi]$, giving exact values in terms of π.

3 marks

CONTINUED
PLEASE TURN OVER

Question 7

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{k} & 2 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $k = 16$.

2 marks

b. Find $\Pr(X > 4)$

2 marks

c. Find the median of X

2 marks

Question 8

The random variable X has the following probability distribution:

x	–1	0	1	2
$Pr(X = x)$	$a + b$	$2a - b$	$3a$	0.4

a. Find the value of a.

1 mark

b. If $E(X) = 0.95$, find the value of b.

2 marks

Question 9

The random variable X is normally distributed with mean 50 and standard deviation 5. The random variable Z is normally distributed with mean 0 and standard deviation 1.

a. If $\Pr(X < 56) = \Pr(Z < a)$, find the value of a.

2 marks

b. If $\Pr(50 < X < b) = 0.5 - \Pr(Z > 2)$, find the value of b.

2 marks

Question 10

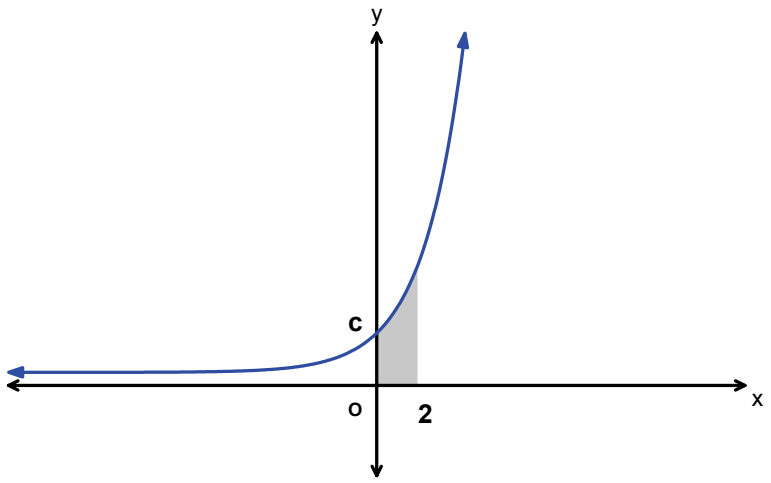
A hemispherical bowl is being filled with water at a constant rate of $150\pi \ \ cm^3/\min$. When the depth of the water in the bowl is h cm, the volume, V cm^3, of the water is given by $V = \pi h^2(30 - \frac{2}{3}h)$. Find the rate at which the depth of the water is increasing when the depth is 5 cm.

4 marks

CONTINUED
PLEASE TURN OVER

Question 11

Part of the graph of the function $f : R \to R,\ f(x) = ae^{2x} + b$ is shown below. If the shaded area is $3e^4 + 1$ square units, find one set of possible values for a, b and c, where c is the y-intercept of the graph $y = f(x)$.



5 marks

END OF QUESTION AND ANSWER BOOK