Trial Examination 2006

VCE Mathematical Methods Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.

Materials supplied

Question and answer booklet of 17 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this book during reading time.

Write **your name** and your **teacher's name** in the space provided above on this page.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Mathematical Methods Units 3 & 4 Written Examination 2.

 ABN 49 910 906 643 58 Pelham St Carlton VIC 3053 Tel: (03) 9663 2523 Fax: (03) 9663 7182 TEVMMU34EX2_QA_06.FM

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The depth of water, d metres, at the end of a pier is given by the equation $d = 12 + 3\cos\left(\frac{\pi}{12}t\right)$, where t is the time in hours after midday. The depth of water at the end of the pier at 3 am is closest to

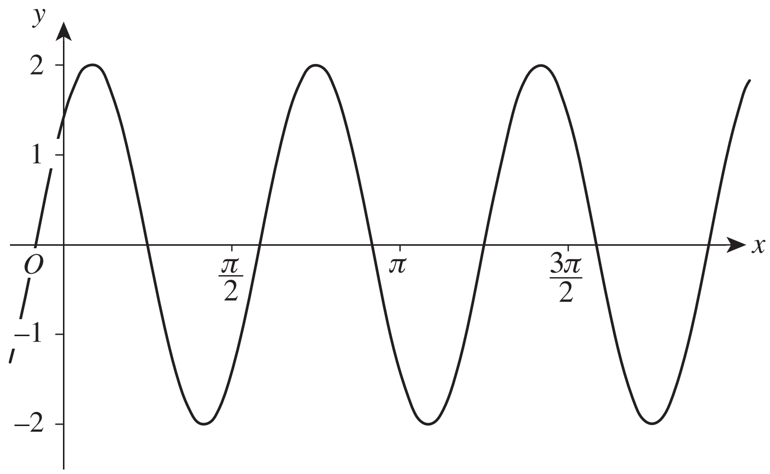
A. 9.10 m
B. 9.88 m
C. 11.8 m
D. 14.12 m
E. 15.0 m

Question 2

The sum of the solutions to the equation $16\sin(3x) = 8$ for the domain $[0, \pi]$ is

A. $\frac{\pi}{18}$
B. $\frac{\pi}{6}$
C. $\frac{\pi}{3}$
D. π
E. 2π

Question 3



The graph shown above has the form $y = a\sin(b(x - c))$. The values of a, b and c respectively could be

A. $2, 3, \frac{3\pi}{4}$

B. $-2, 3, \frac{3\pi}{4}$

C. $2, 3, \frac{\pi}{4}$

D. $-2, 3, \frac{\pi}{4}$

E. $-2, \frac{1}{3}, \frac{\pi}{4}$

Question 4

The function $f : (-\infty, a] \to R$ with rule $f(x) = 3(x + 2)^2 + 4$ will have an inverse function if

A. $a \le -2$
B. $a \le 0$
C. $a \le 2$
D. $a \le 3$
E. $a \le 4$

Question 5

The range of the function $f(x) = \log_3(x)$ where $x \in (0, 3]$ is

A. R

B. $R^+ \cup \{0\}$

C. $R^- \cup \{0\}$

D. $(0, 1]$

E. $(-\infty, 1]$

Question 6

Consider the equation $(x-a)^2(x^2+b)(x^3+c)=0$ where $a \neq b \neq c$ as well as $a > 0$, $b > 0$ and $c > 0$. The number of distinct real solutions that this equation has is

A. 0
B. 1
C. 2
D. 3
E. 4 or more

Question 7

If $f(x) = \sqrt{9-x^2}$ for $x \in [-3, 3]$ and $g(x) = \sqrt{x}$ for $x \in [0, \infty)$, then $f(g(x)) = \sqrt{9-x}$ for

A. $x \in R$
B. $x \in [0, 9]$
C. $x \in [-3, 3]$
D. $x \in [0, \infty)$
E. $x \in (-\infty, 9]$

Question 8

The graph of the function with rule $y = (x+1)^2$ is transformed by the following transformations, in order:

- a reflection in the x-axis;
- a translation of –2 units parallel to the y-axis; and then
- a dilation by a factor of 3 from the x-axis.

The rule of the function corresponding to the transformed graph is

A. $y = -3(x+3)^2 - 6$
B. $y = 3(-x+1)^2 - 6$
C. $y = 3(x+1)^2 - 6$
D. $y = -3(x+1)^2 - 2$
E. $y = -3(x+1)^2 - 6$

Question 9

If $16^x - 4^{x+1} = 32$, then x equals

A. -4
B. -1
C. -1 and 1.5
D. 1.5
E. 8

Question 10

If $4\log_2(x) + 4\log_2(\sqrt{x}) - \log_2(x^3) = -6$, then x equals

A. -2

B. $\frac{-2}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. 4

Question 11

If $f(x) = e^{-2x}\sin(x)$, then $f'(x)$ is equal to

A. $2e^{-2x}\sin(x) + e^{-2x}\cos(x)$

B. $-2e^{-3x}\sin(x) + e^{-2x}\cos(x)$

C. $-2e^{-2x}\cos(x)$

D. $-2e^{-2x}\sin(x) + e^{-2x}\sin(x)$

E. $-2e^{-2x}\sin(x) + e^{-2x}\cos(x)$

Question 12

The derivative of $\log_e\left(\frac{1}{\tan(x)}\right)$ with respect to x is

A. $-\sec^2(x)$

B. $\log_e(\sec^2(x))$

C. $\log_e(-\sec^2(x))$

D. $\frac{-1}{\sin(x)\cos(x)}$

E. $\frac{1}{\sin(x)\cos(x)}$

Question 13

The derivative of $\dfrac{20p}{(1-2p)^4}$ with respect to p is equal to

A. $\dfrac{20}{4(1-2p)^5}$

B. $\dfrac{20(1+6p)}{(1-2p)^5}$

C. $\dfrac{20(1-6p)}{(1-2p)^5}$

D. $\dfrac{20(1+2p)}{(1-2p)^5}$

E. $\dfrac{20}{-8(1-2p)^5}$

Question 14

The equation of the normal to the graph with equation $y = 3x^3 - 6x^2$ at the point where $x = 1$ is

A. $3y = x - 10$

B. $3y = x - 8$

C. $y = -3x$

D. $y = 3x - 6$

E. $3y = -x - 8$

Question 15

The average rate of change of the function $f(x) = (x+1)e^{2x}$ over the interval $[0, 2]$ is

A. $(2x+3)e^{2x}$

B. $\dfrac{7e^2 - 3}{2}$

C. $\dfrac{7e^4 - 3}{2}$

D. $\dfrac{3e^4 - 1}{2}$

E. $7e^4$

Question 16

If $\int_{\frac{\pi}{8}}^{a} \cos(2x)\,dx = 0$ and $0 \le x \le \pi$ then a equals

A. 0

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{8}$

D. $\frac{\pi}{2}$

E. π

Question 17

Using the **right rectangle** approximation with rectangles of width 1 unit, the area of the region bounded by the x-axis, the lines $x = 1$ and $x = 4$, and the curve with equation $y = 1 + \sqrt{x}$ is approximately equal to

A. $2 + \sqrt{2} + \sqrt{3}$

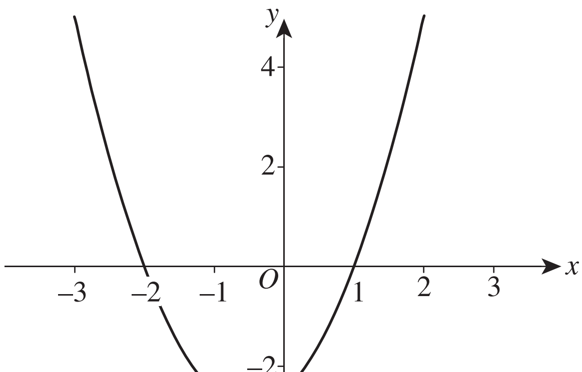
B. $3 + \sqrt{2} + \sqrt{3}$

C. $4 + \sqrt{2} + \sqrt{3}$

D. $5 + \sqrt{2} + \sqrt{3}$

E. $7\frac{2}{3}$

Question 18



The graph of $y = f'(x)$ is shown above. A possible equation for $y = f(x)$ could be

A. $y = (x + 2)(x - 1)$

B. $y = 2x + 1$

C. $y = x^3 + x^2 - 2x + c$ where c is a constant.

D. $y = k(x^3 + x^2 - 2x + c)$ where k and c are constants.

E. $y = k\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + c\right)$ where k and c are constants.

Question 19

A normal random variable, X, has a mean of 50.5 and a standard deviation of 3.5.

$\Pr(X > 48)$ is closest to

A. 0.237

B. 0.238

C. 0.688

D. 0.762

E. 0.763

Question 20

Thirty per cent of men in a company are over the age of 45 years. If five randomly selected men from the company are chosen, the probability that at least two of them are over 45 years of age is closest to

A. 0.013

B. 0.163

C. 0.472

D. 0.528

E. 0.640

Question 21

The volume of soft drink in certain bottles is normally distributed with a mean of 997 mL. If 98% of bottles have less than 1005 mL of soft drink, then the standard deviation is

A. 2.1 mL

B. 3.8 mL

C. 3.9 mL

D. 8.2 mL

E. 9.5 mL

Question 22

The continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} \dfrac{2}{x}, & 1 \le x \le a \\ 0, & x < 1 \text{ or } x > a \end{cases}$$

The value of a is

A. $\sqrt{e}$

B. 2

C. e

D. $2e$

E. e^2

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

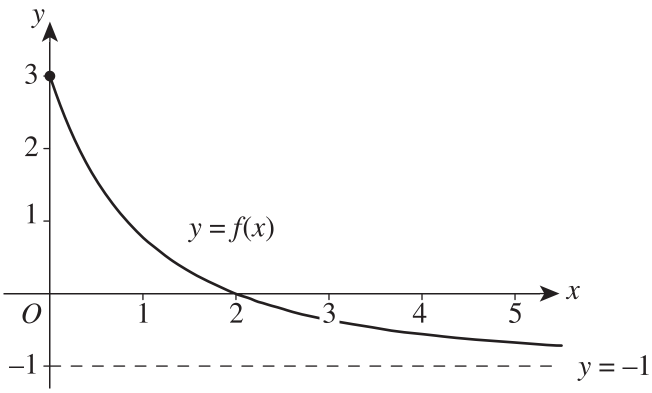
Question 1

The graph shown on the axes below has an equation of the form

$$f(x) = \frac{a}{(x+b)^2} + c, \quad x \geq 0$$

where a, b and c are integers.

The graph has axial intercepts at $x = 2$ and $y = 3$. The line with equation $y = -1$ is a horizontal asymptote.

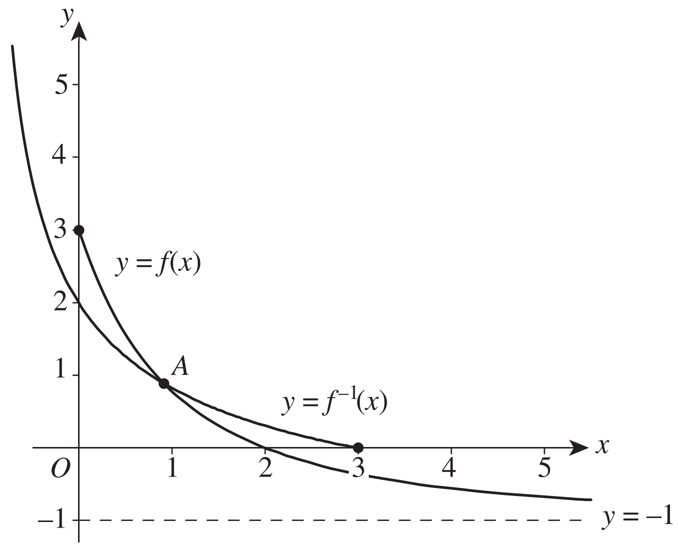


a. **i.** Find the value of c.

ii. Set up and solve suitable equations to show that $a = 16$ and $b = 2$.

1 + 4 = 5 marks

b. The graph below includes the graph of $y = f^{-1}(x)$.



Find the rule for $f^{-1}(x)$ and state the domain and range of f^{-1}.

4 marks

c. Find the x-coordinate of the point of intersection, A. Give your answer correct to four decimal places.

1 mark

Trial Examination 2006

VCE Mathematical Methods Units 3 & 4

Written Examination 2

Formula Sheet

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin(A)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

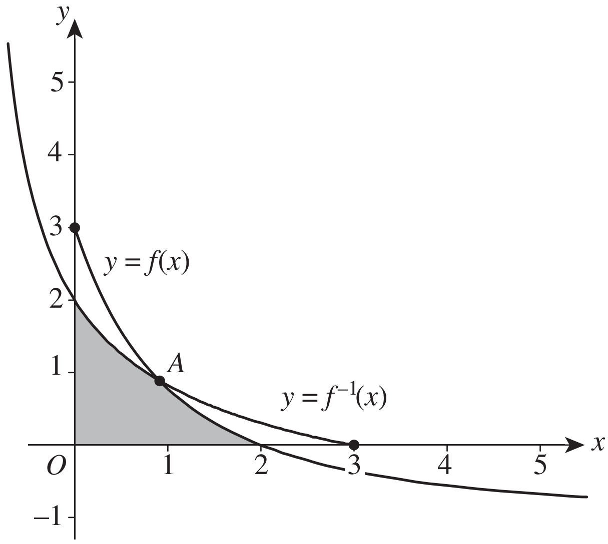
mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{Var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma xp(x)$	$\sigma^2 = \Sigma(x-\sigma)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\ dx$	$\mu = \int_{-\infty}^{\infty} xf(x)\ dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$

END OF FORMULA SHEET

d. A jeweller will use $f(x)$ and $f^{-1}(x)$ to design a pendant with the same shape as the region bounded by the x-axis, the y-axis and the graphs of $y = f(x)$ and $y = f^{-1}(x)$ (the shaded area in the graph below) with 1 unit representing 1 centimetre.



Use calculus to find the area of the pendant in cm^2. Give your answer correct to two decimal places.

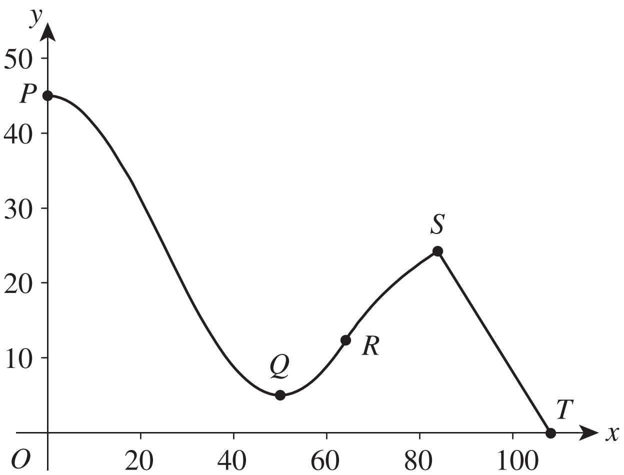
4 marks
Total 14 marks

Question 2

A new waterslide has been built at an amusement park. Riders enter the slide from a platform at P and slide down to a minimum height at Q. They have sufficient energy to slide up the waterslide to S where they abruptly turn and race down a straight path into the water at T. The cross-section of the waterslide ride is shown below. Its function, $f(x)$, is made up of three parts: a trigonometric function from P to R, a logarithmic function from R to S and a straight line ST.

$$f(x) = \begin{cases} A\cos(nx) + B, & 0 \le x \le 64 \\ C\log_e(D(x-50)), & 64 \le x \le p \\ p + q - x, & p \le x \le t \end{cases}$$

where A, B, C, D, n, p, q and t are constants to be found.



Sections of the waterslide from P to R have zero gradient, at P $(0, 45)$ and Q $(50, 5)$.
The waterslide is **smoothly joined** at the point R $(64, r)$: that is, it has **equal gradients** there.
The gradient of the logarithmic curve at S (p, q) is 0.4.
It should be noted that the curve is continuous at S, but it is **not** smoothly joined there.

a. Write down the values of A, B and n for $f(x)$ in the domain $0 \le x \le 64$.

3 marks

b. Using your values of A, B and n, find $f'(x)$ for the domain $0 \le x \le 64$.

1 mark

c. Using $f(x) = A\cos(nx) + B$ at $x = 64$, find (correct to four decimal places) the values of

i. r

ii. $f'(64)$

1 + 1 = 2 marks

d. Hence, using your results from **c.**, find the values of C and D. Give your answers correct to four decimal places.

4 marks

e. Using your values for C and D from **d.**, find the values of p and q to the nearest integer.

3 marks

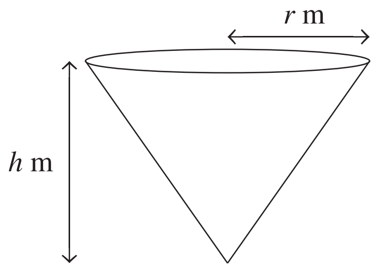
f. Hence find the value of t to the nearest integer.

1 mark

Total 14 marks

Question 3

A grain storage container is shaped like an **open** right-circular cone of height h m and radius r m as shown in the diagram below.



a. If the volume of this container is $4\pi\ \text{m}^3$, show that $h = \frac{12}{r^2}$.

1 mark

b. Given that the curved surface area of a cone of height h and radius r is $\pi r\sqrt{r^2 + h^2}$, show that the curved surface area of this cone, A m^2, can be expressed as $A = \frac{\pi\sqrt{r^6 + 144}}{r}$.

3 marks

c. Find $\frac{dA}{dr}$.

3 marks

d. In order to minimise construction costs, the container is initially designed with a minimal surface area while holding 4π m^3 of grain. Use your answer to **c.** to find the dimensions of such a container in metres. Give your answer correct to two decimal places.

3 marks

e. The manufacturers decide to build a **different** open right-circular cone container with height 3 m and radius 2 m.

Grain is released from this container into trailers below through a small hole at the vertex of the cone. Grain falls from the container at a rate of $0.5\sqrt{H}$ m^3/min where H is the height of the grain above the vertex at any time, t.

Find

i. the volume of grain in the container in terms of H at any time t;

ii. the rate at which the height of the grain is changing when $H = 1.5$. Give your answer correct to two decimal places.

1 + 3 = 4 marks

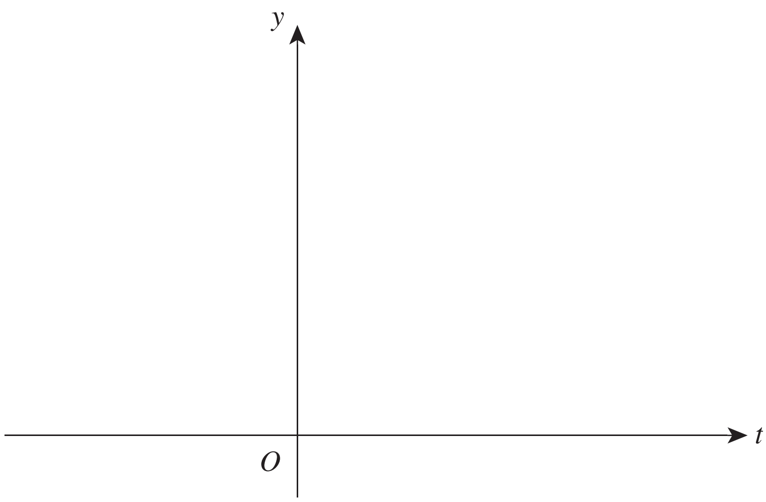
Total 14 marks

Question 4

The time interval, in minutes, between telephone calls to an operator at a particular call centre follows a distribution where the probability density function is given by

$$f(t)=\begin{cases}\frac{1}{2}e^{-\frac{t}{2}}, & t\geq 0\\ 0, & \text{otherwise}\end{cases}$$

a. Sketch the graph of $y=f(t)$ on the axes provided.



3 marks

b. Use calculus to find the exact value of the median time interval.

3 marks

c. Use calculus to find the probability that there will be an interval of at least two minutes between the current call and the next. Give your answer in exact form.

3 marks

d. Time intervals between successive calls are independent events. Find the probability that exactly 3 of the next 5 time intervals between calls will be less than 2 minutes. Give your answer correct to four decimal places.

3 marks

e. Given that the call-centre operator has waited for 2 minutes since the last call, find the exact probability that he will have to wait for at least 1 further minute until the next call.

4 marks

Total 16 marks

END OF QUESTION AND ANSWER BOOKLET

09999 : VICTORIAN SECONDARY COLLEGE

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VCE MATHEMATICAL METHODS (CAS)
Written Examination 2
ANSWER SHEET – 2009

STUDENT NAME: JOHN STUDENT

STUDENT NUMBER

9	9	1	2	3	4	5	6	A
0	0	0	0	0	0	0	0	■
1	1	■	1	1	1	1	1	E
2	2	2	■	2	2	2	2	F
3	3	3	3	■	3	3	3	G
4	4	4	4	4	■	4	4	J
5	5	5	5	5	5	■	5	L
6	6	6	6	6	6	6	■	R
7	7	7	7	7	7	7	7	T
8	8	8	8	8	8	8	8	W
■	■	9	9	9	9	9	9	X

INSTRUCTIONS:

USE PENCIL ONLY

SIGN HERE IF YOUR NAME AND NUMBER ARE PRINTED CORRECTLY.

SIGNATURE: J. Student

If your name or number on this sheet is incorrect, notify the Supervisor.
Use a **PENCIL** for **ALL** entries. For each question, shade the box which indicates your answer.
All answers must be completed like **THIS** example: A ■ C D E
Marks will **NOT** be deducted for incorrect answers.
NO MARK will be given if more than **ONE** answer is completed for any question.
If you make a mistake, **ERASE** the incorrect answer – **DO NOT** cross it out.

SUPERVISOR USE ONLY

USE PENCIL ONLY

Shade the "**ABSENT**" box if the student was absent from the examination.

ABSENT ☐

SUPERVISOR'S INITIALS

ONE ANSWER PER LINE

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

Please DO NOT fold, bend or staple this form.