Trial Examination 2006

VCE Mathematical Methods Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator.

Materials supplied

Question and answer booklet of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Mathematical Methods Units 3 & 4 Written Examination 1.

 ABN 49 910 906 643 58 Pelham St Carlton VIC 3053 Tel: (03) 9663 2523 Fax: (03) 9663 7182

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

A random variable X has the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	$3a$	$4a$	$2a$	a

a. Find the value of a.

b. Find $\Pr(X \leq 2)$.

c. Evaluate $\mathrm{E}(2X + 1)$.

1 + 1 + 2 = 4 marks

Question 2

a. If $f(x) = e^{x+1} - 2$ for $x \in R$, find the rule for the inverse function, f^{-1}.

b. Write down the domain of f^{-1}.

2 + 1 = 3 marks

Question 3

The graph of $f(x) = x^2$ is transformed to the graph of $g(x) = 2x^2 + 4x - 7$.

a. Write $g(x)$ in the form $2(x + a)^2 + b$, where a and b are integers.

b. Describe the transformations which map $f(x)$ to $g(x)$.

2 + 2 = 4 marks

Question 4

Find the exact solutions of $\cos(3\pi x) = -\sin(3\pi x)$ for $0 \le x \le 1$.

3 marks

Question 5

A curve has equation $y = x^2 e^{-3x}$.

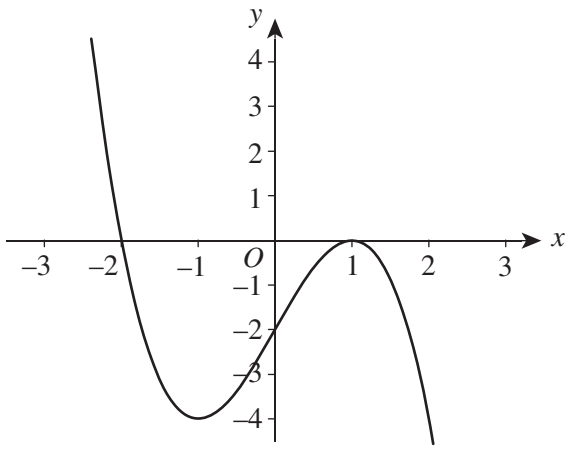
a. Find $\frac{dy}{dx}$.

b. Write down the **exact** coordinates of all stationary points.

2 + 2 = 4 marks

Question 6

Let $f : R \to R, f(x) = -(x + 2)(x - 1)^2$. Part of the graph of $y = f(x)$ is shown below.



a. On the same axes, sketch the graph of $y = g(x)$ where $g(x) = |f(x)|$, clearly labelling any axis intercepts.

b. Write down the domain of g'.

c. Find the area enclosed by the curve with equation $y = g(x)$ and the x-axis.

1 + 1 + 3 = 5 marks

Question 7

The function f is defined as

$$f(x) = \begin{cases} k(x+1), & 0 \leq x \leq 2 \\ 0, & x < 0 \text{ or } x > 2 \end{cases}$$

a. Find k so that $f(x)$ could be used as a probability density function of a continuous random variable X.

b. Find the median of X, giving your answer in exact form.

2 + 3 = 5 marks

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Formula Sheet

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax) \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax) \qquad \int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{Var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\sigma)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\ dx$	$\mu = \int_{-\infty}^{\infty} xf(x)\ dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$

END OF FORMULA SHEET

Question 8

Consider the following functions:

$$f(x) = 3\sin(2x),\ 0 \le x \le \pi$$
$$g(x) = 1 - x^2,\ x \in R$$

a. Show that the composite function with rule $g(f(x))$ exists.

b. Write down the rule for $g(f(x))$ and state the domain of this composite function.

c. What is the range of this composite function?

1 + 2 + 1 = 4 marks

Question 9

If $f(x+h) \approx f(x) + hf'(x)$ and $f(x) = \frac{1}{\sqrt{x}}$, find an approximation to the value of $\frac{1}{\sqrt{3.9}}$.
Give your answer in the form $\frac{a}{b}$, where a and b are integers.

4 marks

Question 10

Consider the curve with equation $y = 4x^3 + 1$. Find the equation of the tangent which touches this curve and passes through the origin.

4 marks

END OF QUESTION AND ANSWER BOOKLET