Insight Publications

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Trial Exam Paper

2006

MATHEMATICAL METHODS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.

Materials provided
- The question and answer book of 25 pages, with a separate sheet of miscellaneous formulas.
- An answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the box provided and on the answer sheet for multiple-choice questions.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam
- Place the answer sheet for multiple-choice question inside the front cover of this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer will be awarded one mark and an incorrect answer will be awarded no marks.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.

Question 1

Which one of the following functions does **not** have an inverse function?

A. $f:[-3,1) \to R,\ f(x) = \sqrt{x+3}$

B. $g: R^+ \to R,\ g(x) = \dfrac{1}{x^3} + 3$

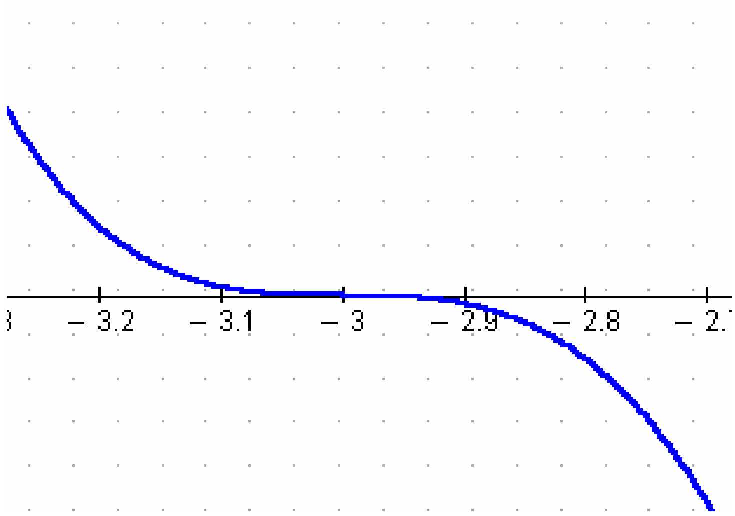
C. $h: R \to R,\ h(x) = x^5$

D. $k:[0,\infty) \to R,\ k(x) = x^2 + 1$

E. $m: R^+ \to R,\ m(x) = |2x-5|$

Question 2

A polynomial function p has degree 4. A part of its graph, near the point on the graph with the coordinates $(-3, 0)$, is shown below.



Which one of the following could be the rule for the fourth degree polynomial p?

A. $p(x) = (x+3)^4$

B. $p(x) = x^3(x+3)$

C. $p(x) = (x-1)(x+3)^3$

D. $p(x) = -x(x+3)^3$

E. $p(x) = x^2(x+3)^2$

Question 3

The maximal domain, D, of the function $f : D \to R$ with the rule $f(x) = \log_2((x-2)^2)$ is

A. $R \setminus \{0\}$

B. $R \setminus \{2\}$

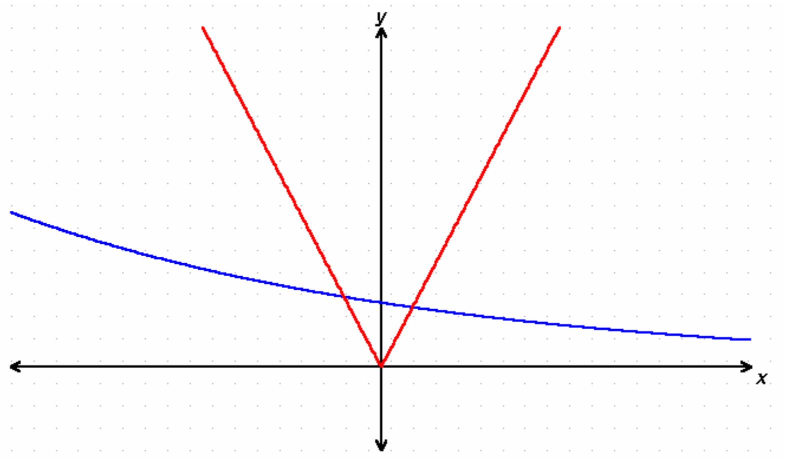
C. R

D. $(2, \infty)$

E. $(-\infty, 2)$

Question 4

Part of the graphs of the functions with equations $y = |3x|$ and $y = e^{-0.4x}$ are shown below.



The solution of the equation $|3x| = e^{-0.4x}$ for $x < 0$ is closest to

A. 0.30

B. 0.29

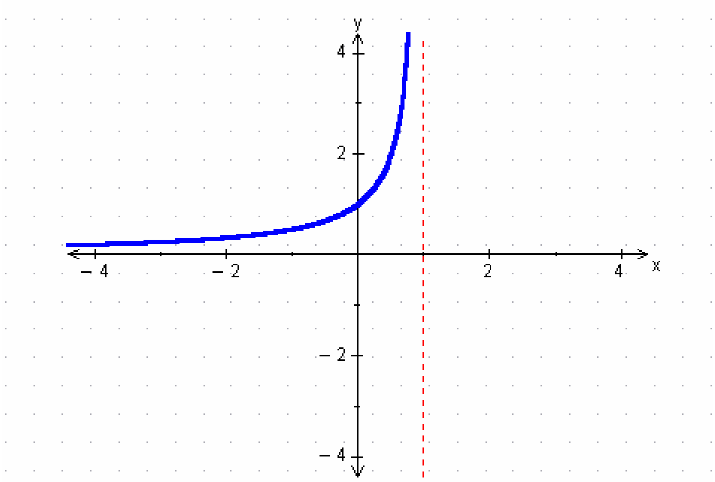
C. 1.17

D. −0.38

E. −0.39

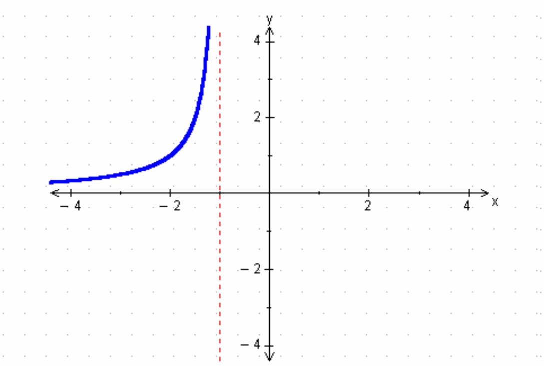
Question 5

Part of the graph with the function with rule $y = f(x)$ is shown below.

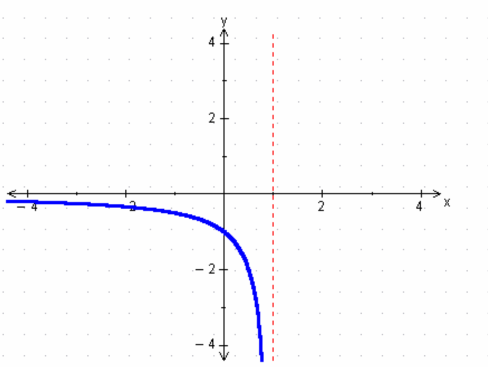


Which one of the following is most likely to be the corresponding part of the graph of the function with rule $y = f(-x)$

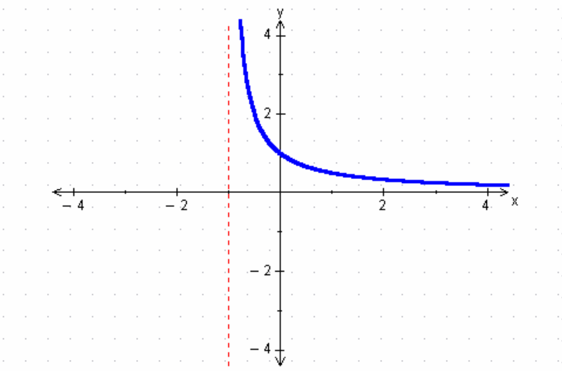
A.



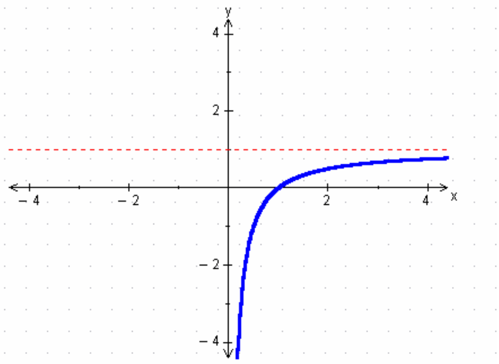
B.



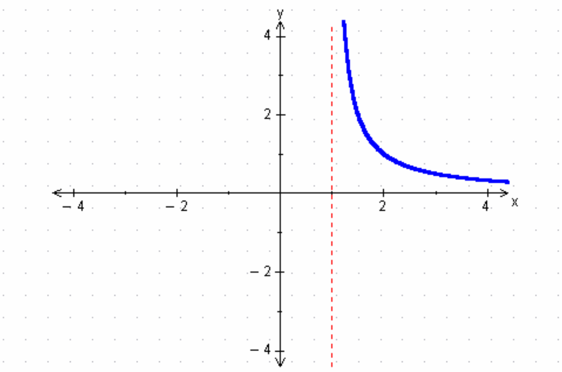
C.



D.



E.



Question 6

The **linear** factors of $x^4 + x^2 - 2$ over R are

A. $x - 1$

B. $x - 1, \; x + 1$

C. $x - 1, \; x + 1, \; x^2 + 2$

D. $x - 1, \; x + 1, \; x - \sqrt{2}, \; x + \sqrt{2}$

E. $x - 1, \; x + 1, \; x, \; x + 2$

Question 7

The number of solutions to the equation $(x^2 + a)(x^3 - b)(x + c) = 0$ where $a, b, c \in R^+$ is

A. 6

B. 5

C. 4

D. 3

E. 2

Question 8

$\log_7 9$ is equal to

A. $\dfrac{\log_e 7}{\log_e 9}$

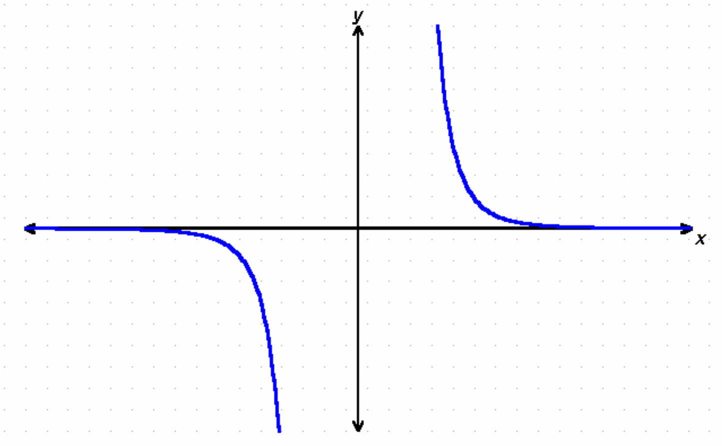
B. $\dfrac{\log_e 9}{\log_e 7}$

C. $\log_e (\dfrac{9}{7})$

D. $\log_e 2$

E. $\log_e 9 - \log_e 7$

Question 9



The rule for the graph shown could be

A. $y = x^{\frac{1}{5}}$

B. $y = x^{\frac{5}{2}}$

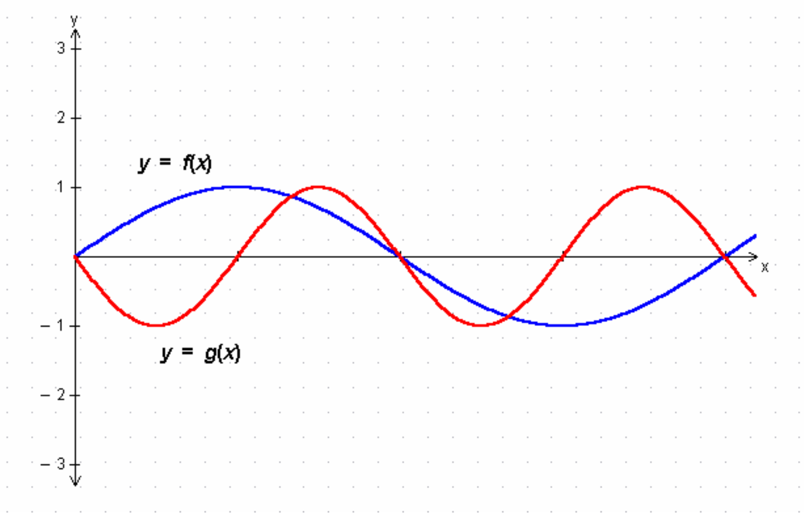
C. $y = x^{\frac{2}{5}}$

D. $y = \dfrac{1}{x^5}$

E. $y = \left| x^{\frac{1}{5}} \right|$

Question 10

The diagram below shows the graphs of two circular functions, *f* and *g*.



The graph of the function with the equation $y = f(x)$ is transformed into the graph of the function with equation $y = g(x)$ by

A. a reflection in the x-axis and a dilation of factor 2 from the y-axis.

B. a reflection in the y-axis and a dilation of factor 2 from the x-axis.

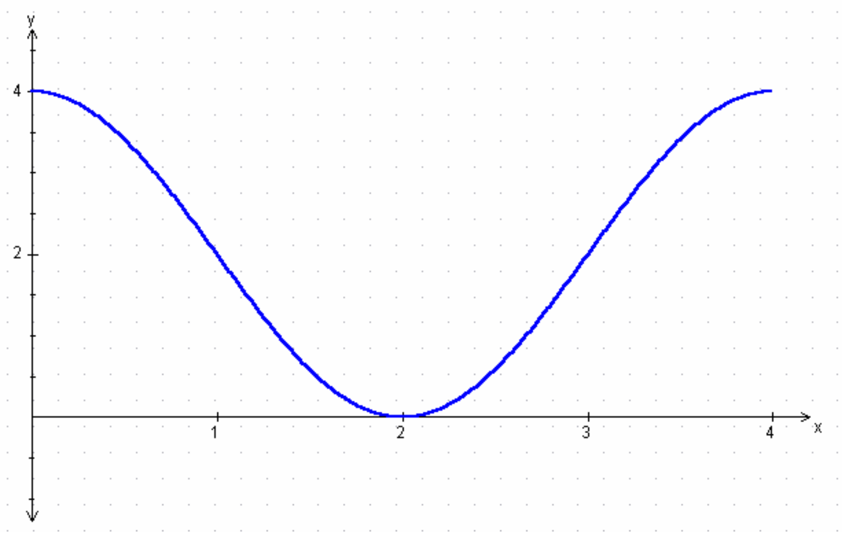
C. a reflection in the x-axis and a dilation of factor $\frac{1}{2}$ from the y-axis.

D. a reflection in the x-axis and a dilation of factor $\frac{1}{2}$ from the y-axis.

E. a reflection in the y-axis and a dilation of factor π from the y-axis.

Question 11

The diagram below shows one cycle of the graph of a circular function.



A possible equation for the function whose graph is shown is

A. $y = 4\cos(4x) + 2$

B. $y = 2\cos(2x) + 2$

C. $y = 2\cos(\frac{1}{2}x) + 2$

D. $y = 2\cos(\frac{\pi}{2}x) + 2$

E. $y = 4\cos(8\pi x) + 2$

Question 12

The depth of water near the Sorrento pier changes with the tides and can be given by the rule

$$d(t) = 8 - 3\sin(\frac{4\pi t}{25} + \frac{3\pi}{2})$$

where t is the time in hours after high tide and d is the depth in metres. A high tide occurred at 3 a.m. The time of the next **low tide** is

A. 9:15 a.m.

B. 3:50 a.m.

C. 9:25 a.m.

D. 3:30 a.m.

E. 3:50 p.m.

Question 13

If $y = |\cos(2x)|$, then the rate of change of y with respect to x at $x = k$, $\frac{\pi}{2} < x < \frac{3\pi}{4}$, is

A. $-2\sin(2k)$

B. $2\sin(2k)$

C. $-\sin(2k)$

D. $-\cos(2k)$

E. $\frac{1}{2}\sin(2k)$

Question 14

An ice-block melts and forms a circular puddle on the floor. The radius of the puddle increases at a rate of 3 cm/min. The rate at which the area is increasing, in cm^2/min, when the radius is 2 cm is

A. 6π

B. 12π

C. 3π

D. $\frac{12}{\pi}$

E. 24π

Question 15

If $y = \log_e(\sin(3x))$, then $\frac{dy}{dx}$ is equal to

A. $3\tan(3x)$

B. $-3\tan(3x)$

C. $\frac{1}{\tan(3x)}$

D. $\frac{3}{\tan(3x)}$

E. $3\cos(3x)$

Question 16

If u is a function of x and if $y = u(x).x^{\frac{2}{3}}$, then the rate of change of y with respect to x when $x = 8$ is equal to

A. $\frac{2}{3}u(8)x^{\frac{-1}{3}}$

B. $\frac{1}{3}u'(8)$

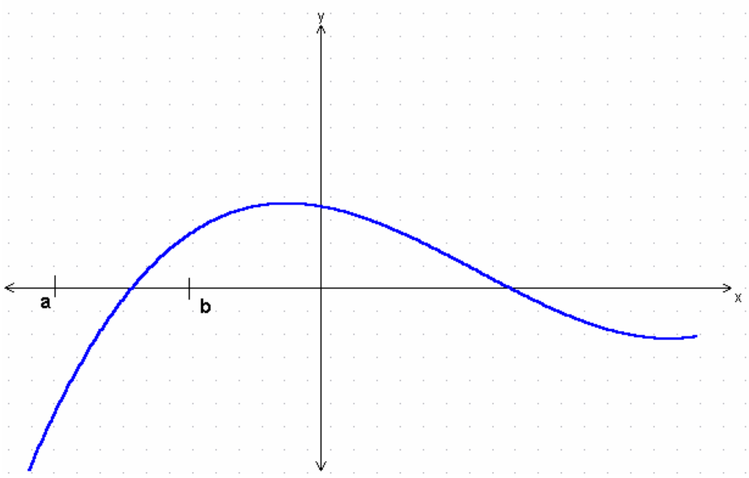
C. $4u'(8)+\frac{1}{3}u(8)$

D. $4u'(8)+4u(8)$

E. $4u(8)$

Question 17

Part of the graph of a function f is shown below.



Let g be a function such that $g'(x) = f(x)$.

Over the interval (a,b), the graph of g will have a

A. point of inflection.

B. positive gradient.

C. local maximum value.

D. local minimum value.

E. negative gradient.

Question 18

If $f'(x) = 3\cos(2x-3)+1$, then $f(x)$ could be equal to

A. $-6\sin(2x-3)+x$

B. $\frac{3}{2}\sin(2x-3)+x$

C. $\frac{2}{3}\sin(2x-3)+x$

D. $\frac{-3}{2}\sin(x^2-3x)+x$

E. $-6\sin(2x)$

Question 19

If $\int_1^2 f(x)dx = 4$, then $\int_2^3 2f(x-1)dx$ is equal to

A. 8

B. 7

C. 6

D. 4

E. 3

Question 20

The number of defective calculators in a box of calculators ready for sale, is a random variable with a binomial distribution with mean 20 and variance of 14.

If a calculator is drawn from the box, the probability that it is defective is

A. 0.1

B. 0.2

C. 0.3

D. 0.5

E. 0.7

Question 21

The random variable X has the following probability distribution, where $0 < p < \frac{1}{2}$.

x	0	1
$\Pr(X = x)$	$2p$	$1-2p$

The standard deviation of X is

A. $1-2p$

B. $p(1-2p)$

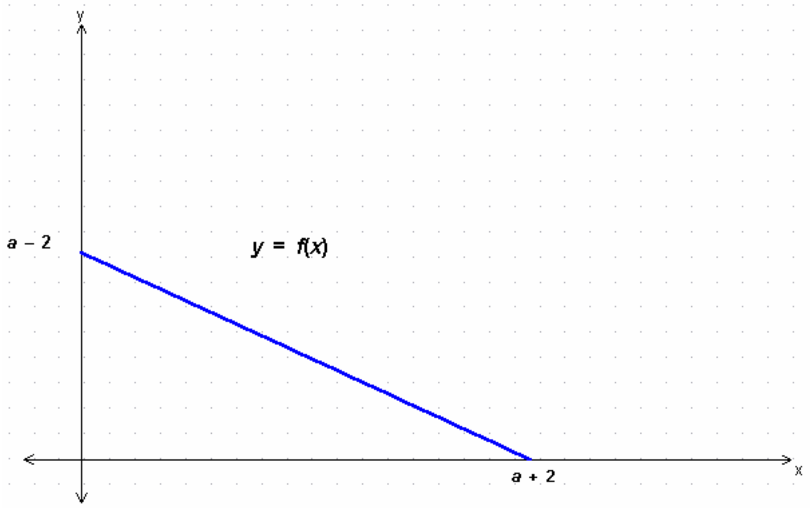
C. $2p(1-2p)$

D. $\sqrt{2p(1-2p)}$

E. $\sqrt{p(1-p)}$

Question 22

The graph shown below represents a probability density function.



The value of a is

A. $\frac{1}{4}$

B. 3

C. $\sqrt{5}$

D. $\sqrt{6}$

E. 6

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal answer will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise stated diagrams are not drawn to scale.

Question 1

The team from 'Frontyard Frenzy', a garden renovation television show, are doing a garden makeover. As part of the makeover they are constructing a wire wall sculpture. Two identical pieces of curved wire are to be used. Each piece has a shape that can be defined by the rule

$$y_w = \frac{1}{2000}x(x-10)(x^2-60x+910) \text{ for } x \in [0,38]$$

where all measurements are in centimetres.

a. Show that $x^2 - 60x + 910 > 0$.

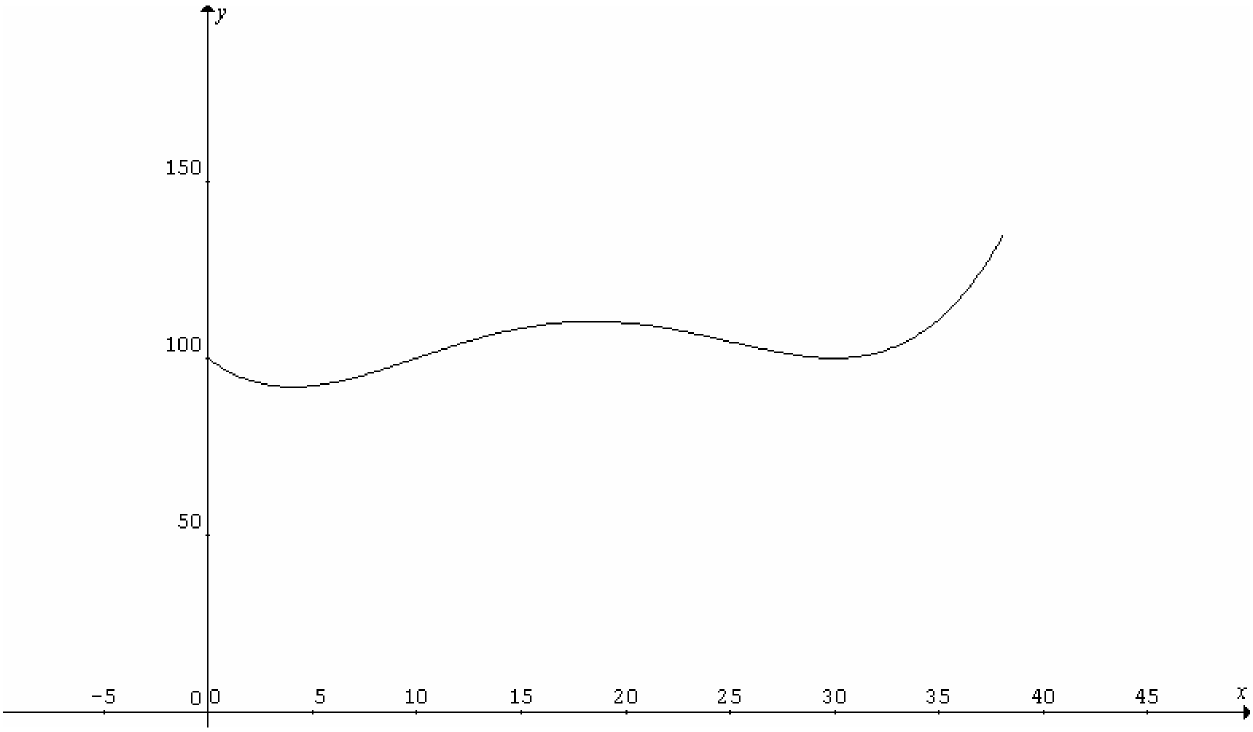
2 marks

b. Find the x-intercepts of the graph of $y_w = \frac{1}{2000}x(x-10)(x^2-60x+910)$.

1 mark

SECTION 2 – Question 1 – continued
TURN OVER

One piece of wire (wire a, shown by the curve y_a) is to be attached to the wall with the starting point raised 100 cm off the ground as shown in the graph below.



c. State the equation of the assembled piece of wire, y_a, as shown in the diagram above.

1 mark

The other piece of wire is attached to the wall and is positioned so that it is a reflection of wire a in the line $y = 120$. Its equation is given as $y_b = 140 - \frac{1}{2000}x(x-10)(x^2 - 60x + 910)$.

d. State the transformations (in the correct order) involved in producing y_b from y_w.

2 marks

e. The two wires are to be secured to the wall at the endpoints. State the coordinates of the endpoints for both pieces of wire (correct to two decimal places).

2 marks

f. The two wires intersect at one point in the domain. Find the point of intersection (correct to two decimal places).

1 mark

It is decided to support the wires by placing vertical slats at selected intervals between the two curved wires.

g. One vertical slat is to be placed at $x = 28\text{ cm}$. Find the minimum length of this slat (correct to two decimal places).

2 marks

h. **i.** Two slats of length 10 cm are used. Where along the wires will slats of this length need to be positioned (correct to two decimal places)?

ii. Find the area of the region enclosed by the two curves and the two vertical slats of length 10 cm (correct to two decimal places).

2 + 2 = 4 marks

Total 15 marks

Question 2

Ethan, a curious young boy, is fascinated by his front-loading washing machine. He likes to watch it go through all the cycles and is particularly interested as to how fast it revolves when on spin cycle. He decides to place a red marker on the inside of the bowl and watches the red marker revolve around the edge of the bowl. The red marker is at its lowest point when the cycle begins, and the bowl is 70 cm in diameter. He times the spin cycle and notes that the red marker has gone around five times in four seconds.

The vertical position of the red marker can be modelled by the equation

$$y = A\cos nt + b$$

where y is the vertical position in centimetres from the starting point and t is the time in seconds.

Assume the bowl spins continuously in one direction.

a. State the period and show that $n = \frac{5\pi}{2}$.

2 marks

b. Show that the value of A is −35 and b is 35

2 marks

SECTION 2 – Question 2 – continued
TURN OVER

c. If the washing machine spins continuously for three minutes, explain why the marker will be at its lowest point at the start of every minute.

2 marks

A tumble dryer with diameter of 40 cm is mounted directly above the washing machine. Ethan continues his experiment by placing a blue marker on the internal drum of the dryer. The internal drum of the dryer moves so that it takes 1.2 seconds to complete one cycle. He starts both machines at the same time with both markers at their lowest points and the blue marker 120 cm above the red marker.

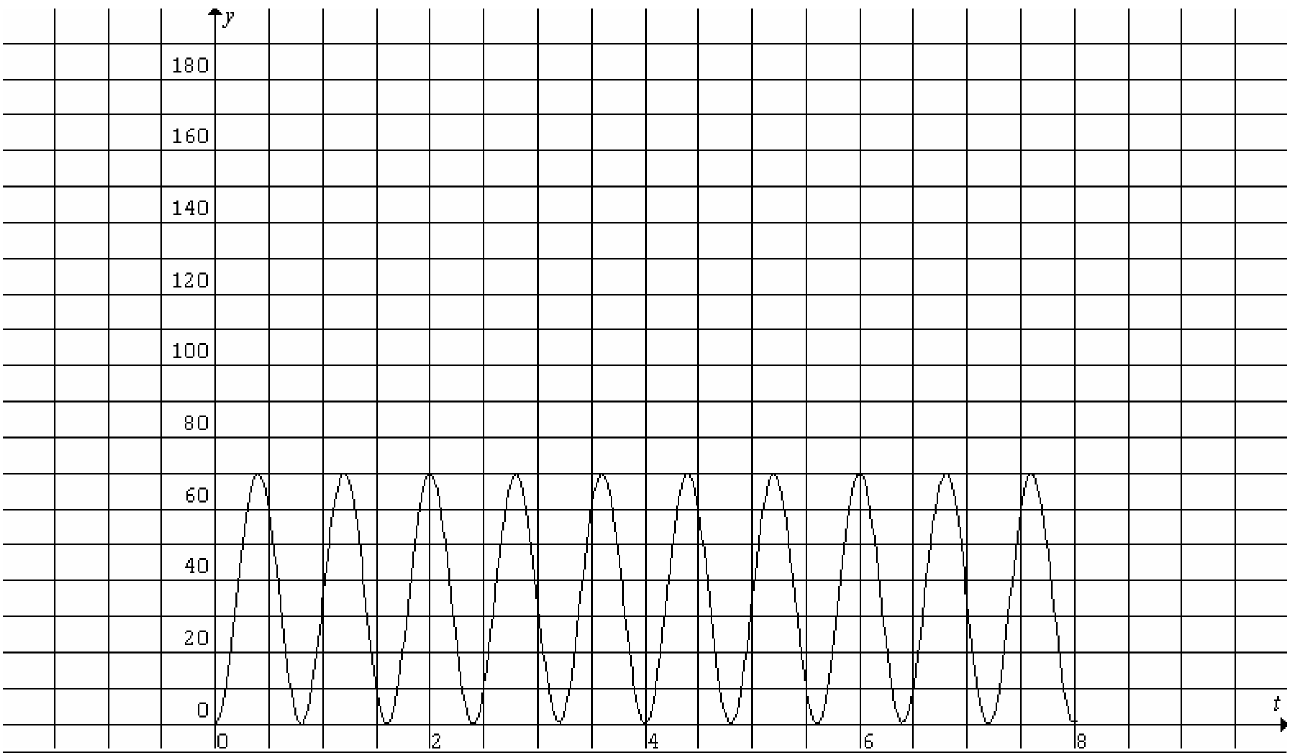
Both machines spin continuously in one direction. The equation for the vertical position of the blue marker, in centimetres, above the starting position of the red marker, is given as

$$y = 140 - 20\cos(\frac{5\pi t}{3})$$

where t is the time in seconds since both machines started to spin.

The graph of the position of the red marker is given below for $t \in [0,8]$.

d. On the axes given, sketch the graph of the position of the blue marker for $t \in [0,8]$



2 marks

SECTION 2 – Question 2 – continued

e. i. Ethan notices that the markers come closest together three times in the first eight seconds. Find these three times.

The rule T_n gives the nth time when the two markers are closest together. T_n is defined as $T_n = a + bn$

ii. Write an equation in terms of *a* and *b* for T_1 and T_2.

iii. Solve the equations simultaneously to find *a* and *b* and hence state the tenth time when the markers are closest together.

2 + 2 + 2 = 6 marks

f. **i.** Write down an expression, in terms of t, for the rate of change of the vertical position of the blue marker with respect to time.

ii. At what rate (in cm/s), correct to two decimal places, is the vertical position of the blue marker changing when $t = 1$?

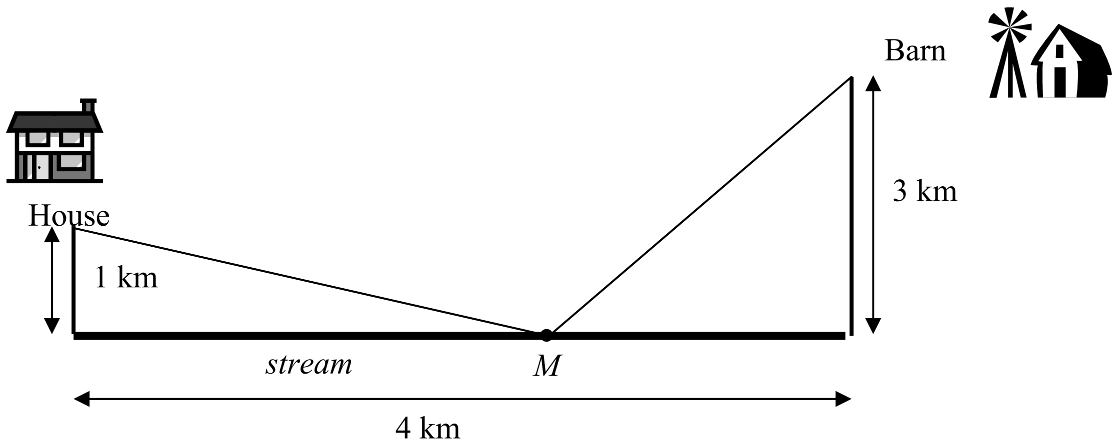
1 + 1 = 2 marks

Total 16 marks

Question 3

Mary, a farmer, needs to collect fresh water from a stream that borders her property. The stream follows a straight line and forms the east–west boundary at the southern end of the property. Mary walks at a constant rate from the house to the stream, fills her pail, and then carries it to the barn to feed the animals.

Initially she walks at a constant speed of 6 km/hr and her path is described in the diagram below.



She walks from the house to a point *M*, 2 km along the stream.

a. i. Find the distance from the house to the stream (correct to three decimal places) along this path.

ii. Find the distance from the stream to the barn (correct to three decimal places) along this path.

1 + 1 = 2 marks

b. Find the total time taken in hours (correct to three decimal places) to complete the task.

2 marks

Suppose that instead of walking to point M, Mary walks to a point P, x km from the house along the stream.

c. Show that the time taken, $T(x)$, to complete the task in terms of x is given by

$$T(x) = \frac{\sqrt{x^2+1}}{6} + \frac{\sqrt{x^2-8x+25}}{6} \text{ for } x \in [0,4]$$

2 marks

d. Find the location of the point on the stream that Mary should walk to in order to complete the task in the shortest time. State the shortest time in hours correct to three decimal places.

2 marks

Realistically, Mary cannot carry the full pail as quickly as the empty pail.

e. If her speed with the empty pail is k times her speed of 6 km/hr with the full pail (where $k \geq 1$), write an expression for the time taken to complete the task in terms of k.

2 marks

f. Show that the quickest path occurs when $k = \dfrac{x}{4-x}\sqrt{\dfrac{(x^2-8x+25)}{(x^2+1)}}$, $x \neq 4$.

4 marks

Total 14 marks

Question 4

Computer modelling for weather forecasting suggests that the probability of rain occurring on a particular day depends strongly on whether it has rained the day before. If it rains on a particular day, the probability of rain on the next day is 0.6. If it doesn't rain on a particular day, then the probability of it not raining the next day is 0.75.

Suppose it rains one Sunday.

a. **i.** What is the probability that it will rain on Monday, Tuesday and Wednesday?

ii. What is the probability that it will rain on exactly one of the days from Monday to Wednesday?

1 + 3 = 4 marks

When it does rain, the time, in minutes, that the rain falls is described by the random variable with the probability density function

$$f(t) = \begin{cases} kt(100 - t^2) & 0 \le t \le 10 \\ 0 & \text{otherwise} \end{cases}$$

b. **i.** Show that the value of k is 0.0004.

ii. On a day when it does rain, what is the probability, correct to three decimal places, that it rains for at least six minutes?

2 + 2 = 4 marks

Suppose it rains each day for the next week.

c. What is the probability, correct to three decimal places, that it rains for at least six minutes on at least three out of the next four days?

2 marks

d. On 21% of occasions when it rains, it rains for more than *n* minutes. Find the value of *n*, correct to two decimal places.

3 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK