Insight Publications

INSIGHT
Trial Exam Paper

2006

MATHEMATICAL METHODS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided
- The question and answer book of 9 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions
- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Use the factor theorem to show that $(x+2)$ is a factor of $9x^3+12x^2-11x+2$.

b. The equation $y=9x^3+12x^2-11x+2$ can be written in the form $y=(x+2)(ax-b)^2$ where $\{a, b\} > 0$. State the values of a and b.

2 + 2 = 4 marks

Question 2

a. The graph of a function g is obtained from the graph of the function f which has the rule $f(x)=2(x-2)^5$ by a performing a translation of -4 units parallel to the x-axis. Write down the rule for g.

b. The graph of a function h is obtained from the graph of g by a reflection in the y-axis. Write down the rule for h.

c. The graph of a function k is obtained from the graph of h by a dilation by a scale factor of $\frac{1}{2}$ along the y-axis. Write down the rule for k.

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1 + 1 + 1 = 3 marks

Question 3

Solve the equation $\sqrt{3}\sin(2x) + \cos(2x) = 0$ for $x \in [0, 2\pi]$, giving exact values in terms of π.

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3 marks

Question 4

Let $f(x) = x^2$ and $g(x) = 3x - 5$.

a. Write down the rule of $f(g(x))$.

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b. Find the derivative of $f(g(x))$.

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c. Hence, find the coordinates of the point P on the curve with the equation $y = f(g(x))$ at which the tangent is parallel to the line $2y - 12x = 7$.

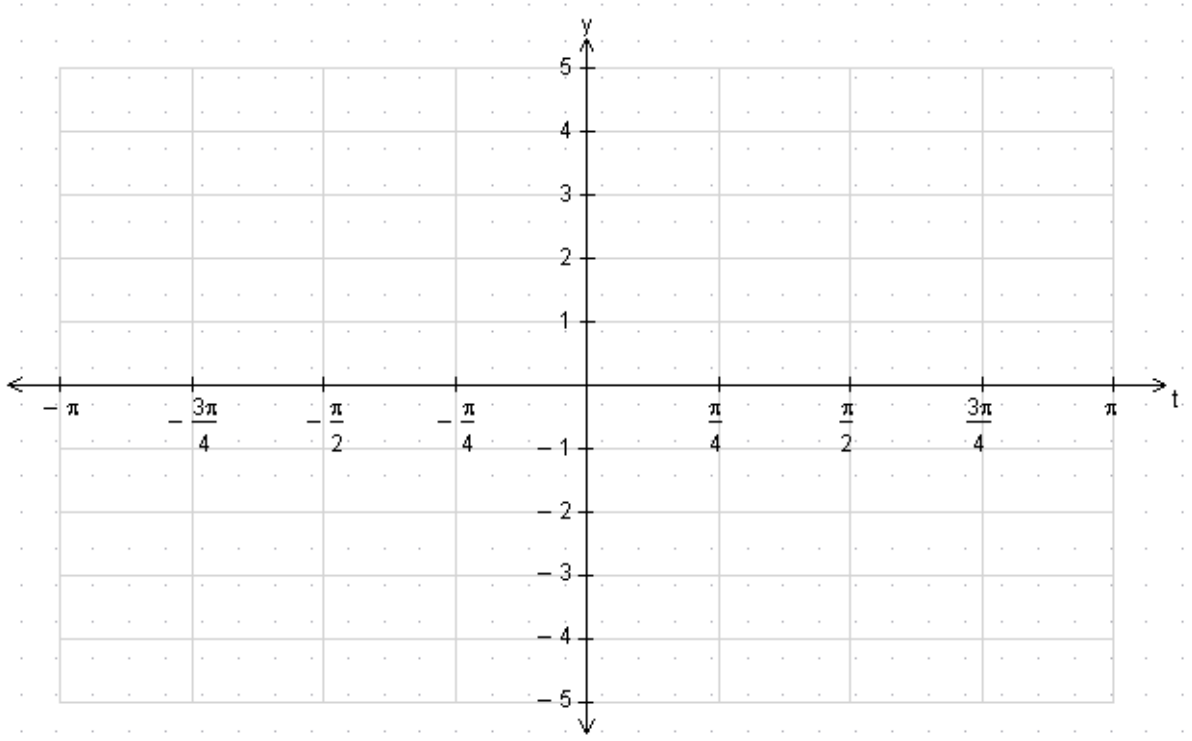
1 + 1 + 2 = 4 marks

Question 5

For the function $f : [-\pi, \pi] \to R,\ f(x) = -2\cos(2t + \frac{\pi}{2})$

a. Write down the period of the function.

b. On the set of axes below, sketch the graph of the function f.



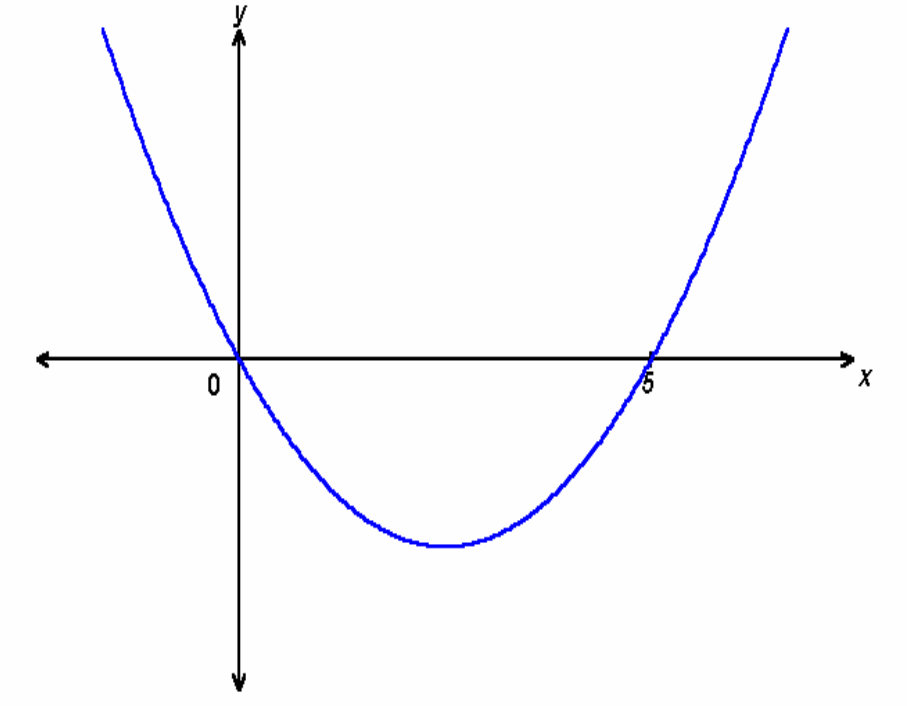
QUESTION 5 – continued

c. State the number of solutions to the equation $\cos(2t + \frac{\pi}{2}) = \frac{1}{2}$, where $-\pi \leq x \leq \pi$.

1 + 2 + 1 = 4 marks

Question 6

Part of the graph of $y = x^2 - 5x$ is shown below.



a. On the same set of axes sketch the graph of $y = |x^2 - 5x|$.

b. Find the set of values of x for which $|x^2 - 5x| \geq 6$.

1 + 2 = 3 marks

Question 7

For the function $f(x) = 2e^{1-x}$,

a. find the rule of the inverse function f^{-1}.

b. find the domain of the inverse function f^{-1}.

2 + 1 = 3 marks

Question 8

The random variable X has the following probability distribution.

x	−1	0	1	2
$Pr(X = x)$	a	2a	3a	0.4

a. Find the value of *a*.

b. If $\Pr(X \leq k) > 0.5$, find the minimum value of *k*.

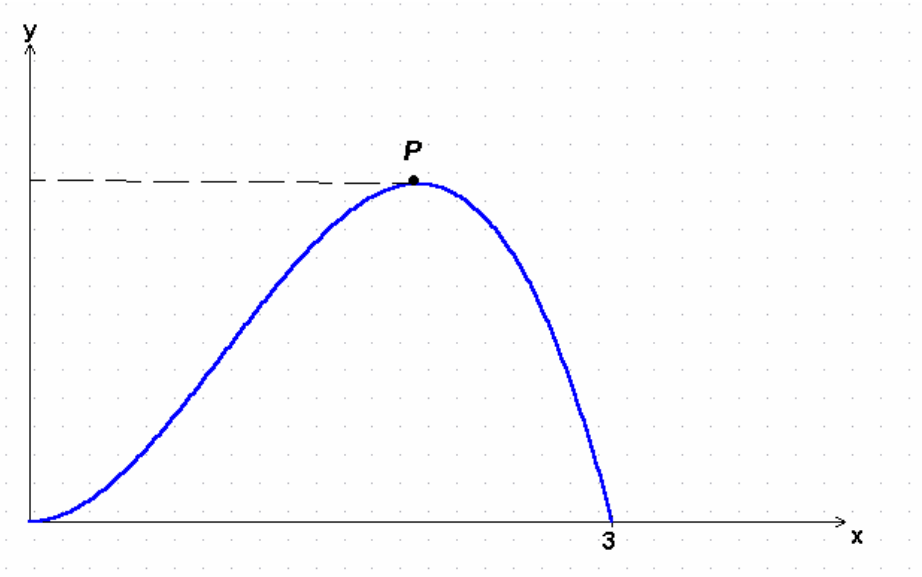
1 + 1 = 2 marks

Question 9

A continuous random variable X has the probability density function given by

$$f(x) = \frac{4}{27}(3x^2 - x^3), \quad 0 \le x \le 3$$

The graph of f, as shown below, has a maximum point at P.



a. Find the value of the x-coordinate of P.

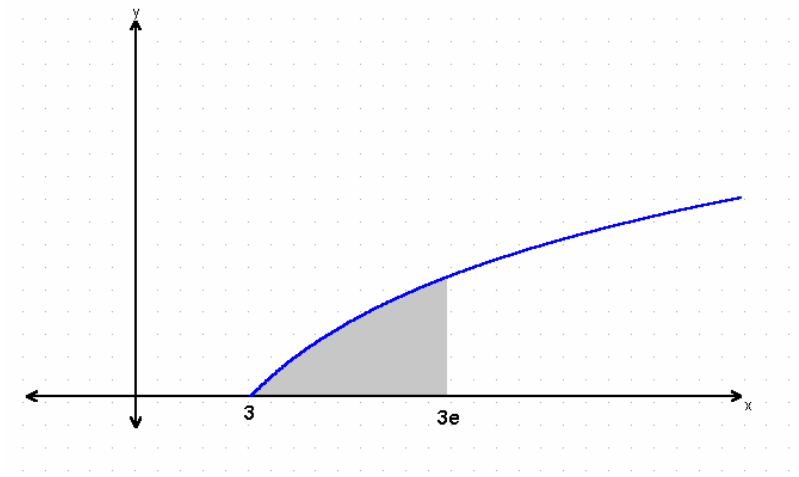
b. Find the $\Pr(0 < X < 2)$.

c. Find the mean value of X.

3 + 3 + 3 = 9 marks

Question 10

The graph of the function $f:[3,\infty) \to R,\ f(x) = \log_e \frac{x}{3}$ is shown below.



a. If $y = x\log_e \frac{x}{3} - x$, find $\frac{dy}{dx}$.

QUESTION 10 – continued

b. Hence**,** find the exact area of the shaded region.

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2 + 3 = 5 marks

END OF QUESTION AND ANSWER BOOK