CSE CENTRE FOR STRATEGIC EDUCATION
Leading educational thinking and practice

IARTV NO. 27 A/S

YEAR 12 – OCTOBER 2006

MATHEMATICAL METHODS

Written test 2

ANSWERS & SOLUTIONS BOOK

SECTION 1 – Multiple choice questions

1 B	6 A	11 A	16 E	21 A
2 B	7 C	12 E	17 B	22 A
3 D	8 C	13 B	18 B	
4 D	9 D	14 C	19 C	
5 A	10 E	15 D	20 B	

SECTION 2

Question 1

a. $x = 4\cos t,\ \dfrac{dx}{dt} = -4\sin t$

$y = 4\sin t,\ \dfrac{dy}{dt} = 4\cos t$

b. $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = -\dfrac{4\cos t}{4\sin t} = -\cot t$

c. $\dfrac{dy}{dx} = -\cot\dfrac{5\pi}{6} = \sqrt{3}$

d. $-1 = -\cot t \Rightarrow \tan t = 1 \Rightarrow t = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

e. $y - 4\sin t = -\cot t(x - 4\cos t)$

$y = -\cot t \times x + \dfrac{4\cos^2 t}{\sin t} + 4\sin t$

$y = -\cot t \times x + \dfrac{4\cos^2 t + 4\sin^2 t}{\sin t}$

$y = -\cot t(x) + \dfrac{4}{\sin t}.$

f. $B(0, \dfrac{4}{\sin t}), C(\dfrac{4}{\cos t}, 0)$

g. $A = \dfrac{1}{2} b \times h = \dfrac{1}{2} \times \dfrac{4}{\sin t} \times \dfrac{4}{\cos t}$

h. $A = 4(\sin 2t)^{-1} \Rightarrow \dfrac{dA}{dt} = -\dfrac{8\cos 2t}{\sin^2 2t}$

i. St. points when

$\dfrac{dA}{dt} = 0 \Rightarrow \cos 2t = 0 \Rightarrow t = \dfrac{\pi}{4},$

Also the gradient changes from negative to zero to positive so that the S.P. is a local minimum. (students should provide the sign diagram)

Question 2

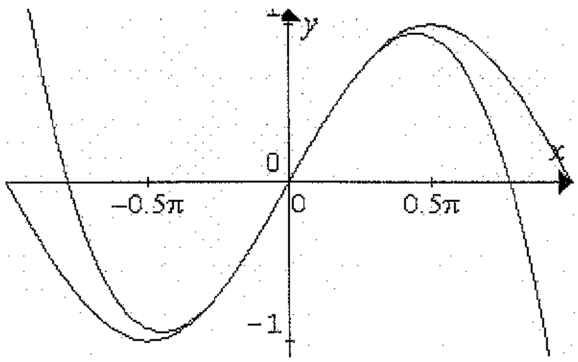
a. $P(x) = \dfrac{x}{6}(6 - x^2) = 0 \Rightarrow x\text{int} = 0, \pm\sqrt{6}$

b. $P'(x) = 1 - \dfrac{x^2}{2}$

c. TPs are when $P'(x) = 0 \Rightarrow x = \pm\sqrt{2}$

TPs are at $(\pm\sqrt{2}, \pm\dfrac{2\sqrt{2}}{3})$

d.



Note that the cubic is above the wave on left of O, and then the cubic is below the wave on the right of O.

Endpoints are $(\mp\pi, \pm 2.026)$

e. Shade the area between curves.

between $-\pi$ and π

Question 2 (continued)

f. $Area = 2\int_0^{\pi} \sin x - (x - \frac{x^3}{6})dx$

$Area = 2\left[-\cos x - \frac{x^2}{2} + \frac{x^4}{24}\right]_0^{\pi}$

$= 4 - \pi^2 + \frac{\pi^4}{12}$

g. $S(\frac{\pi}{4}) - P(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\pi}{4} + \frac{\pi^3}{384}$

h. Total error =Area between curves.

Question 3

a. Normalcdf(105,1e99,100,10)=0.3085

b. Normalcdf(87,115,100,10)=0.8364

c. invnormalcdf(0.9,100,10)=112.82=113

d. 95% means 2 sd from the mean so the interval is [80,120].

e. $\Pr(life < 90) = 0.15866$

If Y=the number of globes lasting less 90 hours, then Y is Binomially distributed

$\Pr(Y = 2) = binomialpdf(12, 0.15866, 2)$
$= 0.2953$

f. $\Pr(Y > 2) = 1 - \Pr(Y \le 2)$

$= 1 - binomialcdf(12, 0.15866, 2)$
$= 1 - 0.7057$
$= 0.2943$

g. $E(profit) = 4 \times 0.7057 - 2 \times 0.2943 = 2.2342$

Expected profit is \$2.23 per box

$Var(profit) = 16 \times 0.7057 + 4 \times 0.2943 - 2.2342^2$
$= 7.7468$

Variance is \$7.75 per box.

h. Expected profit =\$2234

Question 4

a. Endpoints $(\pm 1, \frac{1}{2}(e + e^{-1}))$

b. $y = -\frac{1}{2}(e^x + e^{-x})$

c. Centre (0,0) height is when x=0

$y = 502.3$, so the height is 502 feet.

d. Dilation from the *y*-axis scale factor w.

Reflection in *x*-axis

e.

$height = \frac{1}{2}(e + \frac{1}{e}) - 1 = \frac{1}{2}(\frac{e^2 + 1 - 2e}{e})$

$height = \frac{(e-1)^2}{2e}$

f. $2 = 4 - 0.5(e^x + e^{-x})$

Leads to $e^x + e^{-x} = 4$, let $w = e^x$

Gives $w^2 - 4w + 1 = 0 \Rightarrow w = 2 \pm \sqrt{3} = e^x$

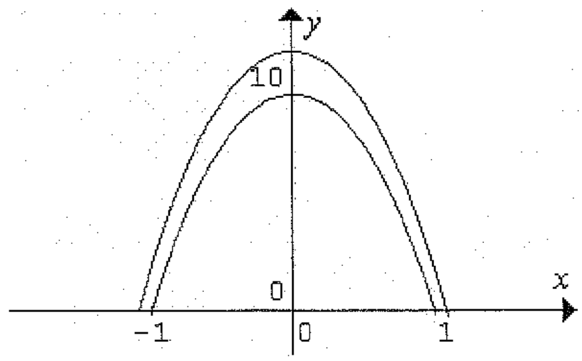
So that $x = \ln(2 \pm \sqrt{3})$

g. $Area = \int_{-c}^{c} b - \frac{a}{2}(e^{\frac{x}{a}} + e^{\frac{-x}{a}})dx$

$= \left[bx - \frac{a^2}{2}(e^{\frac{x}{a}} - e^{\frac{-x}{a}})\right]_{-c}^{c}$

$= 2bc - a^2(e^{\frac{c}{a}} - e^{\frac{-c}{a}})$

h.



The *y* intercepts are 12 and 10.

The *x* intercepts are $\pm 1, \pm 1.087$