STUDENT:	TEACHER:

YEAR 12 – OCTOBER 2006

MATHEMATICAL METHODS

Written test 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
11	11	40

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners and rulers.

- Students are **NOT** permitted to bring into the test room: blank sheets of paper and/or white out liquid/tape.

- No calculator is allowed in this test.

Materials supplied
- Question and answer book of 8 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above.

- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or other electronic communication devices into the test room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1   (3 marks)

Given the functions

$$f: R \setminus \{0\} \to R, \ f(x) = \frac{1}{x} \quad \text{and}$$

$$g: [2, +\infty) \to R, \ g(x) = \sqrt{3x - 4}$$

a.   Find the largest domain for which $g(f(x))$ is defined

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1 mark

b.   Write down the rule of the composite function $g(f(x))$

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1 mark

c.   Find the derivative of $g(f(x))$

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1 mark

Question 2   (4 marks)

Solve the equations

a. $\ln \sqrt{x + 2} - \ln x = 0$

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2 marks

b. $\left| \sin\left(x - \dfrac{\pi}{6} \right) \right| = \dfrac{\sqrt{3}}{2}$ for $x \in \left[0,\, 2\pi \right]$, giving exact answers.

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<div style="text-align: right">2 marks</div>

Question 3 (3 marks)

The graph of the function $y = f(x)$, which passes through the points O (0, 0), A (4, 8) and B (6, 0) is shown
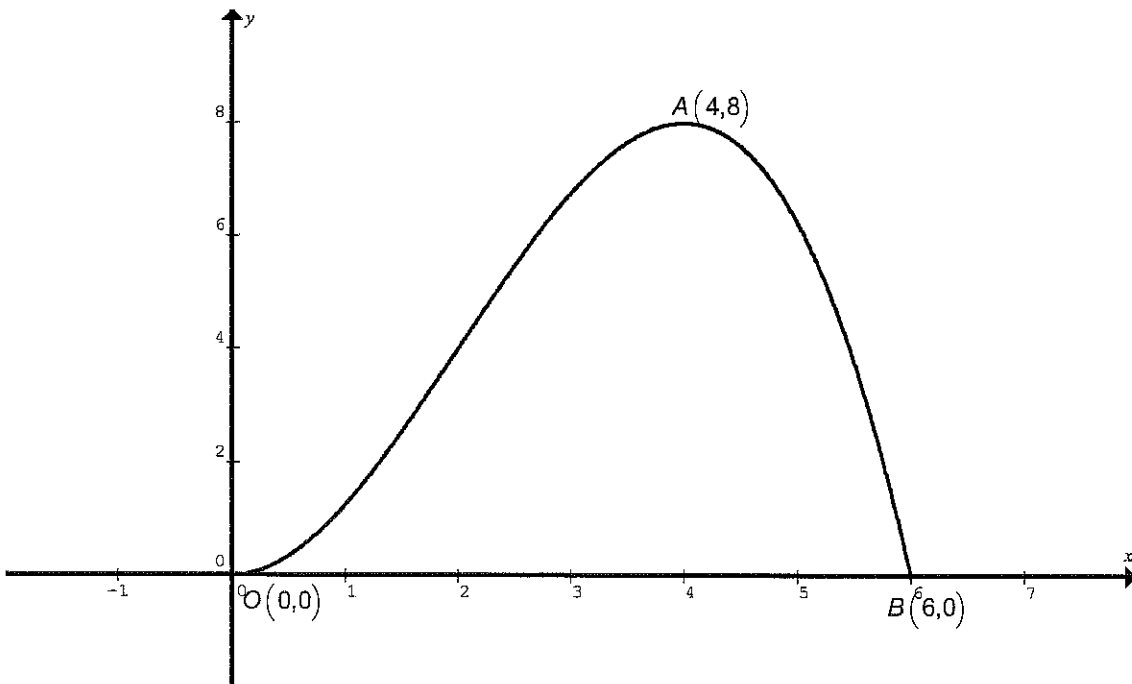
Sketch, on the same set of axes, the graph of $y = f\big(2(x+1)\big)$ with the images of O, A, B clearly shown and their coordinates labelled.

<div style="text-align: right">3 marks</div>

Question 4 (4 marks)

If $f: R \setminus \{2\} \to R$ is such that $f(x) = \dfrac{1}{x-2}$, find the equations of the tangents to $y = f(x)$ which have a gradient of -4.

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4 marks

Question 5 (4 marks)

Considering that the difference between $\left(\dfrac{\pi}{3}\right)^c$ and 1^c is small, use linear approximation to find $\sin 1^c$, giving your answer in exact form.

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4 marks

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\dfrac{1}{2}(a+b)h$ 　　　　volume of a pyramid: $\dfrac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$ 　　　　volume of a sphere: $\dfrac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$ 　　　　area of a triangle: $\dfrac{1}{2}bc\sin A$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1}+c, n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$$

$$\int \frac{1}{x}dx = \log_e|x|+c$$

$$\frac{d}{dx}(\sin(ax))=a\,\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}(\cos(ax))=-a\,\sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\dfrac{d}{dx}(uv)=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$ 　　　　quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$ 　　　　approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$ 　　　　$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A|B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$

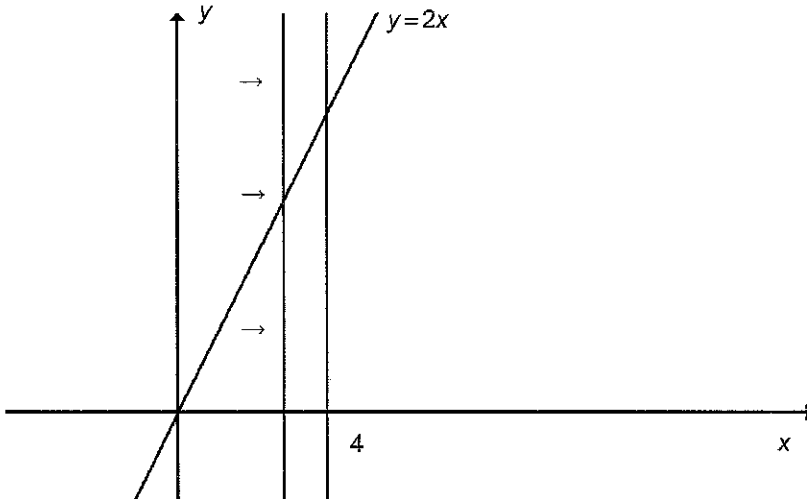mean: 　$\mu=E(X)$ 　　　　variance: 　$\text{var}(X)=\sigma^2=E((X-\mu)^2)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2 p(x)$
continuous	$\Pr(a<X<b)=\displaystyle\int_a^b f(x)dx$	$\mu=\displaystyle\int_{-\infty}^{\infty}x\,f(x)dx$	$\sigma^2=\displaystyle\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$

END OF FORMULA SHEET

Version 2 – March 2006

Question 6   (4 marks)

A right angled triangle is defined by the line $y=2x$, the positive side of the $x-$ axis and a vertical line from

$y=2x$ to the $x-$ axis.

If the vertical line is moving to the right from the origin at a constant rate of 2 units/min, find the rate of increase

of the area of the right angled triangle when $x=4$ units.

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4 marks

Question 7   (4 marks)

a.   Find $\dfrac{dy}{dx}$ if $y=2\log_e\dfrac{1}{|2x-5|}$, $x\neq\dfrac{5}{2}$.
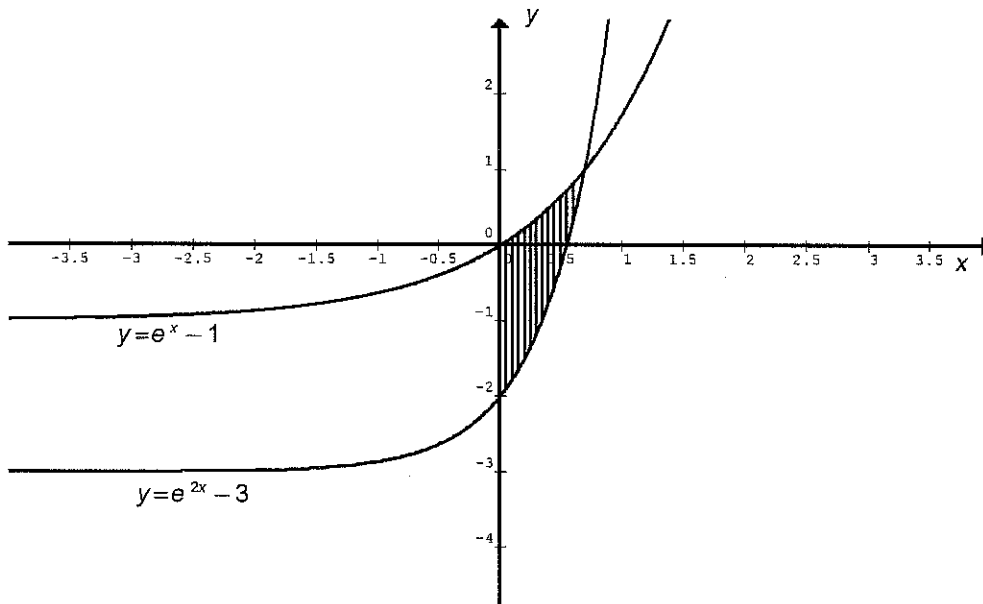
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2 marks

b. If $f: \mathbf{R} \to \mathbf{R}$ is such that $f'(x) = \sin\left(2x - \dfrac{\pi}{3}\right)$ and $f(0) = \dfrac{1}{2}$, find the rule of $f(x)$.

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2 marks

Question 8 (4 marks)

Find the exact area bounded by the curves $y = e^{2x} - 3$, $y = e^x - 1$ and the line $x = 0$.

$y = e^x - 1$

$y = e^{2x} - 3$

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4 marks

Question 9   (3 marks)

A random variable $X = \{1, 2, 3, 4\}$ has a probability distribution function $f(x) = kx$.

a.   Find the value of k.

1 mark

b.   Find the value of $E(X)$.

1 mark

c.   Find the value of $E(2X+3)$.

1 mark

Question 10   (3 marks)

If X is a binomial random variable with parameters n and p.

a.   Find $Pr(X=1)$

1 mark

b.   Show that $Pr\left(X=1 \mid X \leq 1\right) = \dfrac{np}{q+np}$

2 marks

8

Question 11 (4 marks)

The continuous random variable X with a probability density function is given by

$$f(x) = \begin{cases} \dfrac{4}{9}\sqrt{x} & \text{if } 0 \leq x \leq a \\ 0 & \text{if } x < 0 \ \text{or} \ x > a \end{cases}$$

a. Find the value of a.

1 mark

b. Find b such that $Pr(X < b) = 2\,Pr(X > b)$.

3 marks

TOTAL : 40 marks

END OF QUESTION AND ANSWER BOOK