VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2005 Mathematical Methods GA 2: Examination 1

GENERAL COMMENTS

The number of students who sat for the 2005 examination was 16 751, which was 716 fewer than the 17 467 who sat in 2004. Almost 15% scored 90% or more of the available marks, compared with 18% in 2004.

The overall quality of responses was similar to that of recent years. There were many very good responses and it was rewarding to see that quite a substantial number of students had worked through to obtain full marks. It was a pleasant surprise to see a decline in the number of students who scored very few marks and who appeared to attempt little or nothing of Part II. This was a marked improvement on the last few years.

Students need to be aware that the instruction to show working is applied rigorously when marking papers. Failure to show appropriate working and giving only an answer to such a question will result in marks not being awarded. Similarly, a decimal approximation will not be accepted if an exact answer is required. Stating how a calculator was used to obtain the 'exact' answer to a certain number of decimal places is not an appropriate response. Simply using a graphics calculator to determine an area where 'use calculus' is specified will not receive full marks. Many students also ignored the instruction 'hence' in Questions 6b. and 7b.

As noted in previous Assessment Reports, there continues to be problems associated with algebraic skills, setting out of solutions, graphing skills and the use of mathematical notation. This was especially evident this year in Questions 3, 4, 5a. and 5b., 6c. and 7a. and 7b.

SPECIFIC INFORMATION

Part I – Multiple-choice questions

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer
1	6	2	1	90	1	0
2	4	88	4	1	2	0
3	3	6	7	69	15	0
4	7	9	15	4	64	0
5	3	7	19	65	4	1
6	3	3	3	86	5	0
7	4	4	12	8	71	0
8	2	2	9	83	4	0
9	4	5	7	8	75	0
10	85	3	2	1	9	0
11	5	5	4	72	14	0
12	60	19	6	11	3	0
13	45	19	9	11	16	1
14	5	4	85	2	4	0
15	12	5	14	28	41	0
16	92	2	1	2	4	0
17	2	88	2	2	6	0
18	2	2	78	11	7	0
19	7	8	63	18	4	0
20	2	6	63	18	11	0
21	8	18	28	38	7	0
22	21	6	27	21	24	1
23	2	8	8	76	5	0

Question	% A	% B	% C	% D	% E	% No Answer
24	7	67	7	13	6	1
25	6	8	24	7	53	0
26	15	56	8	12	8	0
27	19	34	30	10	6	0

In the multiple-choice section, Question 27 was clearly the most difficult with less than 20 per cent of the students selecting the correct response. Students appeared to have difficulty relating the function and its integral, and so ignored the left hand side of the graph as this was below the x-axis and $G(x) > 0$. However, including $x = 0$ did not seem to pose a problem.

Part II – Short-answer questions

Question 1

Marks	0	1	2	Average
%	20	20	60	**1.5**

Correct response:

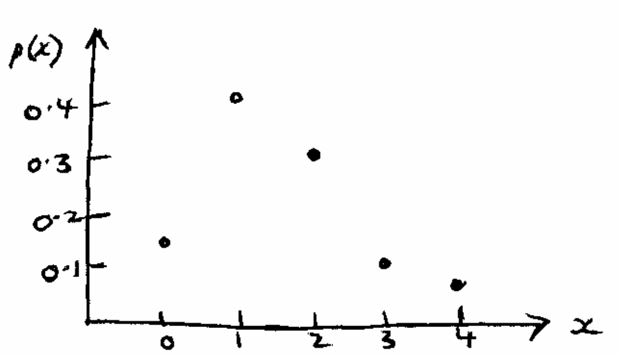
$\Pr(X < 46) = \Pr(Z < 1.667)$
$= 0.952$

Most students were able to obtain the correct answer. However, many did not provide sufficient working to be given the two marks. Answers could have been supported by specifying the required parameters, using a diagram, setting up an equation or providing the parameters used for a graphics calculator program. Very few students interpreted the problem as an approximation of a discrete distribution.

Question 2

Marks	0	1	Average
%	55	45	**0.5**

Correct response:

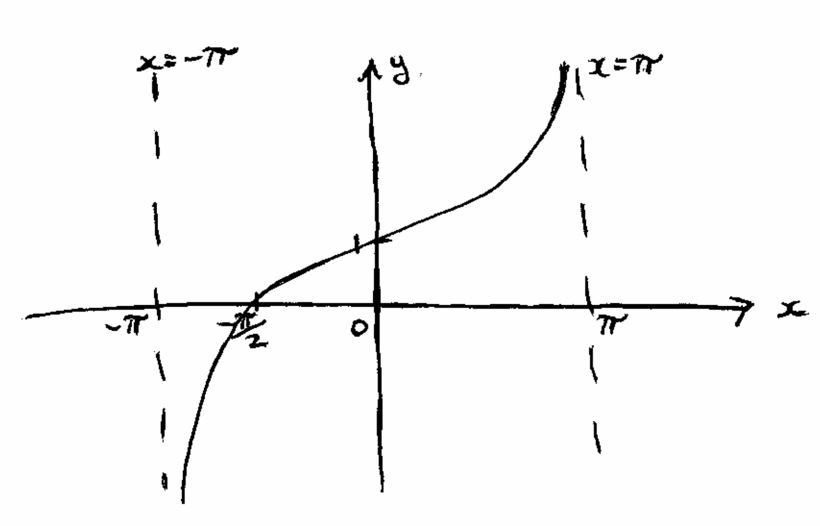


The responses to this question were disappointing. Most students ‘joined the dots’, completely ignoring the fact that it was supposed to be the graph of a discrete distribution. Other common errors included poor or incorrect indication of scale, with 0.15 being less than 0.10 and 0.05 being far less than half of 0.10.

Question 3

Marks	0	1	2	3	Average
%	15	12	26	47	**2.2**

Correct response:



This question was generally well done, with the majority of students sketching a transformed tangent graph with one period. Common errors included:

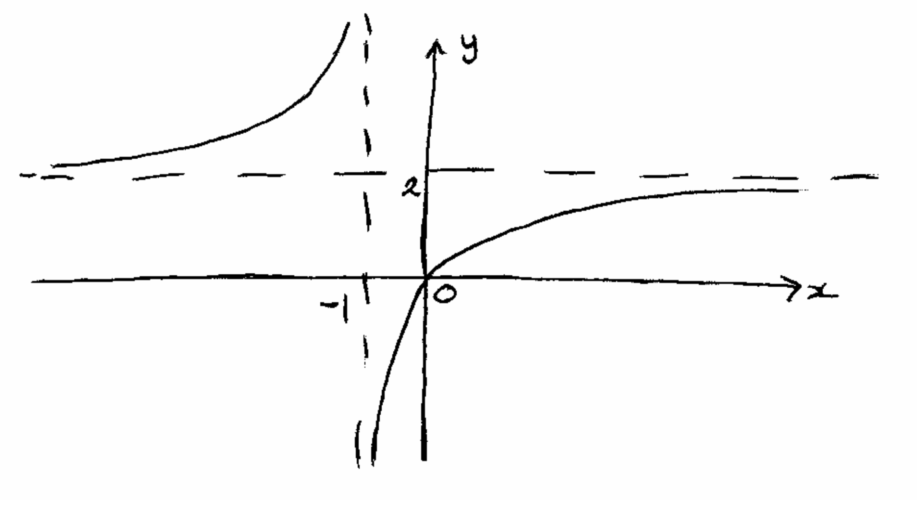
- joining the curve to the asymptotes (indicating the use of a graphics calculator)
- giving approximate instead of exact values for intercepts
- writing the x-intercept as a positive value, even though it was to the left of the origin
- leaving out equations for asymptotes, or in some instances giving them as $y = \pi$.

Clearly showing asymptotic behaviour as the curve approaches the asymptote was problematic for quite a few students. It was pleasing see that the use of degrees rather than radians was limited to a small number of students.

Question 4

Marks	0	1	2	3	Average
%	12	17	31	40	**2.1**

Correct response:



The asymptotes were nearly always drawn in the correct positions, although they were not always labelled. It was disappointing to see the number of responses that did not have curves passing through the origin. This piece of information seemed to be ignored by many students, as was the fact that the gradient was positive for all x values except -1. Other common errors were:

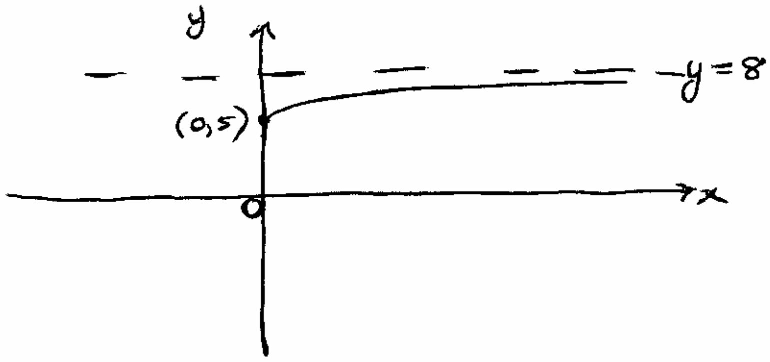
- only including one branch of the hyperbola
- having the curve curling away from the asymptotes.

Alternative correct responses, which showed one branch of the curve crossing the horizontal asymptote or an excluded point on the vertical asymptote on one branch, were rarely seen.

Question 5
5a.

Marks	**0**	**1**	**2**	**Average**
%	25	43	33	**1.2**

Correct response:



Many students ignored the domain and sketched the graph over *R*. Both the *y*-axis intercept and the asymptote were often not labelled, as was required by the question, and occasionally the asymptote was given as $y = 9$. Far too many students failed to show the horizontal asymptote, while others had a vertical asymptote, usually at either $x = 8$ or $x = 0$.

5b.

Marks	**0**	**1**	**2**	**Average**
%	18	28	54	**1.4**

Correct response:

$$x = 8 - 3e^{-y}$$
$$3e^{-y} = 8 - x$$
$$e^{-y} = \frac{8-x}{3}$$
$$y = -\log_e\left(\frac{8-x}{3}\right) \text{ or } \log_e\left(\frac{3}{8-x}\right)$$

While the majority of students recognised the need to interchange *x* and *y* to find the rule of the inverse function, far too many were unable to handle the algebra required to rearrange the corresponding equation correctly. Negative signs appeared and disappeared, and logarithm laws seemed non-existent. A handful of students attempted to differentiate to find the answer.

Question 6
6a.

Marks	**0**	**1**	**Average**
%	27	73	**0.7**

Correct response:

$$y = (x+2)(x^2 - 4x + 3)$$

There was substantial evidence of the use of a graphics calculator program to answer this question. Despite this, a significant number of students were still unable to find the correct quadratic factor. It was surprising to see the number of students who wrote in a linear factor other than $x + 2$.

6b.

Marks	0	1	2	Average
%	27	3	70	**1.5**

Correct response:

$(x+2)(x^2-4x+3)=(x+2)(x-3)(x-1)=0$

$x=1, 3, -2$

The 'hence' instruction was often ignored or not understood by students. Many simply wrote down the three solutions without any attempt to factorise or even show the three linear factors.

6c.

Marks	0	1	2	3	4	Average
%	26	10	12	21	31	**2.4**

Correct response:

$$\text{Area} = -\int_1^3 x^3-2x^2-5x+6\ dx \ + \int_{-2}^{1} x^3-2x^2-5x+6\ \ dx$$

$$= \left[\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x\right]_3^1 + \left[\frac{x^4}{4}-\frac{2x^3}{3}-\frac{5x^2}{2}+6x\right]_{-2}^{1}$$

$= 21\frac{1}{12}$ (or 21.08, correct to two decimal places)

Despite their answers to parts a. and b., most students were able to determine the values of the terminals. '*dx*' was often missing from the integral expressions and students were penalised for this omission. Most students were able to find the correct anti-derivative. Many then proceeded to use their graphics calculator to find the correct area, totally ignoring the fact that their answer did not follow from their original expression, which did not take account of part of the area being below the x-axis. Some students attempted to use modulus signs with limited success, while others swapped terminals but not necessarily on the correct part. Others who attempted to complete the question without their graphics calculator often made arithmetic errors.

Question 7

7a.

Marks	0	1	Average
%	51	49	**0.5**

Correct response:

$x\log_e(3) = k\,x$

so, $k = \log_e(3)$ for all $x \in R$

This question was beyond quite a few students, who could not cope with the logarithm involved. All too often x was left in the expression for k.

7b.

Marks	0	1	2	Average
%	66	12	22	**0.6**

Correct response:

$y = 3^x = e^{x\log_e(3)}$

$\frac{dy}{dx} = \log_e(3)e^{x\log_e(3)} = \log_e(3).3^x$

Again, the 'hence' instruction was ignored by many students, who gave answers without showing any working. Poor mathematical expression was also the downfall of many, with $\log_e(3)x$ becoming $\log_e(3x)$. Those students who left k in terms of x were totally confused about what to do. Some students attempted to use the product rule, while others tried anti-differentiating.