2005 Mathematical Methods (CAS) GA 3: Written examination 2

GENERAL COMMENTS

The number of students that presented for Mathematical Methods (CAS) examination 2 in 2005 was 333, which was 56 fewer than the 389 who sat in 2004. The range of marks was from 0 to 55. Student responses showed that the paper was accessible and provided an opportunity for students to demonstrate their knowledge.

Several students presented excellent papers. The median score was 30 and the mean mark for the paper was 28.9. Fifty-five per cent of the students scored over half of the marks for the paper and 67.3 per cent of the students scored over 40 per cent. Just under 13 per cent of students scored under 20 per cent and two students received no marks on the paper.

Generally the symbolic facility of CAS was used well. As in previous years, there was no discernable advantage seen by the assessors of one computer algebra system over another. The generally high level of proficiency with CAS was demonstrated in particular in Question 4ai., where 87 per cent of students achieved full marks.

Students sometimes did not give answers in their exact form even when this was explicitly asked for. Students must ensure that they do not give numerical approximations when an exact answer is required.

Students must ensure that they show their working – if a question is worth more than one mark, students risk losing all available marks if only the answer is given, and it is incorrect. The instructions at the beginning of the paper state that if more than one mark is available for a question then appropriate working must be shown. In this examination, many students did not show sufficient reasoning for the probability question.

Question 1c. proved difficult, as it appeared that many students did not recognise that the discriminant was to be used. Students should be familiar with the use of the discriminant to determine the number of solutions for a quadratic equation and also of the graphical interpretation.

Questions 2ci. and ii., on probability and transition matrices, and powers of these matrices, were relatively straightforward. Students should be familiar with the use of matrices in this context.

The first few parts of Question 3 were successfully completed by most students. They demonstrated a good understanding of the function, its basic properties and its use in modelling the situation.

Students found the last parts of Question 4 challenging, but it was pleasing to find that a number of students completed the question successfully.

Students lost marks when they:

- did not answer the question asked
- gave decimal answers when exact answers were required
- gave the wrong number of decimal places.

When students present working and develop their solutions, they should use conventional mathematical expressions, symbols, notation and terminology. There was an improvement this year in the notation used by students.

SPECIFIC INFORMATION

Question 1

1ai.

Marks	0	1	Average
%	62	38	0.4

(0, 2]

This question was not well done as many students did not consider the given domain. The best procedure for determining the range of a function is through sketching a graph for the given domain.

1aii.

Marks	0	1	2	Average
%	14	35	51	1.4

$(0, 2]$, $f^{-1}(t) = \log_e\left(\frac{2}{t}\right) = -\log_e\left(\frac{t}{2}\right)$

The majority of students correctly determined the rule for the inverse. It was evident that a large number of students obtained the answer through the use of CAS.

1bi.

Marks	0	1	2	3	Average
%	23	8	6	63	**2.1**

$b = 4$ and $c = -3$
$g'(t) = (-t^2 + 4t - 3)e^{-t}$

This question was well done. CAS would not necessarily have given the answer in this form and students who were successful with the question demonstrated that they could obtain the required form either by hand or by further manipulation with their CAS.

1bii.

Marks	0	1	2	Average
%	19	16	65	**1.5**

$p = 0$ and $m = 3$ and $n = 4e^{-3}$

The majority of students answered this question correctly. Students had obviously taken care to assign values to the correct pronumeral. The question was completed successfully by a greater proportion of students than a similar question in 2004.

1biii.

Marks	0	1	2	Average
%	25	14	62	**1.4**

$(3, 8e^{-3} - 5)$ $(1, -5)$

It was evident that most students achieved the correct answer by applying the transformations to the points obtained in Question 1bii. Determining the new functions that corresponded to the transformed graphs and determining the new coordinates was a time consuming process that often led to errors.

1ci.

Marks	0	1	2	3	Average
%	41	35	5	19	**1.0**

$h'(t) = 0$ implies $-t^2 + (2 - a)t + a - 10 = 0$
For one stationary point, the discriminant is equal to zero.
$(2 - a)^2 + 4(a - 10) = 0$
$a = 6$ or -6

This question required the use of the discriminant of a quadratic expression, which many students did not realise. The derivative was given and the requirement that there is one solution to the equation $h'(t) = 0$ was used to determine the values of a. This was achieved through the discriminant of the quadratic factor after observing $e^{-t} \geq 0$ for all t. Many of the students who completed the question successfully did so by hand.

1cii.

Marks	0	1	2	Average
%	86	3	11	**0.3**

$h'(t) < 0$ for $-6 < a < 6$

Students needed to know that there was no solution to the equation $h'(t) = 0$ when the discriminant of the quadratic factor was less than zero and the coefficient of x^2 was negative. This implies that the graph of $y = h'(t)$ always lies 'below the x-axis'.

Question 2

2ai.

Marks	0	1	2	Average
%	17	18	64	**1.5**

X is the number of customers on dial up.
Binomial, $n = 10$ and $p = 0.8$
$\Pr(X = 8) = 0.3020$

This question was well done, with most students recognising that the distribution was binomial. Some students did not give the answer to the required number of decimal places. Students should take greater care with this aspect of providing numerical answers.

2aii.

Marks	0	1	Average
%	27	73	**0.8**

10 x 0.8 = 8. The expected value of the distribution is 8.

2bi.

Marks	0	1	2	Average
%	50	7	43	**1.0**

$\int_0^\infty t\, f(t)\, dt = 20$

This is applying the definition of the mean of a continuos random variable. Students should take care in writing an integral which will yield the correct answer as part of their working. Students often did not gain marks on this question if they did not show any working.

2bii.

Marks	0	1	2	Average
%	49	6	45	**1.0**

$\int_{30}^\infty f(t)\, dt = e^{-\frac{3}{2}}$. To the nearest per cent, the answer is 22%.

Some students lost marks by not giving the answer to the correct accuracy.

2biii.

Marks	0	1	2	Average
%	40	29	30	**0.9**

$e - \frac{1}{2} = 0.607$, correct to three decimal places.

This question required the use of conditional probability, which is still an area that causes problems. The key word 'given' was included in this question.

2ci.

Marks	0	1	2	3	Average
%	42	31	4	22	**1.1**

$$\begin{bmatrix} 0.8 & 0.05 \\ 0.2 & 0.95 \end{bmatrix}^4 \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.39 \\ 0.61 \end{bmatrix}$$

The proportion is 0.39.

This problem could also be successfully completed using a tree diagram, but very few students achieved a correct answer using this technique. Students should be aware of the convenience of solving such problems using powers of the transition matrix. The transition matrix method is certainly the optimal approach given the availability of CAS.

2cii.

Marks	0	1	2	Average
%	58	19	23	**0.7**

after 9 months

This answer could be determined though the systematic use of trial and error for different values of n with the expression $\begin{bmatrix} 0.8 & 0.05 \\ 0.2 & 0.95 \end{bmatrix}^n \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$.

Question 3

3a.

Marks	0	1	Average
%	10	90	**0.9**

150 m

Students found the initial parts of this question quite accessible and demonstrated an understanding of the model and the mathematics associated with substitution and the solution of equations. Part a. required an understanding of the range of the function. The result could be found by using a graphing facility of a CAS.

3b.

Marks	0	1	Average
%	8	92	**0.9**

50 m

This part required an understanding of the range of the function. The result could be found by using a graphing facility of a CAS.

3ci.

Marks	0	1	Average
%	11	89	**0.9**

800 m

3cii.

Marks	0	1	Average
%	11	89	**0.9**

400 m

Questions 3ci. and cii. required the solution of the equation $\cos\left(\frac{\pi(x-400)}{600}\right) = -\frac{1}{2}$. It is evident that most students completed the questions using CAS.

3d.

Marks	0	1	2	Average
%	28	9	63	**1.4**

716 m

Students were required to form and solve the equation $y = 20$. A number of students had difficulty starting this question. The two numerical solutions to the equation for the set of values under consideration were 758.192 and 41.808. The difference 758.192 – 41.808 could then be found and the answer given to the required accuracy. Some students did not give their answer to the required accuracy.

3ei.

Marks	0	1	2	Average
%	20	34	47	**1.3**

$\int_{1200}^{800} f(x)\, dx$

Students had to recognise that the answer required a reversal of the terminals or the use of a negative sign.

3eii.

Marks	0	1	Average
%	32	68	**0.7**

13 080 m^2

The exact answer is $\dfrac{20000\left(3\sqrt{3}-\pi\right)}{\pi}$ m^2.

3fi.

Marks	0	1	Average
%	64	36	**0.4**

$800 - 2k$

3fii.

Marks	0	1	Average
%	69	31	**0.3**

$400 + 2k$

The simple algebra required for the formulation of the length expression in Questions 3fi. and fii. proved difficult for the majority of students; CAS was not helpful. An understanding of the symmetry of the graph was required.

3fiii.

Marks	0	1	Average
%	59	41	**0.4**

$C = (800 - 2k)^2 + (400 + 2k)^2 = 8k^2 - 1600k + 800\,000$

The mark was awarded if the student used their results from parts i. and ii.

3fiv.

Marks	0	1	2	Average
%	63	13	24	**0.6**

$k = 100$

Students could use calculus or complete the square to answer the question.

Question 4

4ai.

Marks	0	1	Average
%	13	87	**0.9**

$\dfrac{4}{(x-11)^2} - \dfrac{9}{(x+1)^2}$

Students could use CAS directly to obtain this result. Generally the question was well done.

4aii.

Marks	0	1	2	3	Average
%	17	9	9	66	**2.3**

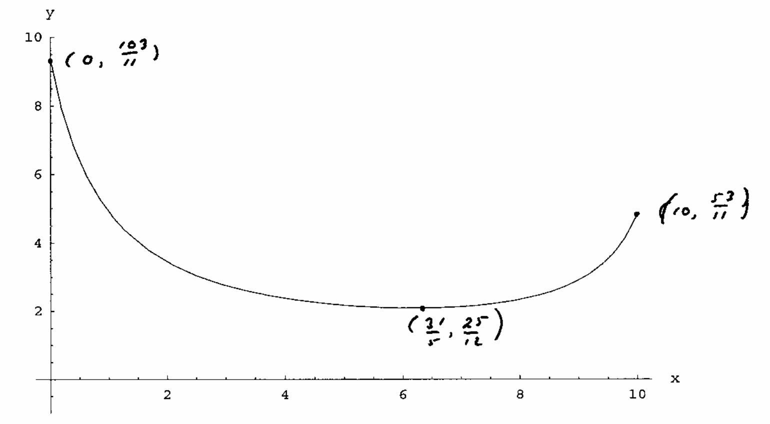
The derivative was not equal to zero to solve

$x = \dfrac{31}{5}$, $y_{\min} = \dfrac{25}{12}$

This question was well done. CAS was used effectively.

4aiii.

Marks	0	1	2	Average
%	8	29	63	**1.6**



Endpoints $\left(0,\ \frac{103}{11}\right),\ \left(10,\ \frac{53}{11}\right)$

Some students lost marks by not giving all the required details on the graph.

4aiv.

Marks	0	1	Average
%	55	45	**0.5**

9044 m = 9.044 km

This question required the numerical solution of an equation. A good approach was to use the graph to interpret the solutions.

4bi.

Marks	0	1	2	3	Average
%	78	11	9	2	**0.4**

The endpoints $\left(0,\ p+\frac{4}{11}\right)$ and $\left(10,\ \frac{p}{11}+4\right)$

$\frac{p}{11}+4 \geq p+\frac{4}{11}$

so $0 < p \leq 4$

Students found this question difficult and only a few obtained two or three marks. A careful consideration of the endpoints and the effect of varying *p* would yield the correct result.

4bii.

Marks	0	1	2	Average
%	73	23	4	**0.3**

$\frac{dy}{dx}=\frac{4}{(x-11)^2}-\frac{p}{(x+1)^2}=0$

implies $x = 11-\frac{24}{\sqrt{p}+2}$ as $x \leq 10$

minimum at $x = 10$ for $p \geq 22^2$

A careful consideration of the endpoints, the local minimum and the effect of varying *p* would yield the correct result. Students found this question difficult and only a few students scored full marks.