2005 Mathematical Methods Written Examination 1

Suggested Answers & Solutions

Part 1 (Multiple-choice) Answers

1. D	**2**. B	**3**. D	**4**. E	**5**. D
6. D	**7**. E	**8**. D	**9**. E	**10**. A
11. D	**12**. A	**13**. A	**14**. C	**15**. E
16. A	**17**. B	**18**. C	**19**. C	**20**. C
21. D	**22**. C	**23**. D	**24**. B	**25**. E
26. B	**27**. A			

Part 1 (Multiple-choice) Solutions

Question 1 [D]
At least two hours means 2 or 3 or 4 hours. The sum of the proportions is

$$\frac{7}{30}+\frac{10}{30}+\frac{4}{30}=\frac{21}{30}$$

Question 2 [B]
Mean = $\sum (x \times \text{proportion})$

$$= 0\times\frac{3}{30}+1\times\frac{6}{30}+...4\times\frac{4}{30}$$

$$=\frac{0+6+14+30+16}{30}$$

$$=\frac{66}{30}$$

Question 3 [D]
InvNorm (0.95, 0, 1) = 1.645 from calculator.

Question 4 [E]
Hypergeometric distribution.
The probability of at least one box with a prize is the same as taking the probability of no prizes from 1.

$$= 1 - \frac{{}^4C_2 \times {}^2C_0}{{}^6C_2}$$

$= 9/15$

Question 5 [D]
Binomial: Bi(n, 0.3)
Pr(John wins no games) = 0.7^n
Therefore $0.7^n = 0.0576$
$n \log(0.7) = \log(0.0576)$
$n = 8.0023$
Answer: 8 games

Question 6 [D]
Using Factor 7 (or 9) the factorised form is $x(x - 2)((x + 2)(x + 3)$.
So (x-3) is not a factor.
Answer: $(x - 3)$

Question 7 [E]
The turning point of this quadratic is at (3, 1). The function needs to be one-to-one for the inverse to exist. Since the domain extended to $+\infty$, it starts from the x value of the turning point. The domain is $[\,3,\ \infty)$ or $a \geq 3$.

Question 8 [D]

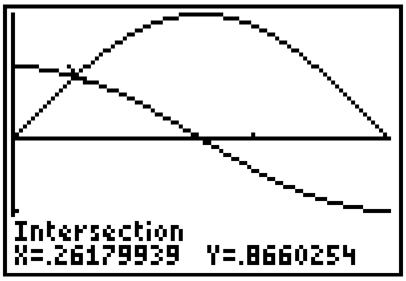
$3e^{2x} = 4$
$e^{2x} = 4/3$
$\ln(e^{2x}) = \ln(4/3)$
$2x = \ln(4/3)$ and so $x = \frac{1}{2}\ln(4/3)$
$x = 0.144$ to 3 decimal places.

Question 9 [E]
$2 \log_{10}(x) + 3 = 5 \log_{10}(x)$
Therefore $3 = 5 \log_{10}(x) - 2 \log_{10}(x)$
$3 = 3 \log_{10}(x)$
$1 = \log_{10}(x)$ and so $x = 10$.

Question 10 [A]
The amplitude is 2, eliminating alternatives C and D.
Period: $\frac{2\pi}{n}=1\div\frac{5}{2}=\frac{2}{5}$ and so $n = 5\pi$.
This eliminates E.
When $t = 0$ the value of $y = 3$. Out of the two alternatives remaining, A fulfils this requirement.

Question 11 [D]
The period of the graph $y = \tan(kx)$ is $\frac{\pi}{k}$ and so the period here is π ÷ ¼ or 4π.

Question 12 [A]
Using the Graphics Calculator with $y_1 = \cos 2x$ and $y_2 = \sqrt{3}\sin 2x$, only one point of intersection is found within the domain.



Dividing this x value shown on the screen by π gives 0.0833333, (the decimal equivalent for $\frac{1}{12}$) and so the exact answer is $\frac{\pi}{12}$.

Or, algebraically,

$\text{Cos}(2x) = \sqrt{3}\sin(2x)$

$\text{Tan}(2x) = \frac{1}{\sqrt{3}}$

$2x = \frac{\pi}{6}$

$x = \frac{\pi}{12}$

Question 13 [A]
If $a\sin(x) + c < 0$ then $c < -a$ as the maximum value of $\sin(x)$ is 1.

Question 14 [C]
The x and y values interchange for the inverse function. Because the original function had a horizontal asymptote with equation $y = 1$, this will become a vertical asymptote with equation $x = 1$ for the inverse. Only alternatives C and E display this feature. Also, a function and its inverse are mirror images in the line $y = x$. Only C displays this.

Question 15 [E]
A translation of −2 parallel to the x axis of $y = x^2$ results in a graph with equation $y = (x + 2)^2$.
This new graph is then dilated by a factor of ½ from the y axis. This means that the x intercept of (−2, 0) is now positioned at (−1, 0), i.e. half the distance from the y axis. E is the only alternative to do this.
Answer: $y = (2x + 2)^2$.

Question 16 [A]
A factor of $(x - 1)^2$ is required because the graph touches the x axis at $x = 1$. This eliminates B and C. It also requires factors of $(x - 3)$ and $(x + 2)$, or the negative of both of these. This eliminates D. If $x = 0$ the value of y is negative. A has this property whereas E does not.

Question 17 [B]
The vertical asymptote has equation $x - 1 = 0$ and so $x - 1 = x + b$. The value of b is −1. Only alternatives A and B are possible answers. The graph's horizontal asymptote has equation $y = 2$. But $y = c$ is the horizontal asymptote in the given equation. Hence $c = 2$. B satisfies these two requirements.

Question 18 [C]

$$y = \frac{x-2}{x+3} = 1 - \frac{5}{x+3}$$

So as $x \to \pm\infty$, $\frac{5}{x+3} \to 0$, so $y \to 1$
So asymptotes occur at $y = 1$ and $x = -3$.

Question 19 [C]
The gradient of $f(x)$ should have the following features: It

- is always positive
- approaches zero as $x \to \infty$
- is large positive as $x \to 0$.

Only C has these properties.

Question 20 [C]

Average rate of change = $\frac{f(1)-f(0)}{1-0}$.

$f(1) = 1^2 + e^1$ and $f(0) = 0^2 + e^0 = 1$

Average rate = $\frac{1+e-1}{1} = e$.

Question 21 [D]

Using Product Rule: $\frac{d}{dp}(10p(1-p)^9)$

$= (1-p)^9\frac{d}{dp}(10p) + 10p\frac{d}{dp}(1-p)^9$

$= 10(1-p)^9 + 10p\,(9)(-1)(1-p)^8$

$= 10(1-p)^8\,[1-p-9p]$

$= 10(1-p)^8\,(1-10p)$

Question 22 [C]

$\frac{d}{dx}f(e^{2x}) = \frac{d}{du}(f(u))\times\frac{du}{dx}$

where $u = e^{2x}$

$= 2e^{2x}f'(u)$

$= 2e^{2x}f'(e^{2x})$

Question 23 [D]

$y = 2x^4 - 4x^3$ and so $\frac{dy}{dx} = 8x^3 - 12x^2$.

At $x = 2$, the values of y and $\frac{dy}{dx}$ are 0 and 16 respectively. The equation of the tangent is

$y - 0 = 16(x - 2)$.

This is alternative D.

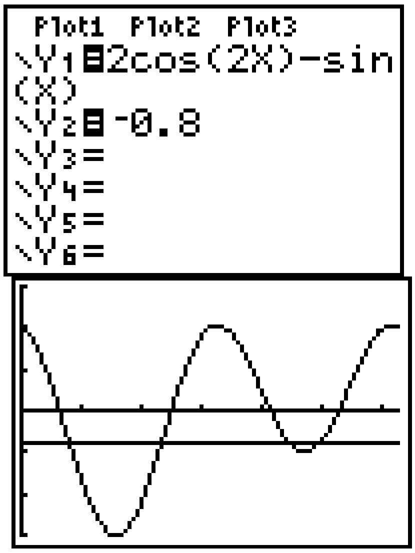
Question 24 [B]

The gradient of this curve is negative for $x > 2$ and positive for all other values of x except at $x = 0$ and $x = 2$ where it is zero.

Alternative B shows these features.

Question 25 [E]

If $f(x) = \sin 2x + \cos x$ then the derivative is $2\cos 2x - \sin x$.



The domain for this window is $[0, 2\pi]$. By observation there are 4 points of intersection.

Question 26 [B]

$\int(ax+b)^n\,dx = \frac{1}{a}\times\frac{1}{n+1}(ax+b)^{n+1} + c.$

In this case $a = 2$, $b = -1$, $n = -3/2$ and so the integral is:

$3\times\frac{1}{2}\times\frac{1}{-1.5+1}(2x-1)^{-1/2} + c$

$= -3(2x-1)^{-1/2} + c$

Question 27 [A]

If any of the sets for x included zero then the definite integral could include $\int_0^0 f(t)dt$ which is zero and so G(x) would not be positive. This eliminates C and D. Alternative E is readily discarded because if $x = a/2$ the area is clearly positive.

Alternative B has certainly got a positive value for G(x) but the word *only* prevents it from being a correct answer.

Alternative A is correct. If $x < 0$ then $\int_0^x f(t)dt$ will be the negative of a negative number, i.e. positive. If $x > 0$ then the integral is clearly positive.

Alternative A is correct.

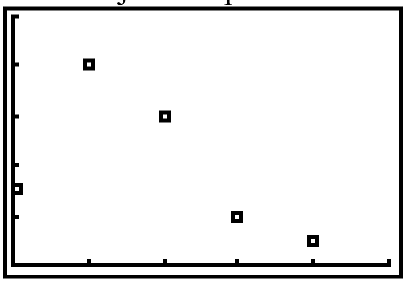
PART II

Question 1

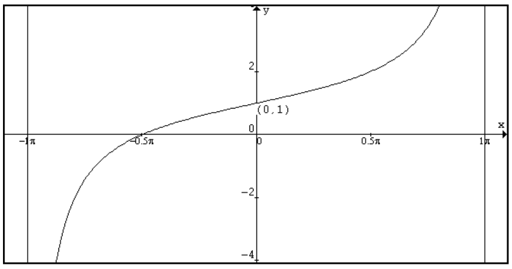
Normalcdf ($-\infty$, 46, 41, 3) = 0.9522.
Answer: 0.952 to three decimal places.

Question 2

The display from a graphics calculator is shown. The x-values of 0,1,2,3,4 need to be placed horizontally and y values of 0.1, 0.2, 0.3, 0.4, 0.5 vertically.
Do NOT join the points.

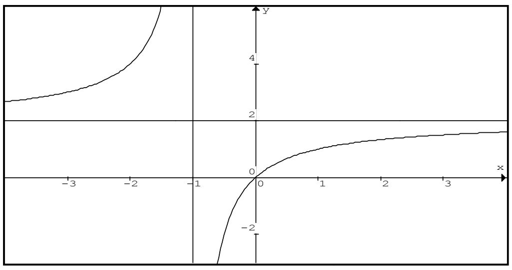


Question 3



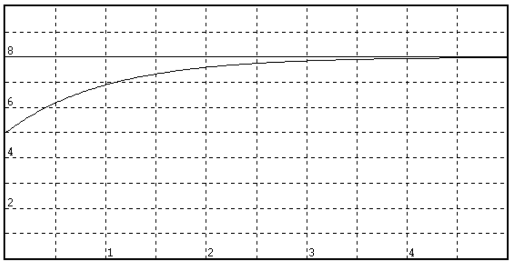
The vertical asymptotes occur at $x = \pi$ and $x = -\pi$.
The intercepts are ($-\pi/2$,0) and (0,1).

Question 4



Question 5

a.



Select a point other than (0,5) to indicate scale along the x axis. (1, 6.90) would suffice.
Horizontal asymptote $y = 8$.
Do NOT sketch anything for $x < 0$.

b.

Interchanging x and y:

$$x = 8 - 3e^{-y}$$

$$3e^{-y} = 8 - x$$

$$e^{-y} = \frac{8-x}{3}$$

$$-y = \log_e\left(\frac{8-x}{3}\right)$$

$$f^{-1}(x) = -\log_e\left(\frac{8-x}{3}\right) \text{ for } 5 \le x < 8$$

Question 6

a. $y = (x + 2)(x^2 - 4x + 3)$

b. $y = (x + 2)(x - 1)(x - 3)$ and so the x intercepts are 1, 3 and -2.

c. $\int_{-2}^{1} (x^3 - 2x^2 - 5x + 6)dx$

$$- \int_{1}^{3} (x^3 - 2x^2 - 5x + 6)dx$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x\right]_{-2}^{1}$$

$$- \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x\right]_{1}^{3}$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6\right) - \left(\frac{16}{4} + \frac{16}{3} - \frac{20}{2} - 12\right)$$

$$- \left(\frac{81}{4} - \frac{54}{3} - \frac{45}{2} + 18\right) + \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6\right)$$

$= 21.08$

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Question 7

a.

$3^x = e^{kx}$

Taking logs to base e of both sides:

$\log_e (3^x) = \log_e (e^{kx})$

$x \log_e 3 = kx$ so $k = \log_e(3)$ for all x.

Answer: $k = \log_e 3$

b.

$$\frac{d}{dx}(3^x) = \frac{d}{dx}(e^{x\log 3})$$

$= \text{loge}(3) \times e^{x\log 3}$

or

$\text{loge}(3) \times 3^x$

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