The Mathematical Association of Victoria

MATHEMATICAL METHODS

Trial written examination 2
(Analysis task)

2005

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	55

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2005 Mathematical Methods Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Instructions

Answer **all** questions in the space provided.

A decimal approximation will not be accepted if an **exact** answer is required for a question.

Where an **exact** answer is required for a question, appropriate working must be shown.

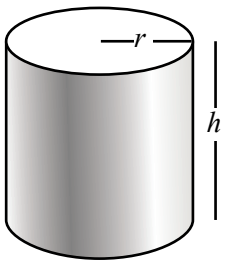
In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A cylindrical can of dog food, Aus Pet, has a radius of r cm and a height of h cm as shown in the diagram at right.



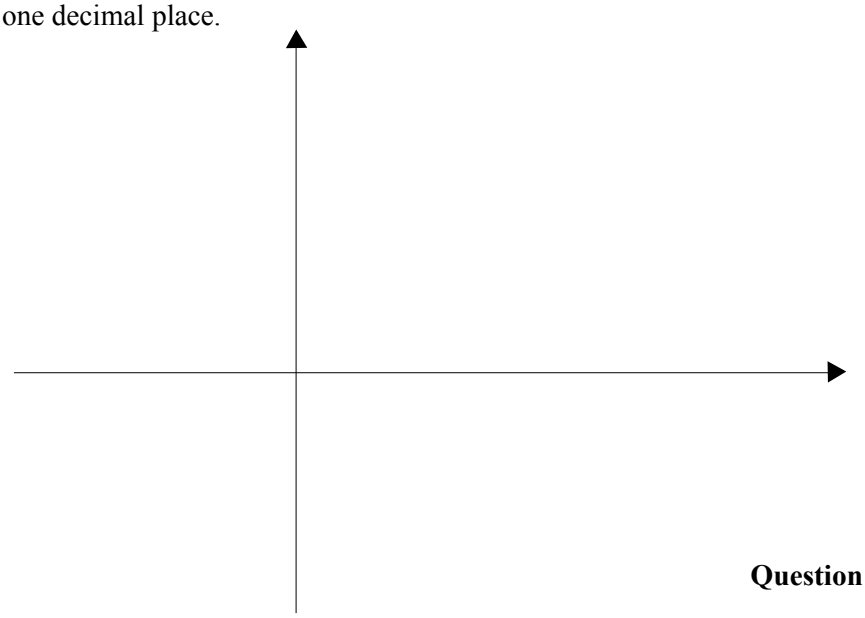
a. If the total surface area of the outside of the can is $98\pi + 140$ cm^2 show that $h = \dfrac{70 + 49\pi - \pi r^2}{\pi r}$ cm.

2 marks

b. Show that the volume of the can, V cm^3 is $V = (70 + 49\pi)r - \pi r^3$

2 marks

c. Sketch the graph of the curve with equation $V = (70 + 49\pi)r - \pi r^3$ for $V > 0$ and $r > 0$.
Give key features correct to one decimal place.



3 marks

d. Use calculus to find the exact radius of the can which will give the maximum volume for the can and hence show that the height of the can which will give the maximum volume is:

$h = 2(10 + 2\pi)\sqrt{\frac{7}{30\pi + 21\pi^2}}$ cm

3 marks

e. The cost of the pet food is 20 c per 100 cm^3 and the materials to make the can cost 50 c per 100 cm^2.

i. Write a cost, $\$C$, equation in terms of V

ii. Describe the transformations that have occurred to the graph of V to get C.

iii. Hence, or otherwise, write down the maximum cost of producing a can of dog food.
Give your answer to the nearest cent.

1 + 2 + 1 = 4 marks

Total 14 marks

Question 2

The length of gum leaves found in an eucalypt forest is normally distributed. 10% of the gum leaves are more than 25 cm in length and 15 % are less than 10 cm.

a. Show that, correct to two decimal places, the mean length is 16.71 cm and the standard deviation is 6.47 cm.

4 marks

b. Find the probability, correct to four decimal places, that a leaf picked up at random is less than 15 cm in length.

2 marks

c. Ten of the leaves are picked up in succession at random.
What is the probability, correct to four decimal places, that at least 3 will be more than 15 cm in length?

3 marks

d. Find the proportion of leaves, correct to four decimal places, that are more than 20 cm in length if all the leaves less than 15 cm are discarded.

3 marks

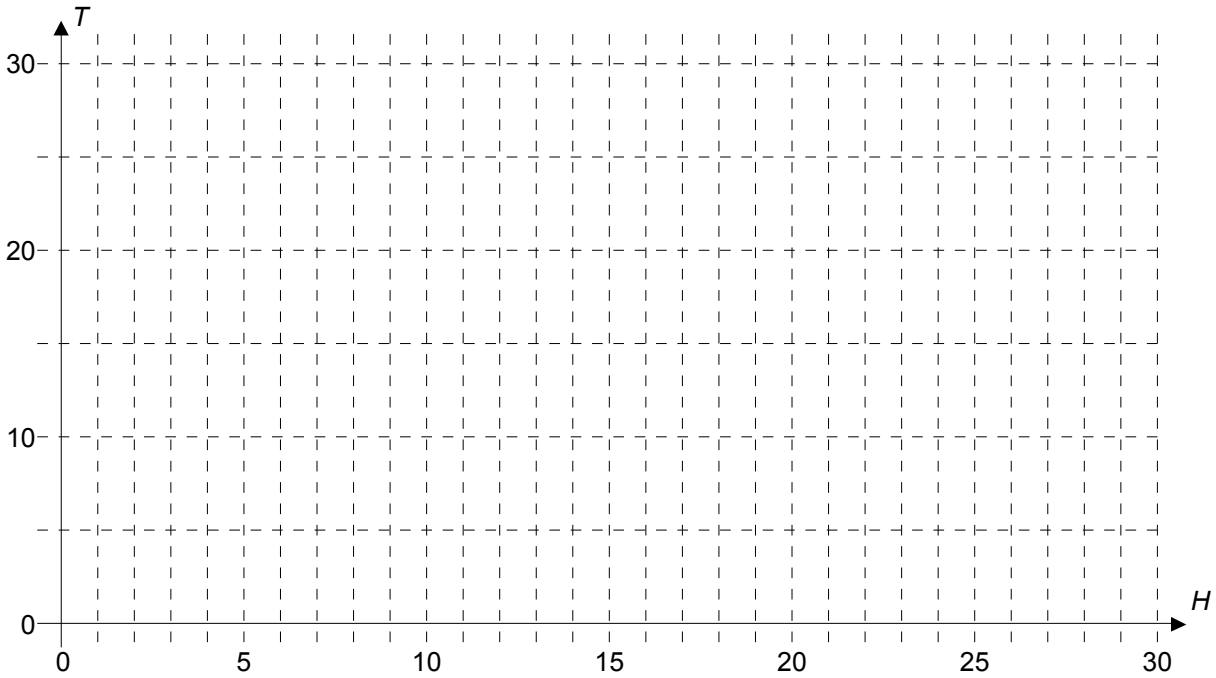
Total 12 marks

Working Space

Question 3

The temperature during a particular summer's day is modelled using a trigonometric equation of the form $T = A \cos(n(H + b)) + c$, where T is the temperature in degrees Celsius and H is the number of hours since midnight. The minimum temperature occurred at 4 a.m. when it was 12 degrees, and the maximum at 4 p.m. when the temperature was 30 degrees.

a. Mark the position of the maximum and minimum temperatures using the axes provided and hence sketch the graph of $T(H)$.



4 marks

b. Find the values of A, n, b and c.

4 marks

c. Find the temperature at midday and at midnight correct to one decimal place.

2 marks

d. Write down an expression for the rate of change of temperature with respect to hours past midnight.

1 mark

e. Hence find when the temperature is rising most rapidly.

3 marks

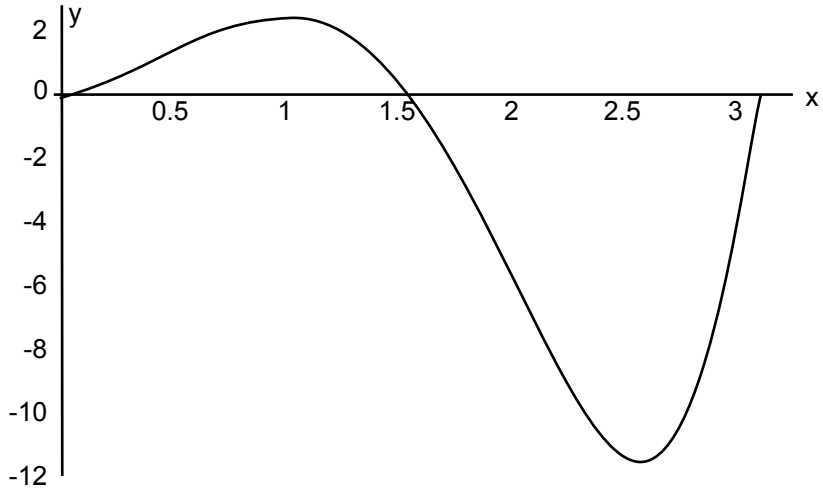
f. Find the exact maximum rate of increase of temperature.

. 1 mark

Total 15 marks

Question 4

The graph of $y = f(x)$ where $f: [0, \pi) \rightarrow R$, and $f(x) = \sin(2x)e^x$ is shown below.



The graph represents an aerial view of farmer Jones' property near Shepparton, where x and y are distances, in arbitrary units.

a. Solve $f(x) = 0$ giving exact solutions.

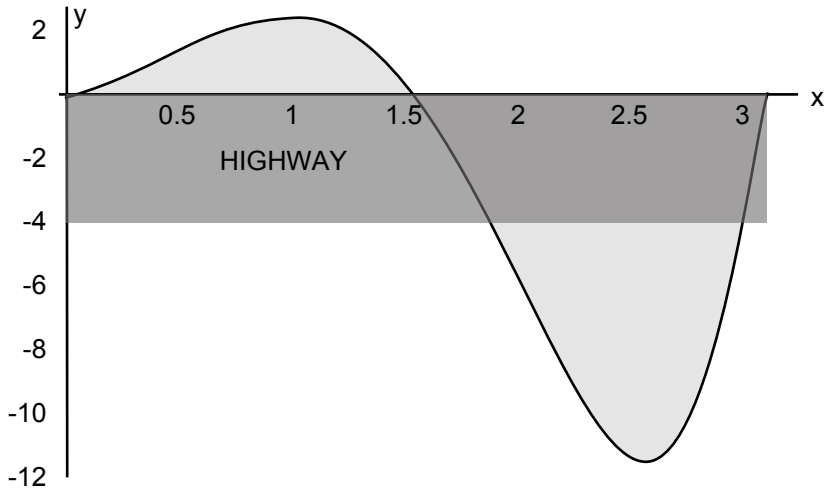
1 mark

b. **i.** Show that $\frac{d}{dx} e^x (\sin(2x) - 2\cos(2x)) = 5e^x \sin(2x)$

ii. Hence, find the exact area bounded by $f(x)$ and the x-axis.

1 + 3 = 4 marks

The government wants to shift the Goulburn Valley highway.
It is to run through the middle of farmer Jones' property as shown in the diagram below.



c. Farmer Jones has two sons, Graham and John. He wants to leave each of his sons the same area of land.

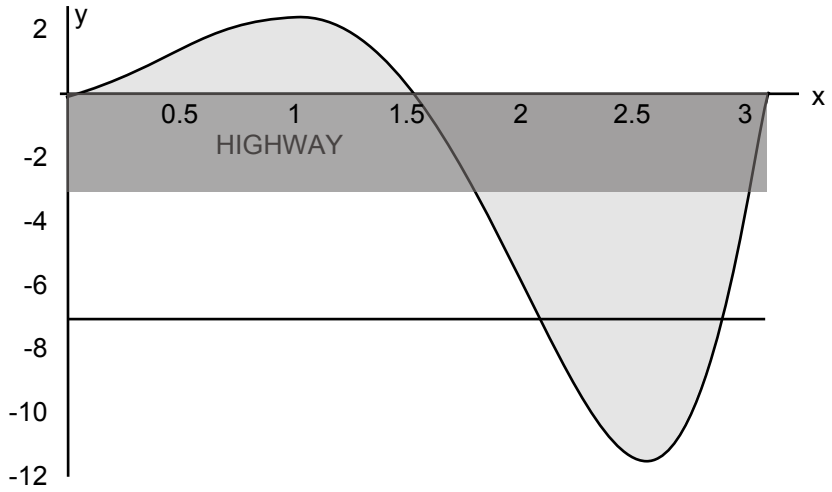
i. Graham is to get the area of land above the *x*-axis.
Calculate the area of this land, in square units, correct to two decimal places.

ii. Use trial and error to find the maximum width of the highway if John is to get the land on the other side of the highway? Give your answer correct to one decimal place.

2 + 3 = 5 marks

d.

Unfortunately, father Jones passed away. His daughter, Ann, decided that she should get an equal share of the land as well. Her brothers kindly agreed to this arrangement so long as Graham still gets the land above the x-axis.
A boundary fence is to be erected between Ann's and John's properties as shown below.



i. Use trial and error to find the maximum width of the highway now. Give your answer correct to the nearest integer.

ii. If the boundary fence line between Ann's and John's properties is at $y = a$, where a is a real constant, find the value of a. Give your answer correct to one decimal place.

3 + 1 = 4 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOK

Working Space

Working Space

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin A$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$ $= \sum x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET