MATHS METHODS EXAM 2: SOLUTIONS

Question 1

a. $2\pi r^2 + 2\pi rh = 98\pi + 140$ **1M**

$2\pi rh = 98\pi + 140 - 2\pi r^2$

$h = \dfrac{98\pi + 140 - 2\pi r^2}{2\pi r}$ **1M**

$= \dfrac{70 + 49\pi - \pi r^2}{\pi r}$ as required

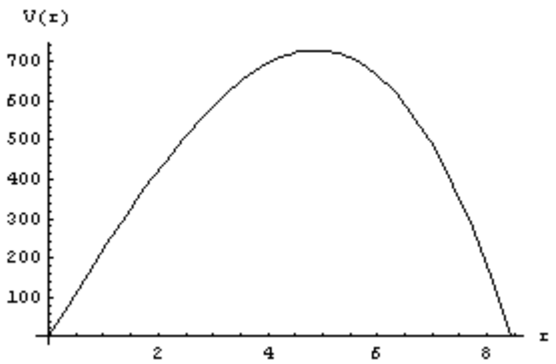
b. $V = \pi r^2 h$

$= \pi r^2 \dfrac{70 + 49\pi - \pi r^2}{\pi r}$ **1M**

$= r(70 + 49\pi - \pi r^2)$ **1M**

$= (70 + 49\pi)r - \pi r^3$ as required.

c.



shape **1M**

open circles at (0, 0) and (8.4, 0) **1M**

maximum marked (4.9, 727.7) **1M**

d. $V'(r) = 0$

$70 + 49\pi - 3\pi r^2 = 0$ **1M**

$r^2 = \dfrac{70 + 49\pi}{3\pi}$

$r = \sqrt{\dfrac{70 + 49\pi}{3\pi}}$ cm **1A**

$h = \dfrac{98\pi + 140 - 2\pi r^2}{2\pi r}$

$= \dfrac{98\pi + 140 - 2\pi \dfrac{70 + 49\pi}{3\pi}}{2\pi\sqrt{\dfrac{70 + 49\pi}{3\pi}}}$ cm **1M**

$= 2(10 + 7\pi)\sqrt{\dfrac{7}{30\pi + 21\pi^2}}$ cm as required.

e. i $C = 0.005(98\pi + 140) + 0.002V$

$= 0.49\pi + 0.7 + 0.002V$ **1A**

ii Dilation of a factor of 0.002 from the r axis, **1A**

followed by a translation of $0.49\pi + 0.7$ cm parallel to the V axis. **1A**

iii $C = 0.49\pi + 0.7 + 0.002V_{max}$

$= 0.49\pi + 0.7 + 0.002 \times 727.721$

$= \$3.69$ **1A**

Question 2

a. 10% > 25 gives $Z_1 = 1.28155$

15% < 10 gives $Z_2 = -1.03643$ **1M**

$\dfrac{25 - \mu}{\sigma} = 1.28155$; $\dfrac{10 - \mu}{\sigma} = -1.03643$ **2A**

solve simultaneously **1M**

$\mu = 16.71$, $\sigma = 6.47$ as required

b. normalcdf(−E99,15,16.71,6.47) **1M**

= 0.3958 **1A**

c. Binomial **1M**

$N = 10$, $p = 1 - 0.3958$, $x \geq 3$ **1A**

$\Pr(X \geq 3) = 0.9886$ **1A**

d. Conditional probability

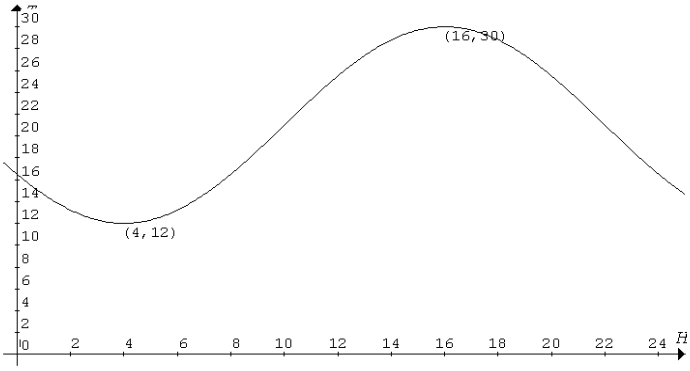
$\Pr(X > 20 \mid X > 15) = \dfrac{\Pr(X > 20)}{\Pr(X > 15)}$ **1M**

$= \dfrac{0.3056}{0.6042}$ **1A**

$= 0.5058$ **1A**

Question 3

a.



Minimum **1A**
Maximum **1A**
Domain [0, 24] **1A**
Axes scaled and shape **1A**

b. amplitude $= \dfrac{30-12}{2}$; $A = 9$ **1A**

period $= \dfrac{2\pi}{n} = 24$; $n = \dfrac{\pi}{12}$ **1A**

median value = 12 + 9; $c = 21$ **1A**

horizontal translation = 16 to the right or 8 to the left; $b = -16$; or + 8 **1A**

c. When $H = 0$, $T = 16.5$;
when $H = 12$, $T = 25.5$ **2A**

d. $\dfrac{dT}{dH} = -\dfrac{3\pi}{4}\sin(\dfrac{\pi}{12}(H+8))$ **1H**

e. $\dfrac{dT}{dH}$ is maximum

when $\sin(\dfrac{\pi}{12}(H+8)) = -1$ **1H**

when $\dfrac{\pi}{12}(H+8) = \dfrac{3\pi}{2}$, $H = 10$ **1A**

Answer: at 10 am **1A**

f. At 10 am $\dfrac{dT}{dH} = \dfrac{3\pi}{4}$ **1A**

Question 4

a. $\sin(2x)e^x = 0$

$e^x \neq 0$,

$\sin(2x) = 0$

$x = 0, \dfrac{\pi}{2}, \pi$ **1A**

b. i $\dfrac{d}{dx}e^x(\sin(2x) - 2\cos(2x)) =$

$e^x(\sin(2x) - 2\cos(2x)) + e^x(2\cos(2x) + 4\sin(2x))$ **1M**

$= 5\sin(2x)e^x$ as required

ii $\int_0^{\frac{\pi}{2}}(\sin(2x)e^x)dx - \int_{\frac{\pi}{2}}^{\pi}(\sin(2x)e^x)dx$ **1M**

$= \dfrac{1}{5}([[(e^x(\sin(2x) - 2\cos(2x))]_0^{\frac{\pi}{2}} - [(e^x(\sin(2x) - 2\cos(2x))]_{\frac{\pi}{2}}^{\pi}$ **1M**

$= \dfrac{2}{5}(e^{\pi} + 2e^{\frac{\pi}{2}} + 1)$ **1A**

$= \dfrac{2}{5}(1 + e^{\frac{\pi}{2}})^2$ units^2

c. i $\int_0^{\frac{\pi}{2}}(\sin(2x)e^x)dx$ **1M**

= 2.32 units^2 **1A**

ii Use trial and error with the calculator.

$\int_{2.09225}^{2.9550}(-7.5 - f(x))dx \approx 2.32687$ units^2 **2M**

7.5 units **1A**

d. i Area of Ann's and John's land

$= 2\int_0^{\frac{\pi}{2}}(\sin(2x)e^x)dx \approx 4.648$ **1M**

Use trial and error with the calculator.

$\int_{1.96101}^{3.01811}(-5 - f(x))dx \approx 4.66728$ **1M**

$\Rightarrow$ 5 units **1A**

ii $a = -7.5$ units from **c. ii** **1A**