Australian Education Academy
2005

MATHMATICAL METHODS

WRITTEN EXAMINATION 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's name: ____

Directions to students

This examination consists of **four** questions.
Answer all questions.
All working and answers should be written in the spaces provided.
The marks allotted to each part of each question appear at the end of each part.
There are **55 marks** available for this task.

These questions have been written and published to assist students in their preparations for the 2005 Mathematical Methods Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victoria Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Written Examination 2: Analysis Task

Question 1

Consider the function $f: D \to R$, for which the rule is $f(x) = \dfrac{24}{x+3} - 6$ where D is the maximal domain for f.

a. Find D

[1 mark]

b. Describe a set of transformations which when applied to the graph of $y = \dfrac{1}{x}$, produce the graph of $y = f(x)$.

[3 marks]

c. Find the coordinates of the points where the graph of f cuts the axes.

[1 mark]

d. Let $g:(-3,\infty)\to R$, $g(x)=f(x)$

(i). State the domain and range of g

[1 mark]

(ii). Find the rule for g^{-1}, the inverse of g.

[2 marks]

(iii). Find the coordinates of the points where the graph of g^{-1} cuts the axes.

[1 mark]

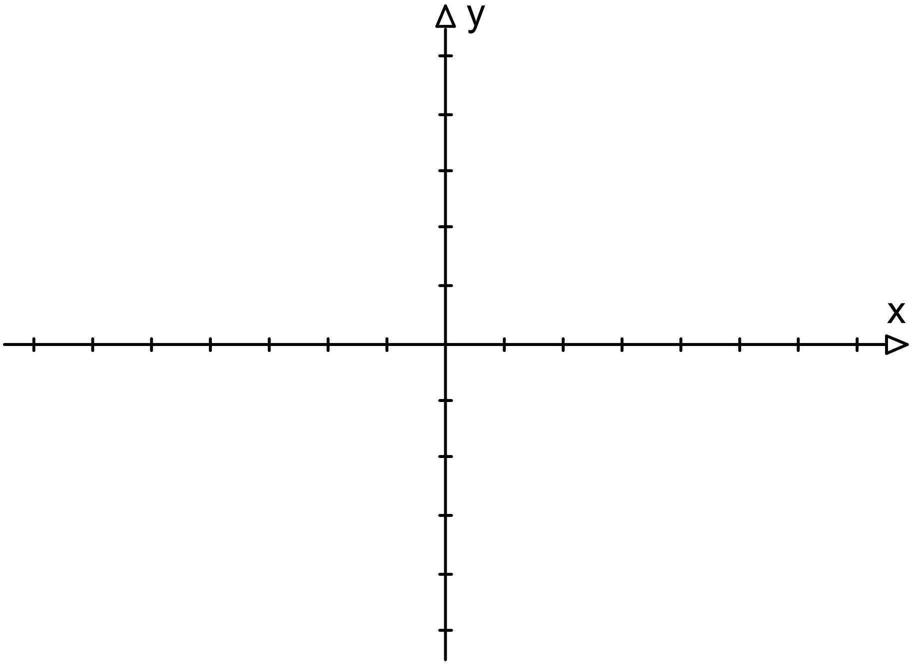
(iv). Write down the domain of g^{-1}.

[1 mark]

(v). Find the values of x for which $g(x)=x$ and hence the values of x for which $g(x)=g^{-1}(x)$.

[3 marks]

(vi). Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the one set of axes.



[2 marks]
[Total 15 marks]

Question 2

The population of rabbits on a particular island t weeks after a virus is introduced is modeled by $P = 1500e^{-0.05\,t}$, where P is the number of rabbits.

a. Find the initial size of the population

[1 mark]

b. Find the time taken for the population to halve (to the nearest week)

[1 mark]

c. Obtain an expression to calculate the rate of decrease of the population at time *t*.

[1 mark]

d. Find the rate of decrease of the population.

(i). after 2 weeks

[1 mark]

(ii). after 10 weeks.

[1 mark]

After 16 weeks the virus has become ineffective and the population of rabbits starts to increase again according to the model $P = P_0 + 12(t-16)\log_e(2t-31)$ where t is the number of weeks since the virus was first introduced.

e. State the domain of P

[1 mark]

f. Determine the value of P_0

[1 mark]

g. Find the size of the population after 32 weeks.

[1 mark]

h. Derive an expression to calculate the rate of increase of the population at time *t*.

[2 marks]

i. Find the rate of change of the population
(i). after 20 weeks

[2 marks]

(ii). and 32 weeks.

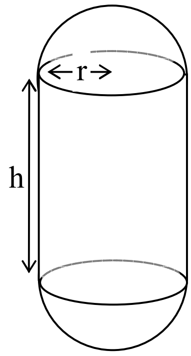
[2 marks]

j. Write down an equation to find how many weeks the population takes to get back to its original number.

[1 mark]
[Total 15 marks]

Question 3

A closed capsule is to be constructed as shown in the diagram. It consists of a circular cylinder of height *h cm* and radius *r cm*, with hemispherical caps of radius *r cm* on each end.



a.

(i). Show that the volume, $V\ cm^3$, the capsule is given by $V = \frac{\pi r^2}{3}(3h + 4r)$.

[2 marks]

(ii). Show that the surface area of the capsule ,$S\,cm^2$, is given by $S = 2\pi r(2r + h)$.

[2 marks]

b.

(i). If $V = \pi a^3$, where *a* is a positive constant find *h* in terms of *a* and *r*.

[1 mark]

(ii) The values which r can take lie in an interval . Find the end points of this interval.

[2 marks]

(iii). Find S in terms of a and r.

[1 mark]

c.

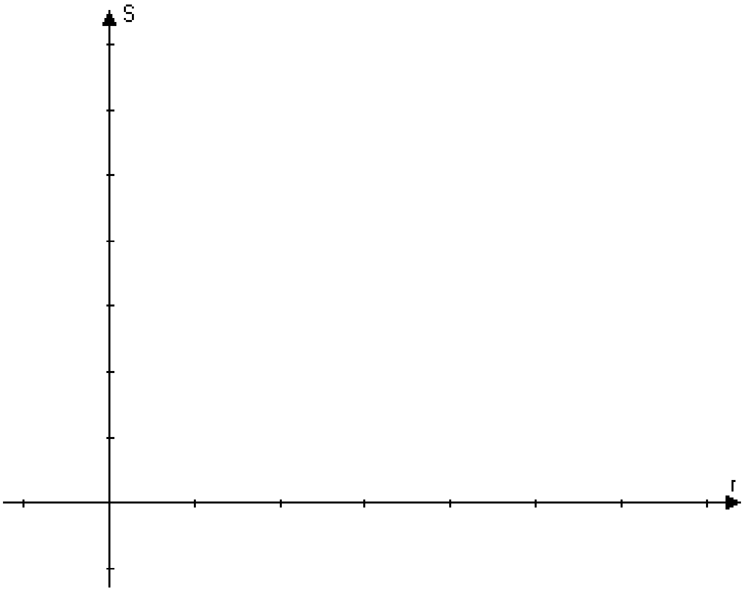
(i). Find an expression for $\frac{dS}{dt}$ and hence find the coordinates of the turning points.

[2 marks]

(ii) Determine the nature of the turning point and the corresponding value of S

[2 marks]

(iii). By using addition of ordinates or otherwise sketch the graph of S against r for suitable domain.



[3 marks]

[Total 15 marks]

Question 4

A multiple choice examination consists of 25 questions, for each of which the candidate is required to tick as correct one of 5 possible answers. Exactly one possible answer to each question is correct. A correct answer gets 1 mark and a wrong answer gets 0 marks. Consider a candidate who has complete ignorance about every question and therefore ticks at random. The random variable X denotes the number of correct answers.

a. What is the probability that he gets a particular answer correct?

____

[1 mark]

b. What is the distribution of X?

____

[1 mark]

c. Write down the probability distribution of X?

____

____

[1 marks]

d. Calculate the mean, the variance and the standard deviation of X.

[3 marks]

e. Calculate the probabilities that he gets zero marks for the paper.

[1 mark]

f. The examiners wish to ensure that not more than 1% of completely ignorant candidates pass the examination. Use Normal approximation to obtain the pass mark that meets this requirement.

[3 marks]
[Total 10 marks]