# 2004 Mathematical Methods (CAS) GA 3: Examination 2

## GENERAL COMMENTS

The number of students presenting for Mathematical Methods (CAS) examination 2 in 2004 was 389, which was 121 more than the 268 who sat in 2003. The range of marks available was 0 to 55. Student responses showed that the paper was accessible and provided an opportunity for students to demonstrate their knowledge.

There were excellent papers presented by several students. Two students achieved a perfect score and 19% of students achieved a score of over 80%. The median score was 27.5 and mean score for the paper was 28.1. Fifty per cent of the students scored over half of the marks for the paper and 60% of the students scored over 40%. There were 20% students who scored under 20% and three students received no marks for the paper.

Generally the symbolic facility of CAS was used well. There was no discernable advantage seen by the assessors of one Computer Algebra System over another. The generally high level of proficiency with the systems was demonstrated in particular in Question 1a**,** where 91% of students achieved full marks.

As in previous years, students sometimes gave their answers as numerical approximations instead of in exact form, although exact answers were asked for explicitly. For example, in Question 1f the students were required to find the exact value of $a$ in the equation $y = (x - 1)^2(x - a)$ for which the local minimum of the graph lies on the straight line with equation $y = -4x$. Some students gave an approximation to this value.

Questions 1d and 1e proved to be difficult for students and revealed some difficulty in understanding transformations.

Students should be familiar with the notation for the equation of the image of a graph under transformation. This includes:

- the equation $y = f(\frac{x}{k})$ of the image, under a dilation of factor $k$ from the $y$ axis, of the graph of $y = f(x)$
- the equation $y = f(x + h)$, where $h > 0$, of the image, under a translation of the graph of $y = f(x)$ of $h$ units in the negative direction of the $x$ axis.

It was apparent that some students were not familiar with this notation.

Questions 2a, b and c on probability were relatively straightforward for students who were familiar with transition matrices and the powers of these matrices. Students should know how to use matrices in this context.

If a question is worth more than one mark, students risk losing all the available marks if an incorrect answer is given and there is no working shown. The instructions at the beginning of the paper state that if more than one mark is available for a question, appropriate working must be shown. Students should take note of this in their responses. This was particularly evident in Questions 1f and 2c, where students lost marks for not showing appropriate working.

Many students lost marks because they:
- did not answer the question asked
- gave decimal answers when an exact answer was required
- gave the wrong number of decimal places.

Students are expected to use conventional mathematical expressions, symbols, notation and terminology when they present working and develop solutions.

## SPECIFIC INFORMATION

**Question 1**

**1a**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 9 | 91 | **0.9** |

**Correct response:** $f'(x) = 3x^2 - 8x + 5$

This question could be completed through a careful and correct entry of the rule for the function and correct application of the differentiation facility.

**1b**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 12 | 23 | 66 | **1.6** |

**Correct response:** $a = 1$ and $b = \frac{5}{3}$

This question was well done, although a number of students reversed the values of *a* and *b*. The question showed good use of CAS.

**1c**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 72 | 11 | 17 | **0.5** |

**Correct response:** $p > 1$ or $p < \frac{23}{27}$

This question required a graphical approach and the use of the local maximum and minimum values given in the previous question, and proved difficult for the majority of students. There was no requirement to graph the function, however a plot of the graph using CAS would have been beneficial. The picture to be formed is considering horizontal lines with equations $y = p$. The required values are those 'beyond' the *y* values of the turning points.

**1di**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 16 | 40 | 44 | **1.3** |

**Correct response:** A dilation of factor *k* from the *y* axis and a translation of 1 unit in the negative direction of the *y* axis.

Descriptions given were often not clear or accurate. Dilations should be described as from a line, in this case the *y* axis. Translations should be given in a form such as 'magnitude one in the negative direction of the *x* axis'. Care should be taken with these written descriptions. In this question order of transformations was not important, although most students gave the conventional order of dilation followed by translation.

**1dii**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 31 | 17 | 52 | **1.2** |

**Correct response:** $x = k$ and $x = 2k$

This could be completed using CAS or simply by noting that $f\left(\frac{x}{k}\right) - 1 = \left(\frac{x}{k} - 1\right)\left(\frac{x}{k} - 2\right)^2$ which clearly has roots *k* and 2*k*. The dilation of factor *k* from the *y* axis could also be used to give the result immediately. A sound understanding of transformations simplifies this problem, although simple experimentation for different values of *k* could also have guided the student to the result.

**1diii**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 58 | 24 | 18 | **0.6** |

**Correct response:** $\frac{k}{12}$

This result can be achieved directly through a suitable integral of $f\left(\frac{x}{k}\right) - 1$. Correct use of CAS would give the result immediately. The area of the region contained between the $x$ axis and the graph of $y = f(x) - 1$ (between $x = 1$ and $x = 2$) is $\frac{1}{12}$ and hence the area of the dilated region is $\frac{k}{12}$.

**1e**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 79 | 15 | 7 | **0.3** |

**Correct response:** $1 \le h < 2$

The question is equivalent to asking when the roots of the equation $f(x + h) - 1 = 0$ are positive and hence when the $x$-axis intercepts of the graph of $y = f(x + h) - 1$ are positive. A consideration of the translations of $y = f(x) - 1$ gives the required result.

**1f**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 44 | 27 | 8 | 21 | **1.1** |

**Correct response:** $\left(\frac{2a+1}{3}, \frac{-4(a-1)^3}{27}\right)$ are the coordinates of the local minimum

$a = \frac{3\sqrt{5}+5}{2}$

This was a challenging question and a good number of students were successful. Many students demonstrated efficient and effective use of CAS. This was a question in which some students lost marks through not presenting working and by using approximations.

**Question 2**

**2a**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 37 | 63 | **0.7** |

**Correct response:** $\frac{4}{5}$

This is obtained from the transition matrix. The majority of students made this interpretation, which is fundamental to understanding the use of the transition matrix.

**2b**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 31 | 20 | 5 | 44 | **1.7** |

**Correct response:** $T^3\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{68}{125} \\ \frac{52}{125} \end{pmatrix}$ where $T = \begin{pmatrix} 0.4 & 0.8 \\ 0.6 & 0.2 \end{pmatrix}$

Therefore the exact probability is $\frac{68}{125} = 0.544$

This problem could also be successfully completed using a tree diagram, but very few students obtained the correct answer using this approach. Students should be aware of the convenience of solving such problems using powers of the transition matrix, which is the most efficient approach given the availability of CAS.

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**2c**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 41 | 6 | 4 | 49 | **1.6** |

**Correct response:** Solve $TS = S$ for $S$

$$S = \begin{pmatrix} \frac{4}{7} \\ \frac{3}{7} \end{pmatrix}$$

57% of nights on the north side (to the closest per cent)

Students could also obtain this value by considering $T^n$ for large $n$. Full marks were awarded for this method if it was demonstrated for sufficiently large $n$. The majority of students who successfully obtained the answer in 2a went on to get this result.

**2d**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 34 | 2 | 64 | **1.3** |

**Correct response:** $\frac{3}{32}\int_3^4 t(4-t)\,\mathrm{d}t = \frac{5}{32}$

This question was well done and it was evident that students who recognised the need for an integral used CAS to successfully obtain the answer. Those who completed the integration by hand were also generally successful.

**2e**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 52 | 13 | 35 | **0.9** |

**Correct response:** Binomial distribution with parameter values $n = 3$ and $p = \frac{5}{32}$

$\Pr(X \geq 2) = 0.066$

Many students did not recognise three **independent** trials and hence the applicability of the binomial distribution.

**2f**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 53 | 20 | 27 | **0.8** |

**Correct response:** Solve $\frac{3}{32}\int_0^T t(4-t)\,\mathrm{d}t = 0.104$ for $T$

The number of minutes $n = 48$ minutes

Students who used the above equation generally obtained the answer, with a small number of students losing a mark for not giving their answer in minutes.

**Question 3**

**3ai**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 65 | 15 | 20 | **0.6** |

**Correct response:** $a + bt \geq 0$ which implies $t \leq -\frac{a}{b}$

Some students did not understand that $-\frac{a}{b}$ is a positive number as $b < 0$.

**3aii**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 87 | 1 | 12 | **0.3** |

**Correct response:** $P(t) = P_0e^{G(t)}$
and $P(0) = P_0$. Therefore $G(0) = 0$.

Students had to establish this result in either 3ai or 3aii. This question required careful thinking for students to successfully present their argument.

**3aiii**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 28 | 3 | 69 | **1.4** |

This was well answered. Students could use the result from 3aii.

**3bi**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 27 | 73 | **0.8** |

Correct substitutions gave the result. Some students lost a mark due to transcription errors.

**3bii**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 55 | 45 | **0.5** |

**Correct response:** Correct substitution for $a$ and $b$ gives $t \leq \frac{-0.02}{-0.002} = 100$ (years).

**3biii**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 48 | 52 | **0.5** |

**Correct response:** $\frac{dP}{dt} = 3(0.02 - 0.0002t)\, e^{0.02t - 0.0001t^2}$

Some students transcribed incorrectly, either from their CAS or their rough work; students must undertake the final recording of the answer with care. The mark was not awarded if 0.001 was used instead of 0.0001.

**3biv**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 49 | 51 | **0.5** |

**Correct response:** $\frac{dP}{dt} = 0.069$ when $t = 20$.

Care must be taken in giving the answer correct to three decimal places.

**3ci**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 36 | 5 | 59 | **1.3** |

**Correct response:** $t = 100 \log_e\left(\frac{p}{3}\right)$

This question was answered well. CAS will give the result directly when applied correctly.

**3cii**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 53 | 47 | **0.5** |

**Correct response:** $t = 100\log_e(2) = 69.3$. Population doubles in 2059.

Solution of the equation $6 = 3e^{0.01t}$ yields the result or direct substitution in the result of 3ci.

**Question 4**

**4a**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 9 | 91 | **0.9** |

**Correct response:** (62 + 60) metres = 122 metres

Maximum when sine takes the value 1. A pleasing demonstration of students' familiarity with the basic properties of circular functions.

**4b**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 7 | 93 | **1.0** |

**Correct response:** (62 – 60) metres = 2 metres

Minimum when sine takes the value –1. Again, a pleasing demonstration of students' familiarity with the basic properties of circular functions.

**4c**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 39 | 61 | **0.6** |

**Correct response:** 1:48 as the period is $2\pi \div \frac{5\pi}{2} = 2\pi \times \frac{2}{5\pi} = \frac{4}{5} = 0.8$

**4di**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 16 | 26 | 58 | **1.4** |

**Correct Response:** Solution of the equation $92 = 62 + 60\sin\left(\frac{(5t-1)\pi}{2}\right)$

$t = \frac{4}{15}$ which gives that the first time at 1:16 pm

This can be done numerically, but an **exact answer** was required.

**4dii**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 43 | 57 | **0.6** |

**Correct response:** 16 minutes

An exact answer in minutes was required.

**4ei**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 35 | 65 | **0.7** |

**Correct response:** $\frac{dh}{dt} = 150\pi\cos\left(\frac{(5t-1)\pi}{2}\right) = 150\pi\sin\left(\frac{5\pi t}{2}\right)$

It was not necessary to write the function in the second, neater form to obtain the mark, and very few students did so. Some students made a mistake when entering the function and thus lost this mark.

**4eii**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 31 | 69 | **0.7** |

**Correct response:** $150\pi\cos(2\pi) = 150\pi$. The exact answer is 150 $\pi$ m/h.
This is obtained by substitution in the result of 4ei.

**4fi**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 37 | 63 | **0.7** |

**Correct response:** (0.4, 122) marked on graph of $y = h(t)$

**4fii**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 41 | 45 | 15 | **0.8** |

Correct use of addition of ordinates after $t = 0.4$. The path of the spider is described by $y = h(t)$ up until the point $P$ reaches its highest point.

**4fiii**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 50 | 30 | 20 | **0.7** |

**Correct response:** Solution of $s(t) = 0$ is required which gives $t = 0.6026$ by numerical solution
Required time = 0.6026 – 0.4 = 0.2026
This result needs to be converted to minutes $0.2026 \times 60 = 12.16$.