2004 Assessment Report

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

GENERAL COMMENTS

The number of students who sat for this examination in 2004 was 389. Marks ranged from 2 to 50 out of a maximum of 50. Student responses showed that the paper was accessible and that it provided the opportunity for students to demonstrate what they knew. Of the whole cohort, 36 students (9.25%) scored 90% or more of the available marks, and 63.5% scored 50% or more of the available marks. The mean score for the paper was 29.9, comprised of a mean of 17.9 for the multiple-choice part (out of 27 marks in total) and a mean of 12.0 for the short answer part (out of 23 marks in total). The median score for the paper was 31 marks. There was a slight decrease in the mean score compared with 2003, but a slight increase in the median score compared to 2003.

Overall, the symbolic facility of CAS was used well. There was no discernable advantage seen by the assessors of one CAS over another, although in some cases techniques for dealing with the mathematics varied according to the CAS. Multiple choice Questions 4, 12, 13, 18 were all answered correctly by at least 80% of students. Multiple choice Questions 3, 11 and 19 were not answered as well, with less than 50% of students obtaining the correct answer to each.

There were many very good responses to the questions in Part II and several students were able to work through questions completely to obtain full marks for this section. There was little evidence to suggest that not making a reasonable attempt at Part II was due to lack of available time.

Students should be familiar with instructions such as:

- a decimal approximation will not be accepted if an exact answer is required to a question
- in questions where more than one mark is available, appropriate working must be shown.

It was pleasing to see that where working was shown, correct mathematical notation was generally used.

Student facility with graphical representation seems to have improved since 2003. The graph of the circular function in Question 4a in Part II was done correctly by over 85% of students. However, care needs to be taken in specifying asymptotes and showing asymptotic behaviour, as in Question 3 of Part II.

SPECIFIC INFORMATION

Part 1 – Multiple-choice questions

Content that needs further attention includes the derivative of the absolute value of a function. This is often most easily done by hand. Since $|\sin(x)| = \sin(x)$ if $\sin(x) \geq 0$ and $-\sin(x)$ if $\sin(x) < 0$ for the interval in question, $\pi < x < 2\pi$, $|\sin(x)| = -\sin(x)$, and so the derivative will simply be $-\cos(x)$. If technology is used, the derivative of $|\sin(x)|$ may be given as $\text{sign}(\sin(x))\cos(x)$, and over the interval in question this will be equal to $-\cos(x)$. Students should be familiar with the sign or signum function, where:

$$sign(x) = \begin{cases} 1 & if \quad x > 0 \\ -1 & if \quad x < 0 \end{cases}$$

as it may appear if technology is used with the absolute value function in particular. Question 24, which tested the definition of the definite integral as the limiting value of a sum and its corresponding representation in integral form (which is fundamental to the understanding of the definite integral), was answered much better this year, but there is still room for improvement. Other areas that would benefit from further attention are simple probability (Question 3), linear factors of a polynomial (Question 9), functional forms (Question 11), transformations of functions (Question 17), **average** rate of change of a function over an interval (Question 20) and definite integrals (Question 24).

This table indicates the number of students who chose each option. The correct answer is indicated by shading.

	A	B	C	D	E	No Answer
Question	**%**	**%**	**%**	**%**	**%**	**%**
1	4	13	79	3	1	0
2	6	16	64	8	5	1
3	14	8	12	19	47	1
4	3	5	82	8	2	0
5	8	68	3	15	6	0
6	13	76	7	2	2	0
7	10	70	13	6	1	0
8	4	7	9	73	7	0
9	24	10	3	11	52	1
10	10	65	4	12	9	0
11	44	36	6	5	9	0
12	3	0	6	90	1	0
13	6	86	3	2	2	0
14	6	11	67	10	5	1
15	79	4	11	6	1	0
16	4	4	79	4	9	0
17	15	12	14	3	56	0
18	2	4	4	3	87	0
19	31	53	6	5	5	1
20	54	3	18	10	14	1
21	14	11	7	62	6	0
22	6	60	8	12	13	0
23	68	4	19	4	5	0
24	4	15	16	53	10	1
25	1	14	79	2	4	0
26	3	7	14	69	7	0
27	51	24	9	11	4	0

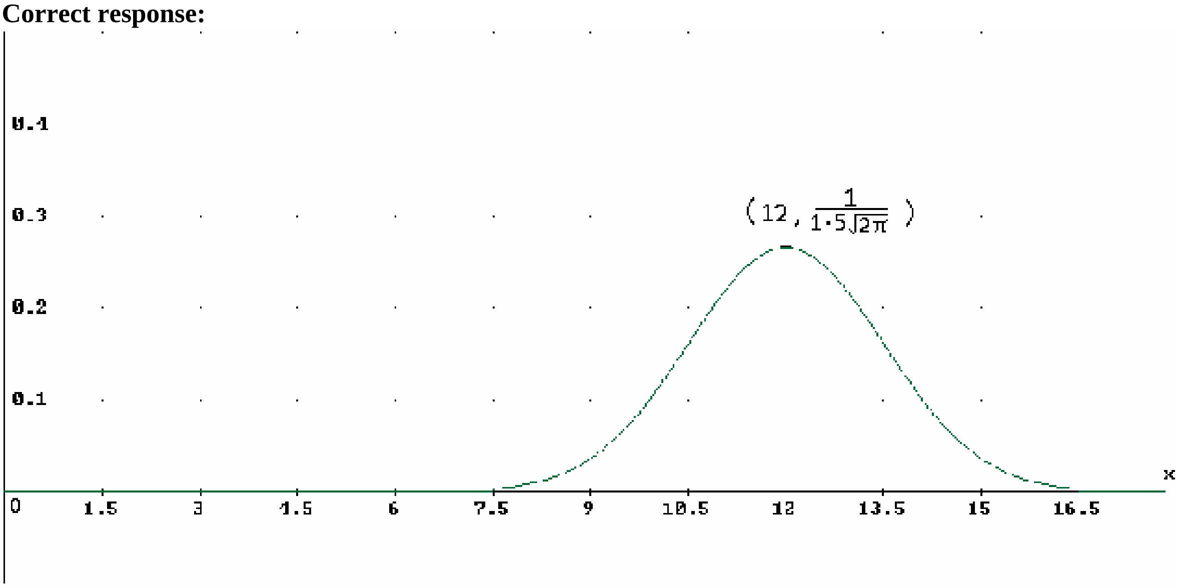
Part 2 – Short Answer

Question 1

a.

Marks	0	1	2	Average
%	11	25	64	**1.5**

Correct response:



Most students were able to sketch an appropriate graph, clearly indicating the mean and up to three standard deviations either side. Students should be reminded that, as this is the graph of a probability density function, some vertical scale should be incorporated. In this case it was appropriate to give the coordinates of the maximum point on the graph,

$(12, \frac{1}{1.5\sqrt{2\pi}}) \approx (12, 0.266)$. Some students correctly indicated the mean, but failed to show the horizontal span of the function. A few students drew a parabolic shape rather than the normal curve.

b.

Marks	0	1	Average
%	39	61	**0.6**

Correct response: $k = 10.5$

This answer could be obtained by realising that 10.5 was one standard deviation below the mean, and so approximately 16% of scores would have been less than this. Alternatively, solve $\int_{-\infty}^{k} \frac{1}{1.5\sqrt{2\pi}} e^{-(x-12)^2/(2\times 1.5^2)} dx = 0.16$ for k, or, use the standard normal tables to find z, with $\Pr(Z < z) = 0.16$, and so $\Pr(Z < -z) = 0.84$, so $-z = 0.99$, $z = -0.99$, and so $k = 12 - 0.99\times 1.5 = 10.5$, correct to one decimal place.

Question 2

Marks	0	1	2	3	Average
%	55	29	10	6	**0.7**

Correct response: The line and curve intersect when $mx = x^2(x - k)$. This can be written $x^3 - kx^2 - mx = 0$ and it has roots 0, $\frac{k+\sqrt{k^2+4m}}{2}$, $\frac{k-\sqrt{k^2+4m}}{2}$. The last two roots will be the same and non-zero provided $k^2 + 4m = 0$ and $k \neq 0$. Graphically, this corresponds to the case when $y = mx$ is tangent to the curve at the point where $x = k/2$. Alternatively, one of the last two roots could be equal to zero, but not the other. For this to happen, $m = 0$ but $k \neq 0$. Graphically, this corresponds to the case where $y = mx$ is tangent to the curve at the origin and again intersects the curve at the point (k, 0). Hence the line and curve intersect at (0, 0) and at one other point when either ($k^2 + 4m = 0$ and $k \neq 0$), or ($m = 0$ and $k \neq 0$).

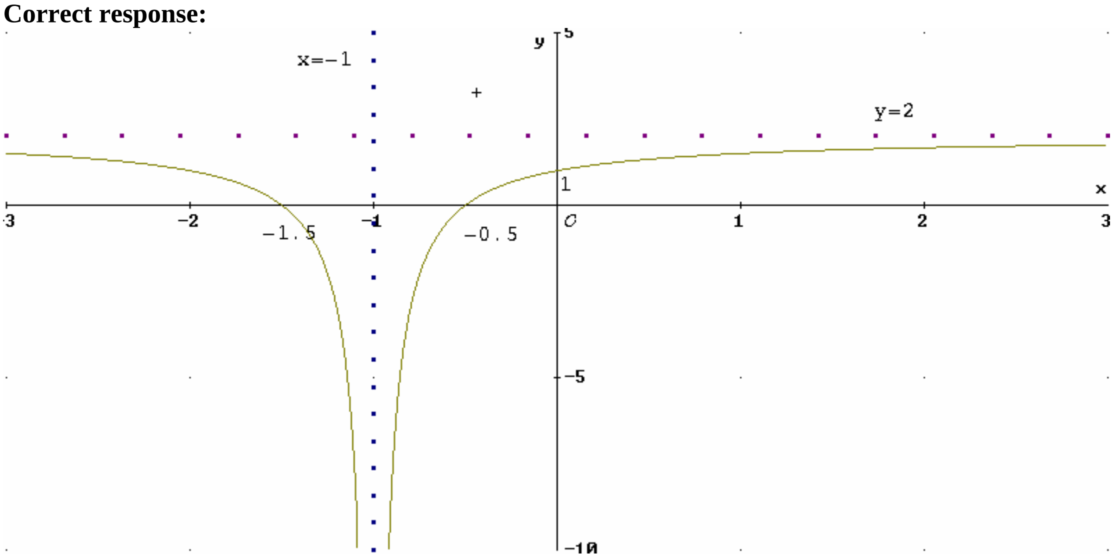
Note that if $k = 0$, then $y = mx$ intersects $y = x^3$ when $x^3 - mx = 0$. Now $x^3 - mx = x(x^2 - m)$, and this has three real distinct solutions if $m > 0$, but only one real solution if $m \leq 0$. Thus $y = mx$ intersects $y = x^3$ either once or three times only.

This question was not well answered. A method mark was awarded if students were able to give the equation to be solved, and another for giving the solutions of the cubic equation.

Question 3

Marks	0	1	2	Average
%	12	27	61	1.5

Correct response:



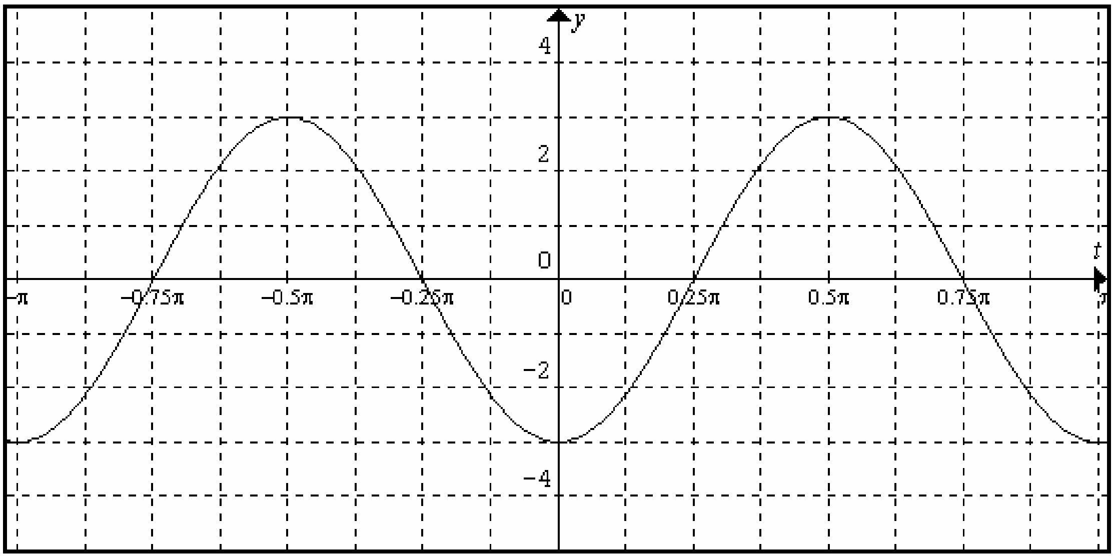
This was reasonably well answered. Some students needlessly lost marks by not labelling the asymptotes with their equations. Some students carelessly wrote $x = 1$ rather than $x = -1$ for the vertical asymptote, and a few labelled the vertical asymptote as $y = -1$ and the horizontal asymptote as $x = 2$. Some students missed the horizontal asymptote, and some had the left branch of the function in the wrong place. It is important to highlight to students the importance of indicating asymptotic behaviour correctly.

Question 4

a.

Marks	0	1	2	Average
%	5	9	85	1.8

Correct response:



Generally well answered. The grid seemed to help students to produce a good sketch graph. A few students were careless when it came to axis intercepts and the endpoints of the graph. Students should also take care to get the smooth curved shape of the graph rather than joining points such as local maxima and minima by straight line segments.

b.

Marks	**0**	**1**	**2**	**3**	**Average**
%	34	34	25	8	**1.1**

Correct response: $t = \dfrac{(3n+1)\pi}{3}$ *or* $\dfrac{(3n+2)\pi}{3}$, $n \in Z$ or $t = \dfrac{(3n-1)\pi}{3}$ *or* $\dfrac{(3n+1)\pi}{3}$, $n \in Z$

This was not as well answered as it could have been. Some students just gave solutions over the interval $[-\pi, \pi]$ rather than the general solution. Some students failed to specify possible values of the parameter in the general form or incorrectly said *R* or the non-negative integers rather than all the integers. Some students wrote *J* instead of *Z*. The standard mathematical notation for the set of integers is *Z*. Some students included CAS output directly in their solution, particularly $\dfrac{(3@n1+1)\pi}{3}$. Students are expected to use correct mathematical notation, not CAS specific terminology.

Some students also wrote things such as $x = \dfrac{\pi(3x+1)}{3}$. This indicates poor interpretation and understanding of the results obtained from a CAS.

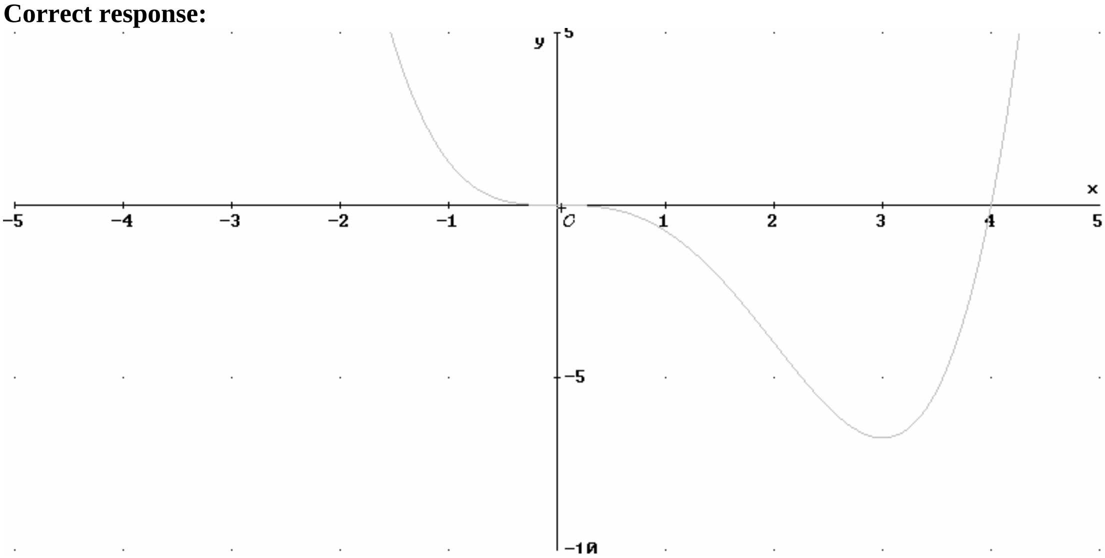
Indeed, teachers should possibly consider teaching the general solution to equations involving circular functions before selection of solutions from a particular domain.

Question 5

a.

Marks	**0**	**1**	**2**	**Average**
%	28	16	56	**1.3**

Correct response:



This question was not as well answered as it could have been. Some students gave upside down graphs, indicating they were consistently misinterpreting inequalities. Some students did not represent the horizontal point of inflection at the origin appropriately, while others failed to have the graph pass through the point with coordinates (4, 0), and some failed to have a turning point at $x = 3$.

b.

Marks	0	1	2	Average
%	44	9	48	**1.1**

Correct response: $b = 4$ since the curve passes through (4, 0).
Since (2, –4) lies on the curve, solve $f(2) = -4$ for a, giving $a = 0.25$.

Many students were unable to do this because they did not take into account the fact that (4, 0) lies on the curve. Students should be encouraged to check answers to questions such as these by drawing the graph of the resulting function and checking that it has the desired properties.

c.

Marks	0	1	2	3	Average
%	41	3	13	43	**1.6**

Correct response: Since $f'(x) = x^2(x-3)$ and $f'(4) = 16$
an equation of the tangent at the point (4, 0) is $y = 16(x - 4)$ or $y = 16x - 64$.

Some students used an automatic tangent finding on their CAS to simply write an equation down, and were unable to obtain full marks as no working was shown. Some students obtained the tangent in the first form given, and then proceeded to express it in the second form, but failed to multiply 16 by 4 correctly, and consequently lost a mark. Some students wrote things like tangent = $16x - 64$ or simply $16x - 64$. A method mark was awarded for calculating the derivative of f with respect to x, and another was awarded for evaluating $f'(4)$.

Question 6

a.

Marks	0	1	Average
%	64	36	**0.4**

Correct response: $4p(1-p)^3$ or ${}^4C_1 p(1-p)^3$

Not as well answered as might have been expected for a straightforward probability formulation. Several students did not seem to understand what a proportion was, and used $\frac{p}{100}$ instead of p in the response. Some students also wrote things like $p = \frac{p}{100}$; students should, as a matter of course, consider the reasonableness of what they write.

b.

Marks	0	1	2	Average
%	68	3	29	**0.6**

Correct response: Solve $\frac{d}{dp}(4p(1-p)^3) = 0$ for p. There are two solutions, at $p = 0.25$ and $p = 1$. A quick check of the graph of the function shows that $p = 0.25$ is the required value.

This was not well answered. Some students put their probability expressions equal to one (for example) and tried to solve for p. A method mark was awarded for indicating that that the derivative of the probability expression was to be found, set to zero, and solved for p. Some students simply wrote down an answer (correct or otherwise), and failed to obtain full marks as the question required working to be shown.