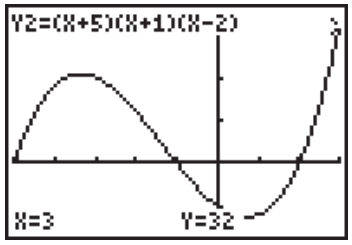
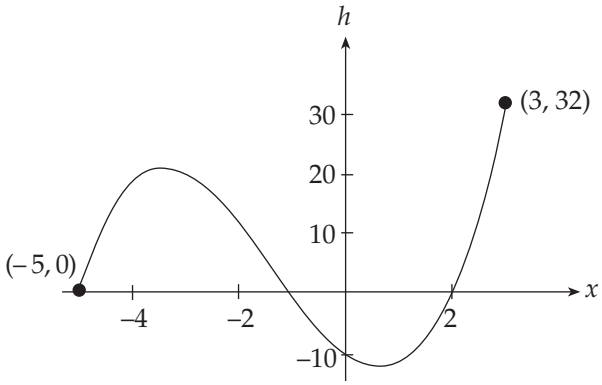
2004 Mathematical Methods
Written Examination 2 (Analysis task)
Suggested answers and solutions

Question 1

a The end points are (–5, 0) and (3, 32). **[2A]**





b $f(\frac{1}{2}) = -\frac{99}{8}$ and $f(-5) = 0$

$$m = \frac{-\frac{99}{8} - 0}{\frac{1}{2} + 5} = -\frac{9}{4}$$ **[1M]**

$$y - 0 = -\frac{9}{4}(x + 5)$$

$$4y = -9x - 45$$ **[1A]**

c $h_1(x) = (x + 5)(x + 1)(x - 2)$

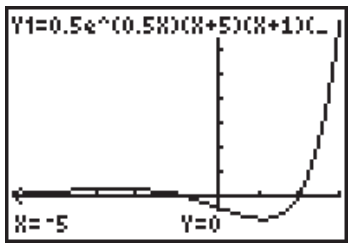
$$= x^3 + 4x^2 - 7x - 10$$

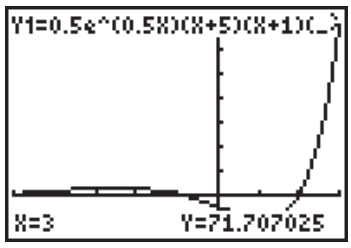
$$h_1'(x) = 3x^2 + 8x - 7$$ **[1M]**

$$h_1'\left(\frac{1}{2}\right) = \frac{3}{4} + 4 - 7 = -\frac{9}{4}$$ **[1M]**

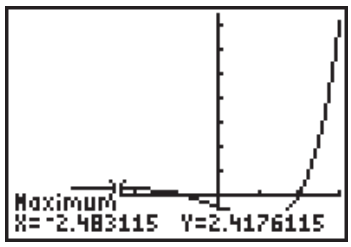
Since the gradient of the tangent to $h_1(x)$ at $x = \frac{1}{2}$ is $-\frac{9}{4}$ then $4y = -9x - 45$ is the equation of the tangent. **[1A]**

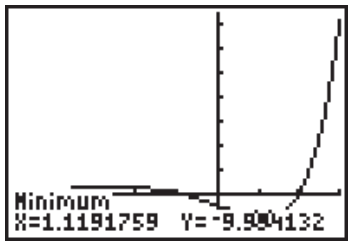
d The end points are (–5, 0) and (3, 71.71). **[1A]**



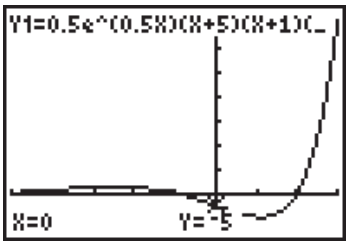


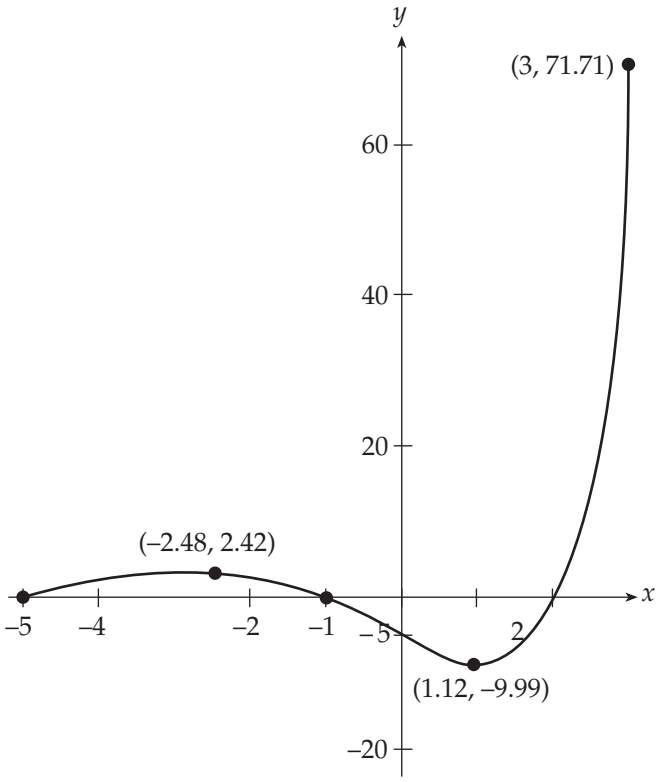
The turning points are (–2.48, 2.42) and (1.12, –9.99). **[1A]**





The x-intercepts are –5, –1 and 2.
The y-intercept is –5. **[1A]**

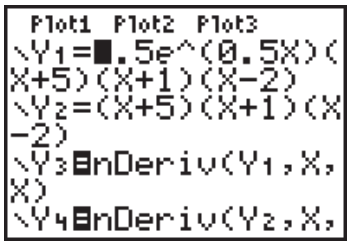


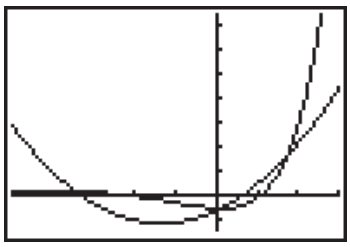


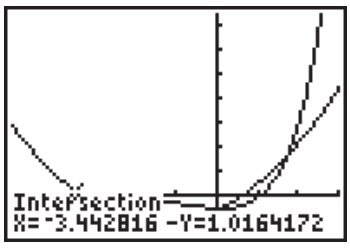
Correct shape. **[1A]**

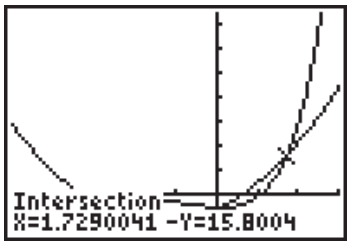
e $h_2(3) - h_1(3) = 71.707 - 32 = 39.71$ **[1A]**

f $\{x: -3.44 < x < 0.11) \cup \{x: 1.73 \le x \le 3\}$ **[2A]**









g $h_1'(x) = \frac{1}{4}e^{\frac{1}{2}x}\left(x^3 + 4x^2 - 7x - 10\right)$

$$+ \frac{1}{2}e^{\frac{1}{2}x}\left(3x^2 + 8x - 7\right)$$

via the product rule **[1M]**

$$= \frac{1}{4}e^{\frac{1}{2}x}\left[\left(x^3 + 4x^2 - 7x - 10\right) + 2\left(3x^2 + 8x - 7\right)\right]$$

$$= \frac{1}{4}e^{\frac{1}{2}x}\left(x^3 + 10x^2 + 9x - 24\right)$$

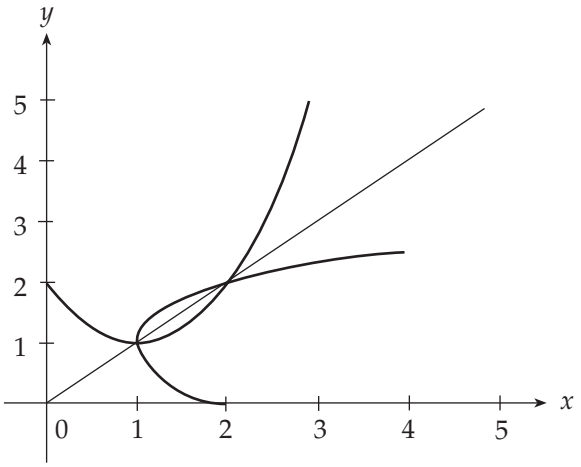
$a = 1$, $b = 10$, $c = 9$ and $d = -24$ **[2A]**

Question 2

a $x = -\frac{b}{2a}$ **[1A]**

b From the graph x is positive.
a is positive as there is a minimum. **[1A]**
Hence b has to be negative.

c



Correct endpoints **[1A]**

Correct shape **[1A]**

d $x = ay^2 + by + 2$ **[1M]**

$0 = ay^2 + by + 2 - x$

$y = \frac{-b \pm \sqrt{b^2 - 4a(2-x)}}{2a}$ **[1M]**

$= \frac{-b \pm \sqrt{b^2 + 4a(x-2)}}{2a}$

$= \frac{-b \pm \sqrt{4a(x-2) + b^2}}{2a}$ as required **[1A]**

e $x = ax^2 + bx + 2$ where $a > 0$ and $b < 0$

$x = -\frac{b}{2a}$ at the turning point.

$-\frac{b}{2a} = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + 2$ **[1M]**

$-2b = b^2 - 2b^2 + 8a$ **[1M]**

$-8a = -b^2 + 2b$

$a = \frac{b^2 - 2b}{8}$ as required

f $1 = \frac{b^2 - 2b}{8}$

$b^2 - 2b - 8 = 0$

$(b-4)(b+2) = 0$

Hence $b = -2$ **[1A]**

g $f(x) = x^2 - 2x + 2$

$x = x^2 - 2x + 2$ at the points of intersection.

$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x = 1$ or 2 **[1M]**

Area $= 2\int_1^2 \left(x - \left(x^2 - 2x + 2\right)\right)dx$ **[1M]**

$= 2\int_1^2 \left(-x^2 + 3x - 2\right)dx$

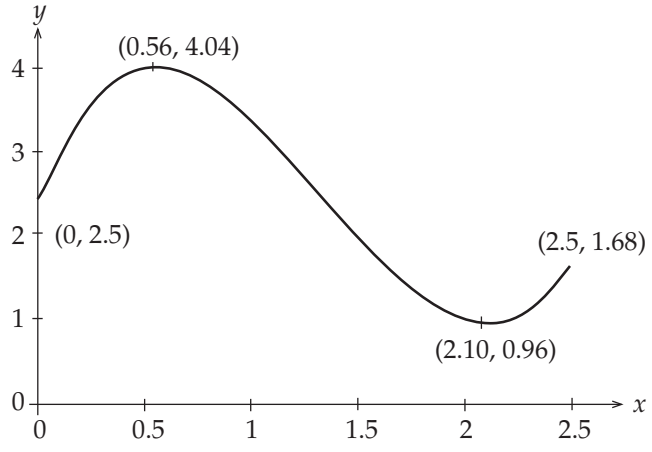
$= 2\left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right]_1^2$ **[1M]**

$= 2\left(\left(-\frac{8}{3} + 6 - 4\right) - \left(-\frac{1}{3} + \frac{3}{2} - 2\right)\right)$

$= \frac{1}{3}$ square unit **[1A]**

Question 3

a



Four correct coordinates **[4A]**

b Amplitude = 0.5 (4 – 1)

$A = 1.5$ **[1A]**

Period = 2(2.25 – 0.75)

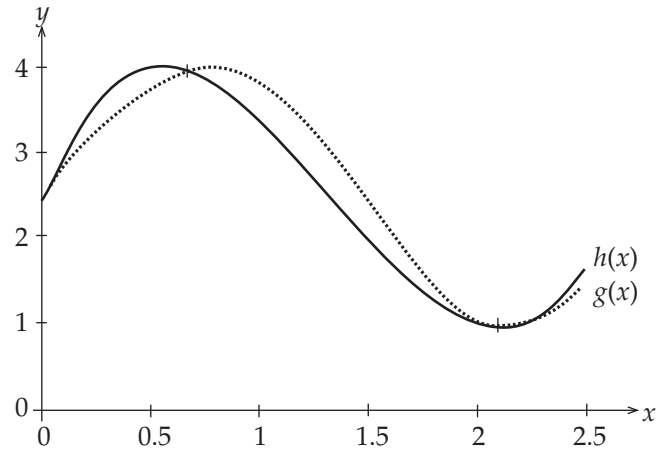
= 3

Period = $\frac{2\pi}{n\pi}$; $n = \frac{2}{3}$ **[1A]**

$B = 2.5$

$h(x) = 1.5\sin\left(\frac{2}{3}\pi x\right) + 2.5$ **[1A]**

c



Correct shape **[1A]**

Labelling $g(x)$ and $h(x)$ **[1A]**

Endpoints at $x = 2.5$ **[1A]**

d $g'(x) = \pi\cos\left(\frac{2\pi}{3}x\right)$ **[1A]**

Maximum occurs where $\cos\left(\frac{2\pi}{3}x\right) = 1$

$\frac{2\pi}{3}x = 0, 2\pi$ **[1A]**

$x = 0$ or 3 (outside domain)

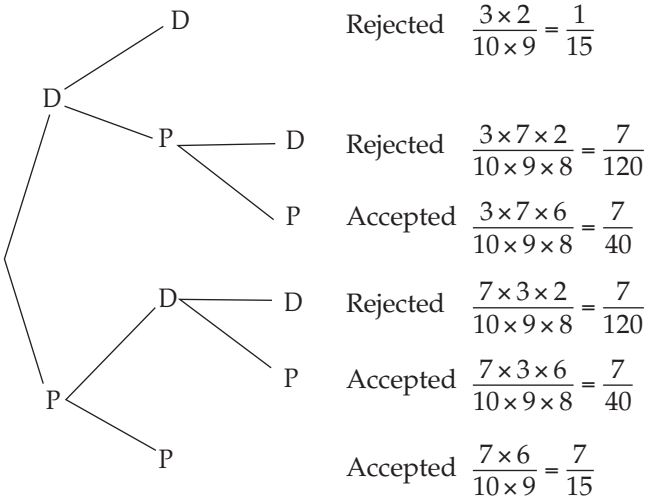
therefore rate of change is greatest at $x = 0$. **[1A]**

e $\int_0^{2.5} h(x)\cdot dx = 6.323 \text{ km}^2$ **[1A]**

$\int_0^{2.5} g(x)dx = 6.608 \text{ km}^2$ **[1A]**

Question 4

a D represents a damaged item and P a perfect team



Partially correct branches **[1M]**

All branches correct **[1A]**

Correct outcomes **[1A]**

b Pr(accepted) = $\frac{7}{40} + \frac{7}{40} + \frac{7}{15} = \frac{49}{60}$ **[1M]**

[1A]

c Pr(only 2 items checked | accepted)

= $\frac{7}{15} \div \frac{49}{60}$ **[1M]**

= $\frac{4}{7}$ **[1A]**

d Binomial $n = 8$, $p = \frac{49}{60}$

Pr (6 boxes accepted)

$= {}^8C_6 \times \left(\frac{49}{60}\right)^6 \times \left(\frac{11}{60}\right)^2$ **[1M]**

$= 0.2792$ **[1A]**

e $\mu = np = 500 \times 0.03$

$= 15$ **[1A]**

$\sigma = \sqrt{np(1-p)} = \sqrt{500 \times 0.03 \times 0.97}$

$= 3.8144$ **[1A]**

normalcdf (6, 16, 15, 3.8144)

$= 0.59$ **[1H]**

or normalcdf (5.5, 16.5, 15, 3.8144)

$= 0.65$