Structure of Booklet

Number of questions	Number of questions to be answered	Marks
27	27	27

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphic calculator (memory may be retained).

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question booklet of 14 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the spaces provided on the cover of the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II).

You may retain this question booklet.

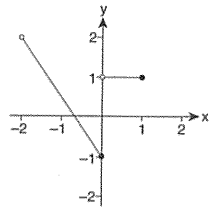
Instructions for Part I

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1



The range of the function shown is

A. $(-2, 1]$

B. $(-2, 0] \cup (0, 1]$

C. $[-1, 1) \cup (1, 2)$

D. $[-1, 2)$

E. $[-1, 1) \cup (1, 2]$

Question 2

In the expansion of $\left(x - \frac{2}{x}\right)^6$, the term independent of x is

A. ${}^6C_3$

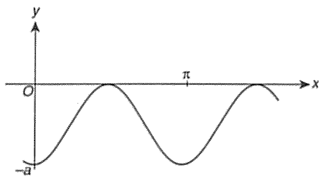
B. $2 \times {}^6C_3$

C. $-2 \times {}^6C_3$

D. $8 \times {}^6C_3$

E. $-8 \times {}^6C_3$

Question 3



The most probable equation to the graph shown is

A. $y = a\sin 2x - a$

B. $y = -\frac{a}{2}\cos 2x - \frac{a}{2}$

C. $y = \frac{a}{2}\cos 2x - \frac{a}{2}$

D. $y = -\frac{a}{2}\sin 2x - \frac{a}{2}$

E. $y = -a\cos 2x - a$

Question 4

If $a^{x-1} \times a^{x+1} = b$, then x is equal to

A. $\log_a\sqrt{b}$

B. $\frac{1}{2}$

C. $\log_a\left(\frac{b}{2}\right)$

D. $\log_b a$

E. $\frac{1}{2}\log_b a$

Question 5

A simplification of the expression $\log_a(a+1) - \log_a(a+1)^2 + \log_a(a^2-1)$ is

A. 1

B. 0

C. $\log_a(a-1)$

D. $\log_a(a+1)$

E. 2

Question 6

The most accurate solution to the equation $2.5^{x+1} = -2x + 1$ is

A. –0.3596

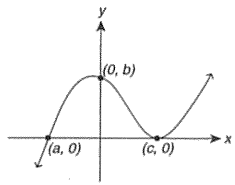
B. –0.3814

C. 1.7193

D. 1.7627

E. 1.7628

Question 7



The equation to this polynomial is

A. $y = \frac{ac^2}{b}(x-c)(x-a)^2$

B. $y = \frac{-b}{ac^2}(x-c)(x-a)^2$

C. $y = \frac{b}{ac^2}(x-a)(x-c)^2$

D. $y = -\frac{b}{ac^2}(x-a)(x-c)^2$

E. $y = -\frac{b}{ac^2}(x-a)(x-c)^2$

Question 8

If the graph of $y = \sin x$ is dilated by a factor of 3 in the x direction and by a factor of 2 in the y direction, then translated $\frac{\pi}{4}$ units in the positive x direction, its equation would become

A. $y = 3\sin\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

B. $y = \frac{1}{2}\sin\frac{1}{3}\left(x + \frac{\pi}{4}\right)$

C. $y = 2\sin\frac{1}{3}\left(x + \frac{\pi}{4}\right)$

D. $y = 2\sin 3\left(x - \frac{\pi}{4}\right)$

E. $y = 2\sin\frac{1}{3}\left(x - \frac{\pi}{4}\right)$

Question 9

The largest possible domain for the function whose rule is given by $f(x) = \frac{-2}{\sqrt{4-x}}$ is

A. $R \setminus \{4\}$

B. $(-\infty, 4)$

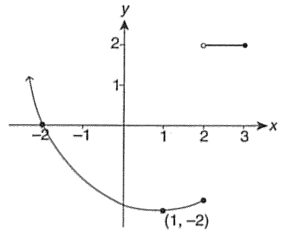
C. $(-\infty, 4]$

D. $(4, \infty)$

E. $[4, \infty)$

Question 10

A graph of $y = f(x)$ is shown.



The largest domain of $y = f(x)$ which allows the inverse function to exist is

A. $(-\infty, 2]$

B. $[-2, \infty)$

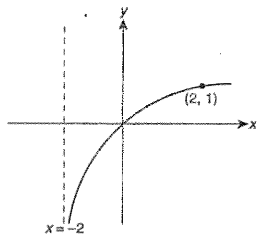
C. $(-\infty, 1]$

D. $(-\infty, 3]$

E. $(-\infty, 1)$

Question 11

The graph of $y = a\log_2(x - b) + c$ is shown.



The values of a, b and c respectively are

A. 1, 2, −1

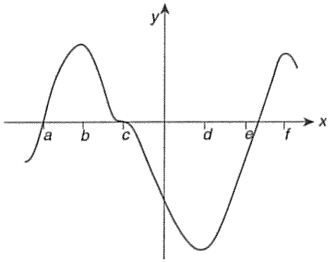
B. 1, 2, 1

C. 2, −2, 1

D. 2, −2, 2

E. 1, −2, −1

The following information relates to Question 12 and Question 13.



Question 12

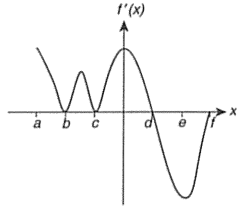
The graph of $f(x)$ has stationary points at $x =$

A. a, c and e only
B. c only
C. b and d only
D. b, c, d and f only
E. b, c and d only

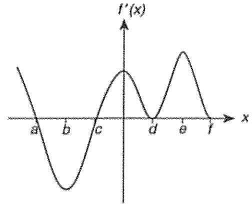
Question 13

The graph of the gradient function $f'(x)$ is

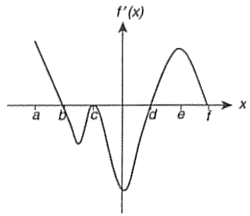
A.



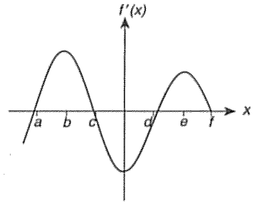
B.



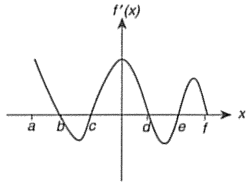
C.



D.

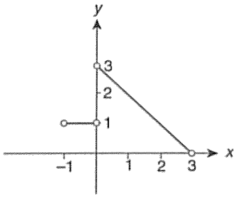


E.



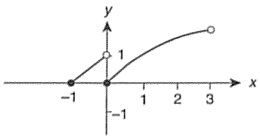
Question 14

The graph of the function $y = g'(x)$ is shown below.

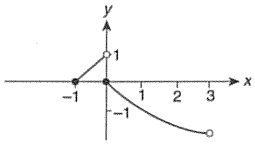


Which one of the following graphs could represent the function g, over the domain $[-1, 3)$?

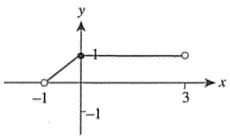
A.



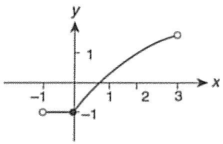
B.



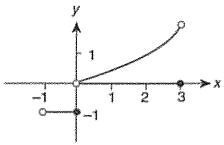
C.



D.



E.



Question 15

If $y = -x^3e^{3x}$, then $\frac{dy}{dx}$ is

A. $-3x^2e^{3x}$

B. $-3x^2e^{3x}(x+1)$

C. $-3x^2e^{3x}(x-1)$

D. $-3x^2e^{3x}(1-x)$

E. $\frac{-3x^2e^{3x}(x-1)}{(x^3e^{3x})^2}$

Question 16

The derivative of $-\sin(\pi x^2)$ is equal to

A. $-\cos(\pi x^2)$

B. $2\pi x\cos(\pi x^2)$

C. $2\pi\cos(\pi x^2)$

D. $-2\pi x\cos(\pi x^2)$

E. $-2\pi\cos(\pi x^2)$

Question 17

The equation of the normal to the curve $y = x(x+2)(x-1)$ at the point where $x = -1$ is

A. $y + x - 1 = 0$

B. $y - x - 3 = 0$

C. $y + x + 3 = 0$

D. $y - x + 1 = 0$

E. $y - x + 3 = 0$

Question 18

Using the right rectangle approximation with rectangles of width 1, the area of the region bounded by the x-axis, the y-axis, the line $x = -3$ and the curve whose equation is $y = e^{-x}$ is approximated by

A. $\frac{1+e+e^2}{e^3}$

B. $\frac{1+e^3}{e^3}$

C. $\frac{e^3-e}{e^3}$

D. $\frac{e^3-1}{e^2}$

E. $\frac{1+e+e^2}{e^2}$

Question 19

An antiderivative of $\frac{1}{2\sqrt{3-x}}$ is

A. $\frac{1}{2}\log_e(3-x)$

B. $3-x$

C. $\sqrt{3-x}$

D. $-\sqrt{3-x}$

E. $\frac{1}{2}\log_e(\sqrt{3-x})$

Question 20

If $\int_1^k \frac{1}{x}dx = \log_e 4$, and $k > 0$ then k equals

A. $\log_e 4 + 4$

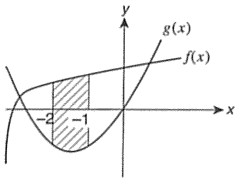
B. $\log_e 4$

C. 4

D. 5

E. $\log_e 5$

Question 21



The area bounded by the curves $f(x)$ and $g(x)$ and the lines $x = -2$ and -1, shown below is

A. $\int_{-1}^{-2} [g(x) - f(x)]dx$

B. $\int_{-2}^{-1} [f(x) + g(x)]dx$

C. $\int_{-1}^{-2} [f(x) - g(x)]dx$

D. $\int_{-2}^{-1} [g(x) - f(x)]dx$

E. $\int_{-1}^{-2} [g(x) + f(x)]dx$

Question 22

Jonathan loves to score a soccer goal. The probability of his kicking a goal is 0.85. If he has 6 kicks at goal, the probability that he will score fewer than 2 goals is

A. ${}^6C_2(0.85)^2(0.15)^4 + {}^6C_1(0.85)^1(0.15)^5 + (0.15)^6$

B. ${}^6C_1(0.85)^1(0.15)^5 + (0.15)^6$

C. $1 - {}^6C_2(0.85)^2(0.15)^4$

D. ${}^6C_2(0.85)^2(0.15)^4$

E. ${}^6C_2(0.85)^2(0.15)^4 + {}^6C_1(0.85)^1(0.15)^5$

Question 23

Jessica is decorating a Christmas tree. She has purchased a box of lights from the $3 shop. The box is clearly labelled:

BOX CONTAINS A MIXTURE OF LABELLED FLASHING AND NON-FLASHING LIGHTS

and

SOME LIGHTS MAY BE FAULTY

She takes a sample of 5 lights from the box and wishes to test them.

The following scenarios could occur:

X: The number of labelled flashing lights is noted.

Y: The number of faulty lights is noted.

Z: The number of faulty non-flashing lights is noted.

Which of the scenarios could be described by a hypergeometric probability distribution?

A. X, Y and Z

B. X and Y

C. X and Z

D. only Z

E. none of X, Y or Z.

Question 24

Rowan designs fibreglass model boats. The masses of these boats have a normal distribution with a mean of 35 kg. If 15% of the boats weigh more than 42 kg, then the variance of the their masses is closest to

A. 45.62

B. 35

C. 6.75

D. 5.92

E. 2.6

Question 25

A random variable has the following probability distribution shown the table below.

P	0	1	2	3
$\Pr(P = p)$	$4k$	$3k$	$2k$	k

If k is 0.1, the mean and variance of p are

A. 1 and 1

B. 1 and 1.4

C. 1.4 and 1

D. 1 and 2

E. 1.4 and 1.4

Question 26

Rowan was out on the bay early one morning fishing for Barramundi. He caught 12 fish, 3 of which were Barramundi.

He returned to land in order in order to inspect his prize catch and noticed that 4 of the bigger fish had been removed.

The variance of the number of Barramundi in the removed fish is

A. $\frac{6}{11}$

B. $\frac{3}{4}$

C. $\frac{5}{11}$

D. $\frac{1}{2}$

E. 1

Question 27

A local gymnasium holds weekly aerobics classes for beginners. Attendance records have shown that each class will contain boys on 25% of occasions. The probability that there will be boys in at least of one of the next three classes is

A. $\frac{{}^{25}C_1 \, {}^{75}C_2}{{}^{100}C_3}$

B. ${}^{3}C_1(0.25)^1(0.75)^2$

C. $1-(0.75)^3$

D. $\frac{{}^{25}C_2 \, {}^{75}C_1}{{}^{100}C_3}$

E. $1-{}^{3}C_1(0.25)^1(0.75)^2$

END OF QUESTION BOOKLET

Structure of Booklet

Number of questions	Number of questions to be answered	Marks
4	4	23

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 6 pages.

Instructions

Detach the formula sheet from the centre of the Part I booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the spaces provided on the cover of this booklet.

All written responses should be in English.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer booklet (Part II).

Instructions for Part II

Answer **all** questions in this part in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown.

In questions where more than 1 mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Consider the function $f:[-1, 2) \to R,\ f(x) = 2e^{x-1} - 3$.

a. Determine the equation of $f^{-1}(x)$.

2 marks

b. State the exact domain of $y = f^{-1}(x)$.

1 marks

Total 3 marks

Question 2

Let $f(x) = \log_e(\sin x)$, where $0 \le x \le 2\pi$.

a. What is the implied domain of $f(x)$?

1 mark

b. i Calculate $f'(x)$

1 mark

ii. Calculate the exact value of the gradient of $f(x)$ when $x = \frac{\pi}{3}$.

2 marks

c. i As $\cot x = \frac{\cos x}{\sin x}$, write an integral expression for the area (A) enclosed between the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$, the x-axis and the curve $y = \cot x + 1$.

1 mark

ii. If this area can be written as $\log_e a + b$, where $a, b \in R$, find the exact values of a and b by evaluating the integral you wrote in **part c.i.**

4 marks
Total 9 marks

VCE Mathematical Methods Units 3 & 4 Trial Examination 1 Question and Answer Booklet

Question 3

At a local festival, a game is played involving loaded dice. When one die is rolled, the probability of obtaining a particular number of dots (the score), X, on the uppermost face is shown in the table below.

X	1	2	3	4	5	6
$\Pr(X = x)$	p	$2p$	$3p$	$4p$	$5p$	q

a. If $p = \frac{1}{20}$, find the exact value of q.

1 mark

If two dice are rolled, find the exact probability that

b. both scores were sixes.

1 mark

c. the sum of the score was 9.

2 marks

d. if the sum of the scores was 9 then one of the scores was a 6

2 marks
Total 6 marks

Question 4

Terry is a keen golfer. The distance he can hit a golf ball is normally distributed with a mean of 150 m and standard deviation of 20 metres.

a. What probability has Terry got of hitting the ball between 120 and 180 metres? Express your answer as a percentage.

2 marks

b. What probability has Terry got of hitting the ball greater than 180 metres given that he hits the ball greater than 150 m? Express your answer as a percentage.

2 marks

c. Interpret the results of **part a.** and **part b.**

1 mark

Total 5 marks

END OF QUESTION AND ANSWER BOOKLET

Neap.
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Trial Examination 2003

VCE Mathematical Methods
Units 3 & 4

Examination 1: Facts, Skills and Applications Task

Suggested Solutions

1	A	B	C	**D**	E
2	A	B	C	D	**E**
3	A	**B**	C	D	E
4	**A**	B	C	D	E
5	A	B	**C**	D	E
6	A	**B**	C	D	E
7	A	B	C	**D**	E
8	A	B	C	D	**E**
9	A	**B**	C	D	E
10	A	B	**C**	D	E
11	A	B	C	D	**E**
12	A	B	C	**D**	E
13	A	B	**C**	D	E
14	**A**	B	C	D	E

15	A	**B**	C	D	E
16	A	B	C	**D**	E
17	A	**B**	C	D	E
18	A	B	C	D	**E**
19	A	B	C	**D**	E
20	A	B	**C**	D	E
21	**A**	B	C	D	E
22	A	**B**	C	D	E
23	A	B	**C**	D	E
24	**A**	B	C	D	E
25	**A**	B	C	D	E
26	**A**	B	C	D	E
27	A	B	**C**	D	E

PART I

Question 1

The range indicates all the y values covered by the graph. Note that this includes –1 but does not include 2.

Answer D

Question 2

The term independent of x must contain $x^3\left(\frac{-2}{x}\right)^3$.

Its binomial coefficient is ${}^6C_3$, so its value is $(-2)^3 \times {}^6C_3$.

Answer E

Question 3

This is the graph of a negative cos function translated down by its amplitude.

Its amplitude is $\frac{a}{2}$ units and its period is π, (so $n = \frac{2\pi}{\pi} = 2$).

Its equation must therefore be $y = -\frac{a}{2}\cos 2x - \frac{a}{2}$.

Answer B

Question 4

Using the first index law gives $a^{2x} = b$. Taking $\log_a$ of both sides gives $\log_a a^{2x} = \log_a b$.

Thus $2x\log_a a = \log_a b$ as $\log_a a = 1$.

$x = \frac{1}{2}\log_a b = \log_a b^{\frac{1}{2}} = \log_a \sqrt{b}$.

Answer A

Question 5

Using the law of logs gives

$$\log_a\left\{\frac{(a+1)(a^2-1)}{(a+1)^2}\right\} = \log_a\left\{\frac{\cancel{(a+1)}\cancel{(a+1)}(a-1)}{\cancel{(a+1)}\cancel{(a+1)}}\right\}$$

$$= \log_a(a-1)$$

Answer C

Question 6

Using a graphic calculator with $Y1 = 2.5^{X+1}$ and $Y2 = {}^-2X + 1$, intersection gives $x = -0.3814$. Alternatively, use the equation solver.

Answer B

Question 7

The linear factor here is $x-a$ (the intersection of graph through $x=a$).

The repeated factor is $(x-c)^2$ (graph is a tangent to x-axis at $x=c$). This gives the equation $y=A\ (x-a)(x-c)^2$. To calculate the constant A we substitute the y-intercept, i.e. $x=0,\ y=b$ which gives

$b=A(-a)(-c)^2$

$\therefore A=\dfrac{b}{-ac^2}$

i.e. $y=-\dfrac{b}{ac^2}(x-a)(x-c)^2$

Note: This will generate a positive cubic function as the value of a is negative.

Answer D

Question 8

A dilation by a factor of 3 in the x direction creates $y=\sin\frac{1}{3}x$.

The dilation by a factor of 2 in the y direction creates $y=2\sin\frac{1}{3}x$.

The translation in the positive x direction of $\frac{\pi}{4}$, then produces $y=2\sin\frac{1}{3}\left(x-\frac{\pi}{4}\right)$.

Answer E

Question 9

As $4-x>0$, $4>x$. That gives a domain of $(-\infty, 4)$.

Answer B

Question 10

For an inverse function to exist, the original must be one to one. This means the largest domain for which $y=f(x)$ remains one to one is $(-\infty, 1]$.

Answer C

Question 11

The basic curve for $y=\log_2 x$ has been translated 2 units to the left. This suggests $b=-2$.

The equation is therefore $y=a\log_2(x+2)+c$.

Substituting (0, 0) gives $0=a\log_2 2+c$

i.e. $0=a+c \quad \ldots(1)$

Substituting (2, 1) gives $1=a\log_2 2+c$

i.e. $1=2a+c \quad \ldots(2)$

Subtracting (1) from (2) gives $a=1$

$\therefore c=-1$

Answer E

Question 12

Stationary points include turning points and points of inflexion, so therefore b, c, d and f only.

Answer D

Question 13

Stationary points of $f(x)$ will be x-intercepts of $f'(x)$.

Point of inflexion **negative** translates to an inverted 'toucher' on $f'(x)$.

Answer C

Question 14

The graph of $g'(x)$ should be the derivative graph of A to E. By taking each option in turn, the incorrect ones will be eliminated.

For graph A, for $-1 < x < 0$, the gradient is +1 and for $0 < x < 3$, the gradient is positive but decreasing. Also, as $g'(x)$ is linear over (0, 3), we expect $g(x)$ to be quadratic.

Answer A

Question 15

Using the product rule,

$$\frac{dy}{dx} = -x^3(3e^{3x}) + e^{3x}(-3x^2)$$

$$= -3x^2e^{3x}(x+1)$$

Answer B

Question 16

Using the chain rule, let $y = -\sin(\pi x^2)$

$$\frac{dy}{dx} = -\cos(\pi x^2)(2\pi x)$$

$$= -2\pi x\cos(\pi x^2)$$

Answer D

Question 17

$$y = x(x+2)(x-1)$$
$$= x^3 + x^2 - 2x$$
$$\therefore \frac{dy}{dx} = 3x^2 + 2x - 2$$

When $x = -1$, $y = 2$ and $\frac{dy}{dx} = -1$.

$\frac{dy}{dx}$ is the gradient of the tangent. Therefore the gradient of the normal is 1

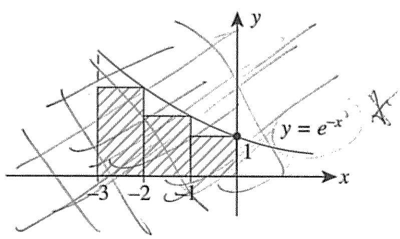
$\therefore$ The equation for the normal is found from

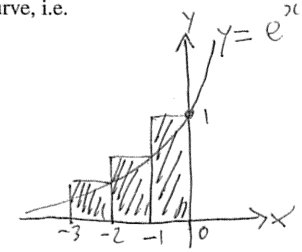
$$y - 2 = 1(x - (-1))$$
$$\therefore y = x + 3$$
$$\therefore y - x - 3 = 0$$

Answer B

Question 18

The right rectangle approximation corresponds to the lower rectangles for this curve, i.e.





By applying the restrictions and when $x = 0, y = 1$

$$x = -1, y = e^{-1} = \frac{1}{e}$$

$$x = -2, y = e^{-2} = \frac{1}{e^2}$$

it follows that the area of the region is

$$\left(1 \times \frac{1}{e^2}\right) + \left(1 \times \frac{1}{e}\right) + (1 \times 1) = \frac{1}{e^2} + \frac{1}{e} + 1$$
$$= \frac{1 + e + e^2}{e^2}$$

Answer E

Question 19

$$\int \frac{1}{2\sqrt{3-x}} = \frac{1}{2}\int (3-x)^{-\frac{1}{2}}dx$$

$$= \frac{\frac{1}{2}(3-x)^{\frac{1}{2}}}{\frac{1}{2}(-1)}$$

$$= -\sqrt{3-x}$$

Answer D

Question 20

If $\int_1^k \frac{1}{x}dx = \log_e 4$, $[\log_e x]_1^k = \log_e 4$ and $\log_e k - \log_e 1 = \log_e 4$

So, $\log_e k = \log_e 4$

$\therefore k = 4$

Answer C

Question 21

The area bounded by the two curves is equal to the integral, between –2 and –1 of the top curve (*f*) minus the bottom curve (*g*), i.e.

$$\int_{-2}^{-1} [f(x) - g(x)]dx = \int_{-1}^{-2} [g(x) - f(x)]dx$$

Answer A

Question 22

If Jonathan scores fewer than 2 goals, he scores nil or 1. If X is 'the number of goals scored', this binomial situation is represented by $\Pr(X = 1) + \Pr(X = 0)$.

That is, ${}^6C_1(0.85)^1(0.15)^5 + {}^6C_0(0.85)^0(0.15)^6$

Answer B

Question 23

The hypergeometric distribution has three distinct characteristics:

1. Each event has two possible outcomes defined as a 'success' or 'failure'.
2. The probability changes because sampling occurs **without** replacement.
3. The events are dependent on each other.

Only scenarios X and Z satisfy these requirements.

Answer C

Question 24

$\mu = 35$ and $X \sim N(35, \sigma^2)$.

Let X = the mass of the boats.

$$\begin{aligned} \text{So, } \Pr(X > 42) &= 0.15 \\ \Pr(Z > z) &= 0.15 \\ \therefore 1 - \Pr(Z < z) &= 0.15 \\ \therefore \Pr(Z < z) &= 0.85 \\ \therefore z &= 1.0364 \end{aligned}$$

As $z = \frac{x - \mu}{\sigma}$, then $\frac{42 - 35}{\sigma} = 1.0364$.

$$\begin{aligned} \text{Therefore, } \sigma &= \frac{7}{1.0364} \\ &= 6.7539 \\ \therefore \sigma^2 &= 45.62 \end{aligned}$$

Answer A

Question 25

$$\begin{aligned} \text{Mean is } \mathrm{E(P)} &= \Sigma p \Pr(P = p) \\ &= (0 \times 0.4) + (1 \times 0.3) + (2 \times 0.2) + (3 \times 0.1) \\ &= 0 + 0.3 + 0.4 + 0.3 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{Variance is } \mathrm{Var(P)} &= E(P^2) - [E(P)]^2 \\ &= (1^2 \times 0.3) + (2^2 \times 0.2) + (3^2 \times 0.1) - (1)^2 \\ &= 0.3 + 0.8 + 0.9 - (1)^2 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

Answer A

Question 26

This is a hypergeometric distribution. Rowan caught 12 fish; 3 Barramundi and 9 other than Barramundi, i.e. $N = 12$, $n = 4$, $D = 3$

$$\begin{aligned} \sigma^2 &= \frac{nD}{N}\left(1 - \frac{D}{N}\right)\left(\frac{N-n}{N-1}\right) \\ &= \frac{4}{12}\left(1 - \frac{3}{12}\right)\left(\frac{12-4}{12-1}\right) \\ &= \frac{6}{11} \end{aligned}$$

Answer A

Question 27

Letting X be the number of times boys are present, and following a binomial distribution with $n = 3$ and $p = 0.25$,

$$\begin{aligned}\Pr(X \geq 1) &= 1 - \Pr(X = 0) \\ &= 1 - {}^3C_0(0.25)^0(0.75)^3 \\ &= 1 - 0.75^3\end{aligned}$$

Answer C

PART II

Question 1

a. For the inverse $x = 2e^{y-1} - 3$ [M]

$$\frac{x+3}{2} = e^{y-1}$$

Taking $\log_e$ of both sides gives $\log_e\left(\frac{x+3}{2}\right) = y - 1$

so $y = f^{-1}(x) = \log_e\left(\frac{x+3}{2}\right) + 1$ [A]

b. The domain of $f(x)$ is $[-1, 2)$.

When $x = -1$, $f(x) = 2e^{-2} - 3$. When $x = 2$, $f(x) = 2e - 3$.

The range of $f(x)$ is $\left[\frac{2}{e^2} - 3, 2e - 3\right)$.

As the domain and range are interchanged for the inverse the domain of

$f^{-1}(x)$ is $\left[\frac{2}{e^2} - 3, 2e - 3\right)$. [A]

Question 2

a. For $f(x)$ to exist, $\sin x > 0$ $\therefore 0 < x < \pi$.

so the implied domain of $f(x)$ is $(0, \pi)$. [A]

b. **i** Using the chain rule, $f'(x) = \frac{\cos x}{\sin x}$ (or $\cot x$) [A]

ii. The gradient at $x = \frac{\pi}{3}$ is $f'\left(\frac{\pi}{3}\right) = \frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}}$ [M]

$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}} \left(\text{or } \frac{\sqrt{3}}{3}\right)$$ [A]

c. **i** $A = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\cot x + 1)dx$ [A]

ii. $A = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\cot x + 1)dx$

$= [\log_e(\sin x) + x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ [M][A]

$= \left(\log_e\left(\sin\frac{\pi}{3}\right) + \frac{\pi}{3}\right) - \left(\log_e\left(\sin\frac{x}{6}\right) + \frac{\pi}{6}\right)$

$= \log_e\frac{\sqrt{3}}{2} + \frac{\pi}{3} - \log_e\frac{1}{2} - \frac{\pi}{6}$

$= \log_e\left(\frac{\sqrt{3}}{2} \div \frac{1}{2}\right) + \frac{\pi}{6}$

$= \log_e\sqrt{3} + \frac{\pi}{6}$

So $a = \sqrt{3}$ and $b = \frac{\pi}{6}$ [A][A]

Question 3

a. $q = 1 - \left(\frac{1}{20} + \frac{2}{20} + \frac{3}{20} + \frac{4}{20} + \frac{5}{20}\right) = \frac{5}{20} = \frac{1}{4}$ [A]

b. Pr(both sixes) $= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$ [A]

c. By using a tree diagram or similar

Pr(sum of scores is 9) = Pr(6, 3) + Pr(5, 4) + Pr(4, 5) + Pr(3, 6) [M]

$= \left(\frac{1}{4}\right)\left(\frac{3}{20}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{20}\right)\left(\frac{1}{4}\right) = \frac{7}{40}$ [A]

d. This is an example of conditional probability.

Pr(one die shows a 6 if given sum of scores is 9) $= \frac{\Pr(6, 3) + \Pr(3, 6)}{\Pr(\text{sum of score is 9})}$ [M]

$= \frac{\frac{3}{80} + \frac{3}{80}}{\frac{7}{40}}$

$= \frac{3}{7}$ [A]

Question 4

a. Let X be the distance Terry can hit the golf ball and $X \sim N(150, 400)$,
we want $\Pr(120 < X < 180)$.

This is

$$\Pr\left(\frac{120-150}{20} < Z < \frac{180-150}{20}\right) = 2\Pr(Z < 1.5) - 1 \qquad \text{[M}\}$$

$$= \Pr(-1.5 < Z < 1.5)$$
$$= 2\Pr(0 < Z < 1.5)$$
$$= 2(0.9332) - 1$$
$$= 0.8664 = 87\ \% \qquad \text{[A]}$$

Alternatively, using a graphic calculator

```
= normal cdf(120,180,150,20)
= 0.8664 (= 87%)
```

b. $$\Pr(X > 180 | X > 150) = \frac{\Pr(X > 180)}{\Pr(X > 150)} = \frac{1 - \Pr(X < 180)}{0.5} \qquad \text{[M]}$$

$$= \frac{1 - 0.9332}{0.5}$$

$$= 0.133 = 13\% \qquad \text{[A]}$$

c. Terry has an excellent chance of hitting the ball between 120 and 180 metres, however, given that he hits it over 150 metres, his chances of hitting the ball more than 180 metres are slim. [A]