2003 Mathematical Methods Written Examination 1 (facts, skills and applications) Suggested answers and solutions

Part 1 (Multiple-choice) Answers

1. **D**	2. **B**	3. **B**	4. **D**	5. **E**
6. **A**	7. **C**	8. **D**	9. **D**	10. **C**
11. **C**	12. **A**	13. **A**	14. **E**	15. **E**
16. **C**	17. **E**	18. **A**	19. **E**	20. **C**
21. **A**	22. **E**	23. **D**	24. **D**	25. **B**
26. **B**	27. **B**			

Part 1 (Multiple-choice) Solutions

Question 1 [D]

Because the graph touches the x-axis at $x = 2$ with a minimum value, there needs to be a factor of $(x-2)^2$.

If $x > 2$, the graph is positive. The only alternative satisfying both of these conditions is D.

Question 2 [B]

If $x = 2$ then $\log_2 2$ is 1. All other alternatives are true.

Question 3 [B]

$\frac{x^3+1}{x} = \frac{x^3}{x} + \frac{1}{x} = x^2 + \frac{1}{x}$ and so the two rules that could have been added are $g(x) = x^2$ and $h(x) = \frac{1}{x}$.

Question 4 [D]

The value of P in $y = e^{kx} + P$ represents the horizontal asymptote that the graph approaches as x tends to negative infinity (k is positive). Since it is positive, the horizontal asymptote must be above the x-axis.

The y-intercept is 1 unit (e^0) above

$y = P$

The graph also tends to ∞ as x increases.

Question 5 [E]

For reflection in the y-axis, change any x to $-x$.

Therefore $\sqrt{x}$ becomes $\sqrt{-x}$.

A dilation factor of 2 from the x-axis doubles each y-value. The new equation becomes $y = 2\sqrt{-x}$.

Question 6 [A]

Dividing both sides of $0.5\cos(2x)= 1$ by 0.5 gives $\cos(2x)= 2$. There are no real solutions to this equation, regardless of the domain.

Question 7 [C]

The graph repeats itself after π . This can be seen from the difference of the x-intercepts .

Question 8 [D]

The period of the function is 4.

$\therefore \frac{2\pi}{k\pi} = 4$ and so $k = 0.5$

The graph oscillates around $y = 1$ and so Q is 1. Substituting $(-1, 3)$ into the equation gives

$$P\sin\left(\frac{-\pi}{2}\right) + 1 = 3$$

so $-P + 1 = 3$ giving $P = -2$.

Question 9 [D]

If $y = 2\tan(2x)$ then $\frac{dy}{dx} = 4\sec^2(2x)$.

$$4\sec^2(2x) = 4 \times \frac{1}{\cos^2(2x)}$$

$$= \frac{4}{\cos^2(2x)}$$

Question 10 **[C]**

Using the Product Rule

$$\frac{dy}{dx} = u'(x)\log_e(x) + u(x)\cdot\frac{1}{x}$$

At $x = 2$ this becomes

$$\frac{dy}{dx} = u'(2)\log_e(2) + \frac{u(2)}{2}$$

Question 11 **[C]**

At $x = -1$ the graph has a stationary value because the gradient is zero. The gradient is positive on both sides of $x = -1$, that is , it does not change signs. It must be a stationary point of inflexion.

Question 12 **[A]**

Since $g'(x) = f(x)$ the curve of $f(x)$ represents the gradient of $g(x)$. On the interval (a, b) the curve of $f(x)$ is negative. Hence the gradient of $g(x)$ is negative in this interval.

Question 13 **[A]**

$$f(x) = \int 4e^{2x}dx$$

$$= \frac{4}{2}e^{2x} + c \text{ where } c \text{ is a constant.}$$

$$= 2e^{2x} + c$$

The first alternative satisfies this equation with $c = 3$.

Question 14 **[E]**

The first integral can be divided in the following way:

$$\int_1^4 (2f(x)+3)dx = \int_1^4 2f(x)dx + \int_1^4 3dx$$

Now $\int_1^4 2f(x)dx = 2 \times \int_1^4 f(x)dx = 4$

and $\int_1^4 3dx = [3x]_1^4$

$$= (12 - 3)$$

The answer is therefore $4 + 9 = 13$.

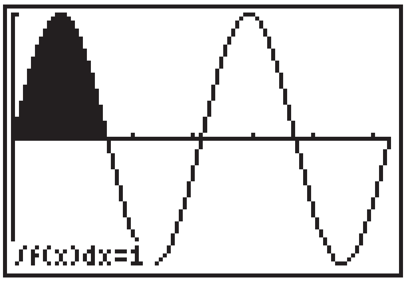
Question 15 **[E]**

$$\int_0^5 g(x)dx = \int_0^5 f'(x)dx$$

Now the integral of a derivative takes us back to the function itself and so the integral on the RHS of this equation is $[f(x)]_0^5$, that is $f(5) - f(0)$.

Question 16 **[C]**

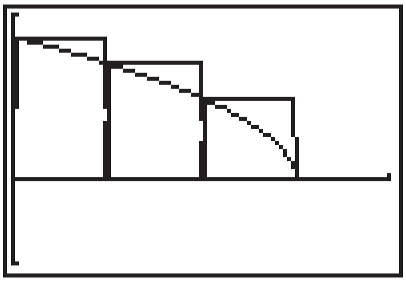
The graph of $y = \sin(2x)$ between 0 and 2π is shown below. The area between 0 and $\pi/2$ has been highlighted and the value is 1.



The total area between 0 and 2π is therefore 4.

Question 17 **[E]**

The left-rectangles are shown on the curve. From left to right, their heights are: $\sqrt{3}$, $\sqrt{2}$ and $\sqrt{1}$ respectively.



The widths are all 1 unit and so the area is $\sqrt{3} + \sqrt{2} + 1$

Question 18 **[A]**

$f(x) = 2(x + 3)^2 - 8$ is already written in turning-point format. The co-ordinates of the minimum value is $(-3, -8)$.

Question 19 [E]

The expansion of $(2x - 3)^5$ is:
$(2x)^5 - {}^5C_1\,(2x)^4(3)^1 + {}^5C_2\,(2x)^3(3)^2 - {}^5C_3\,(2x)^2(3)^3 + \ldots$.

The last term here is the x^2 term and the coefficient is ${}^5C_3\,(2)^2(3)^3 = -1080$.

Question 20 [C]

If a is a positive number then $x^2 + a$ will not reach the x-axis. Therefore, for $y = 0$ the values of x will be c and $-b$.

Question 21 [A]

$f(x)$ has a vertical and horizontal asymptote of $x = 3$ and $y = 1$ respectively.

Interchange the x and y variables to find the inverse function.

Therefore the inverse function has a vertical and horizontal asymptote of $x = 1$ and $y = 3$ respectively.

Question 22 [E]

$2\log_e(x) - \log_e(x+2) = 1 + \log_e(y)$

$2\log_e(x) - \log_e(x+2) - 1 = \log_e(y)$

Now $1 = \log_e e$ and $2\log_e(x) = \log_e x^2$ and so

$\log_e(y) = \log_e x^2 - \log_e(x+2) - \log_e e$

$\log_e(y) = \log_e \dfrac{x^2}{e(x+2)}$

$\therefore \quad y = \dfrac{x^2}{e(x+2)}$

Question 23 [D]

Normal distributions are symmetrical about the mean. Both of these are symmetrical about the same line and so $\mu_1 = \mu_2$.

X_2 is wider than X_1 and so $\sigma_1 < \sigma_2$.

Question 24 [D]

Probability distribution functions must have both of the following properties:

$\sum \Pr(X) = 1$ and $0 \le \Pr(X) \le 1$.

Alternative I has probabilities adding to 1.2 and Alternative IV has negative probabilities.

The rest fulfil the requirements.

Question 25 [B]

$\text{Var}(X) = E(X^2) - [E(X)]^2$ and the standard deviation is $\sqrt{(\text{Variance})}$.

$E(X) = 1 - p$ and $E(X^2) = 1^2(1 - p)$.

$$\begin{aligned}\therefore \text{Var}(X) &= 1 - p - (1 - p)^2 \\ &= 1 - p - (1 - 2p + p^2) \\ &= p - p^2 \\ &= p(1 - p)\end{aligned}$$

The standard deviation is $\sqrt{p(1-p)}$.

Question 26 [B]

Binomial distribution.

$n = 20$ and $p = 0.6$

$\Pr(x = 12) = {}^{20}C_{12}\,(0.6)^{12}(0.4)^8$

Question 27 [B]

Hypergeometric distribution.

$N = 12$, $D = 4$, $n = 2$

Pr(At least 1 multigrain) is the same as 1 – Pr (0 multigrain)

Pr (0 multigrain) means that both rolls chosen are white.

Pr(Both white) = $\dfrac{{}^8C_2}{{}^{12}C_2}$.

The answer is $1 - \dfrac{{}^8C_2}{{}^{12}C_2}$.

PART II

Question 1

If the polynomial is divisible by the factor $(x + 1)$ then $f(-1) = 0$.

$f(-1) = 2((-1)^4 - 3(-1)^3 + 7(-1) + 11$

$= 2 + 3 - 7 + 11$

$= 9$

It is not divisible by this factor $(x + 1)$.

Question 2

a The point on the curve is where the gradient is 3 since it is parallel to the given straight line.

$\frac{dy}{dx} = 2x - 2 = 3$

Therefore $x = 2.5$

$y = (2.5)^2 - 2(2.5) - 1 = 0.25$.

Point P is (2.5 , 0.25).

b The gradient of the normal $= -\frac{1}{3}$ as the product of the gradients of the normal and tangent is -1.

Equation: $y - 0.25 = -\frac{1}{3}(x - 2.5)$

Answer: $12y + 4x = 13$

Question 3

$\sin(2\pi x) = -\sqrt{3}\cos(2\pi x)$

Dividing both sides by $\cos(2\pi x)$ gives:

$\tan(2\pi x) = -\sqrt{3}$ for $0 \le x \le 1$.

Tan is negative in the 2nd and 4th quadrants.

Hence $2\pi x = \pi - \frac{\pi}{3}$ and $2\pi - \frac{\pi}{3}$

$2\pi x = \frac{2\pi}{3}$ and $\frac{5\pi}{3}$

$x = \frac{1}{3}$ and $\frac{5}{6}$

Question 4

a When $y = 0$: $2\log_e(a + 3) = -1$

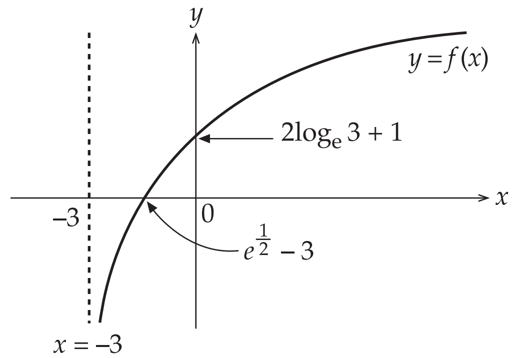
$a + 3 = e^{-0.5}$

$a = e^{-0.5} - 3$

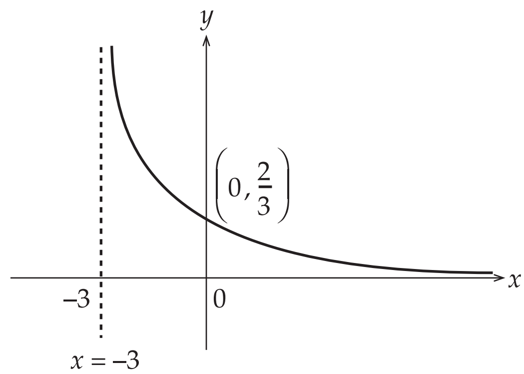
When $x = 0$: $b = 2\log_e 3 + 1$

b The vertical asymptote below must be labelled with its equation: $x = -3$ and the two intercepts should be labelled with their co-ordinates

$(0, 2\log_e 3 + 1)$ and $(e^{-0.5} - 3, 0)$



c The derivative function is shown below. Two asymptotes need to have their equations placed on the graph: vertical asymptote $x = -3$ and horizontal asymptote $y = 0$. The y-intercept has co-ordinates $(0, \frac{2}{3})$



Question 5

a Solving the intersection of

$y_1 = -x + 1$ and

$y_2 = 1 - e^{-x}$

on the graphics calculator gives the x value of 0.567 to three decimal places.

b $$\int_0^{0.567} (-x+1) - (1 - e^{-x})dx$$

$$= \int_0^{0.567} (e^{-x} - x)dx$$

$$= \left[-e^{-x} - \frac{x^2}{2}\right]_0^{0.567}$$

$= 0.272$

The answer is 0.27 to two decimal places.

Question 5

a Normalcdf (-E99,10,20,5) = 0.0228

Normalcdf (10,30,20,5) = 0.9545

Normalcdf (30,E99,20,5) = 0.0228

give the proportions for small, medium and large respectively.

b Expected cost per plant is found from multiplying the proportions by the item costs:

$= \$1.50 \times 0.0228 + \$2.50 \times 0.9545 + \$4.00 \times 0.0228$

$= \$2.5115$

The expected cost of 100 plants is: $\$2.5115 \times 100$

= $ 251 to the nearest dollar.