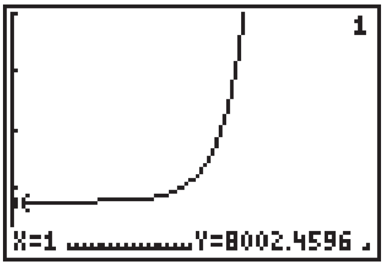
2003 Mathematical Methods Written Examination 2 (Analysis task) Suggested answers and solutions

Question 1

a $t = 1$



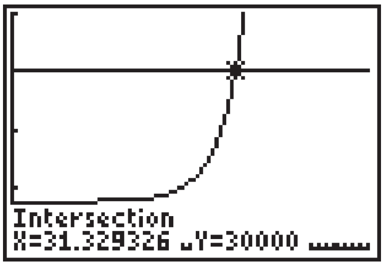
$$\frac{dA}{dt} = 8\,000 + e^{0.3(t+2)}$$

$$= 8\,000 + e^{0.9}$$

$$\approx 8002.46$$

≈ 8002 insects per day **[1A]**

b $\frac{dA}{dt} = 30\,000$



$30\,000 = 8\,000 + e^{0.3(t+2)}$ **[1M]**

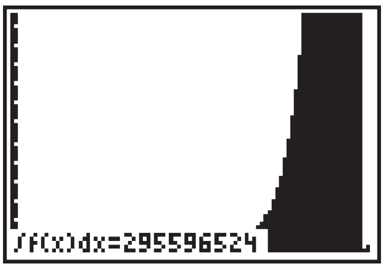
$$t = \frac{10}{3}\log_e(30\,000 - 8\,000) - 2$$

$t \approx 31.33$ days

The date is the 1st February 2002. **[1A]**

c $\int_{31}^{59} \frac{dA}{dt}\, dt$

$= 295596524$



d $\int_{31}^{59} \frac{dA}{dt}\, dt$ calculates the increase in the number of insects for February. **[1A]**

e $A = \int (8000 + e^{0.3(t+2)})dt$

$= 8000t + \frac{e^{0.3(t+2)}}{0.3} + C$ **[1M]**

When $t = 3$, $A = 50\,000$

$50\,000 = 24\,000 + \frac{e^{1.5}}{0.3} + C$ **[1M]**

$$C = 26\,000 - \frac{e^{1.5}}{0.3}$$

$A = 8000t + \frac{e^{0.3(t+2)}}{0.3} + 26\,000 - \frac{e^{1.5}}{0.3}$ **[1A]**

f $t_2 = 30 + 31 + 30 + 21 = 112$ days

$A_2 = a\,(t_2 - 112)^2 + 8\,000$

$$A(90) = 720\,000 + \frac{e^{27.6}}{0.3} + 26\,000 - \frac{e^{1.5}}{0.3}$$

$$\approx 3.23 \times 10^{12}$$

$$746\,000 + \frac{e^{27.6}}{0.3} - \frac{e^{1.5}}{0.3} = a(-112)^2 + 8\,000$$

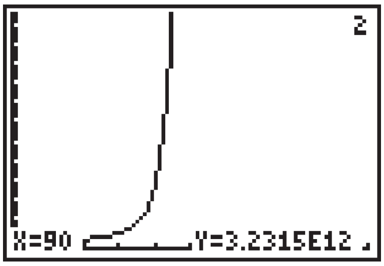
$a = 257\,614\,616$ **[1A]**

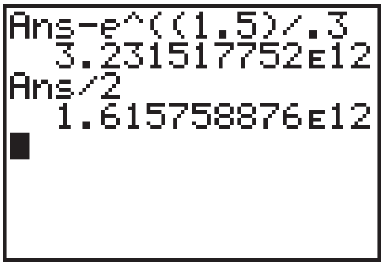
$b = 112$ **[1A]**

$c = 8\,000$ **[1A]**

g The maximum population occurs at midnight on 31 March.

$t = 90$ for model 1 or $t_2 = 0$ for model 2.





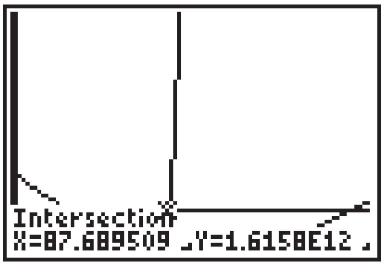
Half the maximum is $373\,000 + \frac{e^{27.6}}{0.6} - \frac{e^{1.5}}{0.6}$

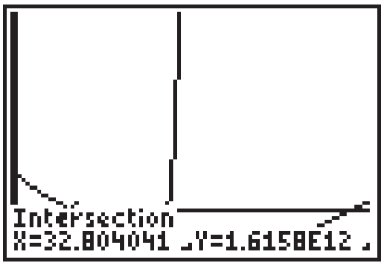
$\approx 1.62 \times 10^{12}$.

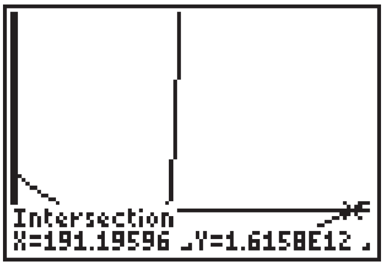
For model 1 this occurs on day 87.7, March 29 2002. **[1A]**

For model 2 this occurs on day 32.8, May 3 2002 **[1A]**

and day 191.2, October 9 2002. **[1A]**







Question 2

a From graph, period = 200 mm **[1A]**

b Amplitude = 50 mm **[1A]**

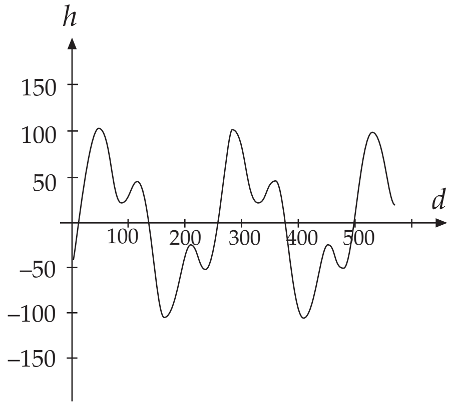
c Various; either 150 mm to right or 50 mm to left **[1A]**

d As period = 200, $\therefore n = \frac{\pi}{100}$

$h = 50 \sin \frac{\pi}{100}(d + 50)$ **[1A]**

e 5 km/200mm = 25000 **[1A]**

f



Correct shape, domain, range **[3A]**

g $h = 0$ at 12.08, 132.08, 252.08 …….

Period = 252.08 – 12.08 = 240 **[1A]**

h Absolute maximum value = 106.61

Absolute minimum value = –106.61 **[1A]**

i Local maxima at (43.39, 106.61), (111.65,47.97) and (204.96, –22.55) **[1A]**

j $\left|\int_0^{12.08} h(d)dd\right| + \int_{12.08}^{50} h(d)dd$ **[1M]**

$= 271.83 + 2755.28 = 3027.11 \text{ mm}^2$ **[1A]**

Question 3

a Let $y = 2 + \frac{3}{x+1}$

Inverse $x = 2 + \frac{3}{y+1}$

$x - 2 = \frac{3}{y+1}$

$(x - 2)(y + 1) = 3$

$y = \frac{3}{x-2} - 1$

$f^{-1}: R \setminus \{2\} \to R$, where $f^{-1}(x) = \frac{3}{x-2} - 1$ **[1A]**

b $f(x)$ has been translated 3 units parallel to the x-axis **[1A]**

and -3 units parallel to the y-axis. **[1A]**

c $f(x) = f^{-1}(x) = x$ at the intersection.

$x = 2 + \frac{3}{x+1}$ **[1M]**

$x - 2 = \frac{3}{x+1}$

$(x-2)(x+1) = 3$

$x^2 - x - 5 = 0$ **[1M]**

$x = \frac{1 \pm \sqrt{21}}{2}$

The coordinates are:

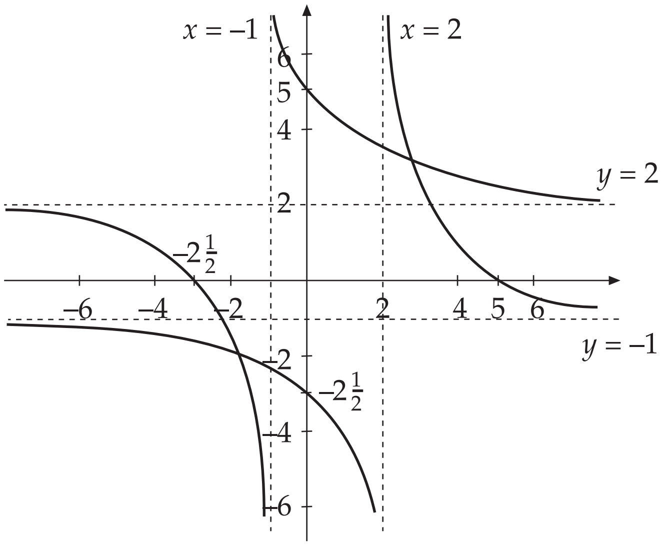
$\left(\frac{1+\sqrt{21}}{2}, \frac{1+\sqrt{21}}{2}\right)$ and $\left(\frac{1-\sqrt{21}}{2}, \frac{1-\sqrt{21}}{2}\right)$

d Correct intercepts **[1A]**

Correct asymptotes **[1A]**

Correct shape for $f(x)$ **[1A]**

Correct shape for $f^{-1}(x)$ **[1A]**



e Area of the plaque = 3 × 3 = 9 units squared **[1A]**

f i $A + \frac{B}{x - C} = x$

$(x - C)(x - A) = B$

$x^2 - Ax - Cx + CA - B = 0$

$x^2 + (-A - C)x + CA - B = 0$

$$x = \frac{A + C \pm \sqrt{A^2 - 2AC + C^2 + 4B}}{2}$$ **[2A]**

ii $A^2 - 2AC + C^2 + 4B > 0$ **[1A]**

or $(A - C)^2 > -4B$

iii $\int_{\frac{A+C-\sqrt{A^2-2AC+C^2+4B}}{2}}^{\frac{A+C+\sqrt{A^2-2AC+C^2+4B}}{2}} (g(x) - g^{-1}(x))dx$ **[1A]**

Question 4

a $E(X) = 0 + 0.33 + 0.34 + 0.66 + 0.32 + 0.1$

$= 1.75$ **[1A]**

$E(\text{fee}) = 5 + 2 \times E(X) = \$8.50$ **[1A]**

b $n = 18, p = .18$ **[1M]**

$\Pr(V \geq 4) = 1 - [\Pr(V = 0) + \Pr(V = 1) + \Pr(V = 2) + \Pr(V = 3)] = 1 - 0.589 = 0.411$ **[1A]**

c $N = 21, D = 7, n = 10, x = 5$ **[1M]**

$\Pr(X = 5) = \frac{77}{646} = 0.119$ **[1A]**

d $\Pr(z \leq \frac{5.1 - \mu}{\sigma}) = .90;$

$\Pr(z \leq \frac{3.6 - \mu}{\sigma}) = 0.05$ **[1A]**

$\frac{5.1 - \mu}{\sigma} = 1.2816;\ \frac{3.6 - \mu}{\sigma} = -1.6449$ **[1A]**

solve simultaneously **[1M]**

correct solutions **[1A]**

e Pr (length ≥ 4.5) = 0.4557;

Pr (length ≥ 4.8) = 0.2428 **[1A]**

Pr (5 vehicles ≥ 4.5) = 0.4557^5

As Pr (4 out of 5 vehicles ≥ 4.8) ⊂ Pr (5 vehicles ≥ 4.5)

we require 4 in the inner set and 1 in the "donut"

Pr (4 out of 5 vehicles ≥ 4.8 ∩ one vehicle being ≥ 4.5

but ≤ 4.8) = ${}^5C_4\, 0.2428^4\ (0.4557 - 0.2428)$ **[1M]**

Pr (4 out of 5 vehicles ≥ 4.8 | all 5 vehicles ≥ 4.5)

$= {}^5C_4\ 0.2428^4\ \frac{0.2129}{0.4557^5}$

$= 0.1883$ **[1A]**