

Year 2003

VCE

Mathematical Methods Trial Examination 2

Suggested Solutions

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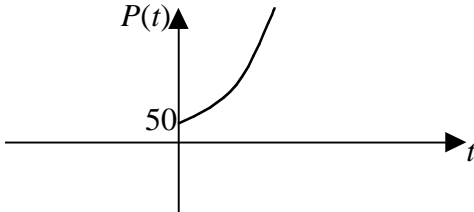
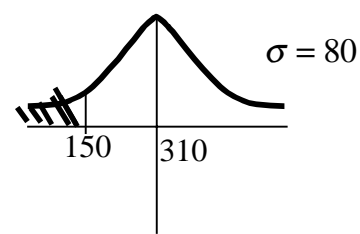


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<p>These solutions are suggested solutions only. Teachers and students should carefully read the answers and comments supplied by the Mathematics Examiners.</p>
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<p>Question 1</p> <p>a. i.</p>  <p>1 mark for shape of graph 1 mark for point (0,50)</p>	<p>ii.</p> <p>$t = 5$</p> <p>$P(t) = 50 \times e^{0.05 \times 5}$</p> <p>$P(t) = 50 \times e^{0.25}$</p> <p>$P(t) = 64$ million people to the nearest million (1 mark)</p>
<p>iii.</p> <p>Average rate of increase = $\frac{\Delta P}{\Delta t}$ (1 mark)</p> <p>In 2003, when $t = 0$, $P = 50 \times e^0 = 50$ (1 mark)</p> <p>$\therefore \frac{\Delta P}{\Delta t} = \frac{64 - 50}{2008 - 2003} = \frac{14}{5} = 2.8$ million / year (1 mark)</p>	<p>iv.</p> <p>$\frac{dP}{dt} = 50 \times 0.05 e^{0.05t}$</p> <p>$\frac{dP}{dt} = 2.5 e^{0.05t}$ (1 mark)</p> <p>When $t = 20$</p> <p>$\frac{dP}{dt} = 2.5 \times e^{0.05 \times 20}$</p> <p>$\frac{dP}{dt} = 2.5e$ million / year (1 mark)</p>
<p>b.i.</p> <p>$R = \log_{10} \frac{3 \times 10^6}{3 \times 10^{12}} + 12$</p> <p>$R = \log_{10} 10^{-6} + 12$ (1 mark)</p> <p>$R = -6 + 12$</p> <p>$R = 6$ (1 mark)</p>	<p>ii</p> <p>$R = \log_{10} \frac{100a}{3 \times 10^{12}} + 12$ (1 mark)</p> <p>$\Rightarrow R = \log_{10} \frac{a}{3 \times 10^{10}} + 12$</p>
<p>b. iii</p> <p>$R_1 = \log_{10} \left(\frac{a}{T} \right) + B$</p> <p>$R_2 = \log_{10} \left(\frac{100a}{T} \right) + B$</p> <p>Change in intensity = $R_2 - R_1$</p> <p>$= \log_{10} \left(\frac{100a}{T} \right) + B - \left[\log_{10} \left(\frac{a}{T} \right) + B \right]$ (1 mark)</p> <p>$= \log_{10} 100a - \log_{10} T + B - [\log_{10} a - \log_{10} T + B]$</p> <p>$= \log_{10} 100a - \log_{10} T + B - \log_{10} a + \log_{10} T - B$ (1 mark)</p> <p>$= \log_{10} 100a - \log_{10} a = \log_{10} \frac{100a}{a}$ (1 mark)</p> <p>$= \log_{10} 100 = 2$ (1 mark)</p>	<p>Question 2</p> <p>a.</p>  <p>$\Pr(X < 150) = \Pr(Z < -2)$ $Z = \frac{x - \mu}{\sigma}$</p> <p>$\Pr(X < 150) = \Pr(Z > 2)$ $Z = \frac{150 - 310}{80} = -2$</p> <p>$\Pr(X < 150) = 1 - \Pr(Z < 2)$ (1 mark)</p> <p>$\Pr(X < 150) = 1 - 0.9772$</p> <p>$\Pr(X < 150) = 0.0228$ (1 mark)</p>

<p>Question 2</p> <p>b. Binomial with $p = 0.0228$ $q = 0.9772$ $n = 10$ $x = 4$ (1 mark)</p> $\Pr(X = 4) = \binom{10}{4} (0.0228)^4 (0.9772)^6$ $\Rightarrow \Pr(X = 4) = 5 \times 10^{-5} \quad (1 \text{ mark})$	<p>c. Conditional probability $\Pr x < 150 / x < 310$ 310 is the mean $\therefore \Pr x < 310 = 0.5$ (1 mark)</p> $\Pr x < 150 / x < 310 = \frac{\Pr x < 150}{\Pr x < 310} \quad (1 \text{ mark})$ $= \frac{0.0228}{0.5}$ $= 0.0456 \quad (1 \text{ mark})$
<p>d. In 2110, $t = 7$ (1 mark) $r(7) = 30 \times 7 + 220 = 430$ (1 mark)</p> $Z = \frac{x - \mu}{\sigma}$ $Z = \frac{430 - 310}{80} = 1.5 \quad (1 \text{ mark})$ $\Pr(X > 430) = \Pr(Z > 1.5)$ $\Pr(X > 430) = 1 - \Pr(Z < 1.5)$ $\Pr(X > 430) = 1 - 0.9332$ $\Pr(X > 430) = 0.0668 \quad (1 \text{ mark})$	<p>e. $\Pr(X < -a) = 0.346$ $\Pr(X > a) = 0.346$ by symmetry $\Pr(X < a) = 1 - 0.346 = 0.654$ (1 mark) $a = 0.396$ (from tables) $\therefore -a = -0.396$ (1 mark)</p> $Z = \frac{x - \mu}{\sigma}$ $-0.396 = \frac{x - 310}{80}$ $x = -0.396 \times 80 + 310 = 278.32 \text{ cm} \quad (1 \text{ mark})$
<p>Question 3</p> <p>a.</p> $C = a + \frac{b}{x^2}$ $200 = a + \frac{b}{1,000,000}$ $140 = a + \frac{b}{25,000,000}$ $\Rightarrow 200 - \frac{b}{1,000,000} = 140 - \frac{b}{25,000,000}$ $\Rightarrow 60 = \frac{b}{1,000,000} - \frac{b}{25,000,000}$ $\Rightarrow 60 = \frac{25b}{25,000,000} - \frac{b}{25,000,000}$ $60 \times 25,000,000 = 24b$ $\Rightarrow b = \frac{60 \times 25,000,000}{24} \quad (1 \text{ mark})$ $\Rightarrow b = 62,500,000$	<p>b.</p> $200 = a + \frac{62,500,000}{1,000,000}$ $200 = a + 62.5$ $a = 200 - 62.5$ $a = 137.5 \quad (1 \text{ mark})$

Question 3

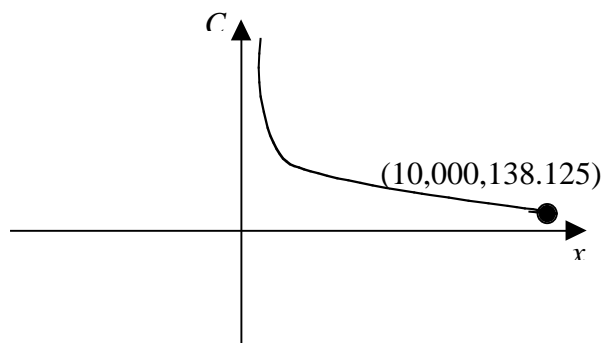
c.

$$C = 137.5 + \frac{62,500,000}{x^2}$$

When $x = 1000$, $C = 200$

When $x = 5000$, $C = 140$

When $x = 10,000$, $C = 137.5 + 0.625 = 138.125$



1 mark for shape, 1 mark for end point

d.

$$R = Cx = \left(a + \frac{b}{x^2}\right)x$$

$$R = ax + \frac{b}{x}$$

$$\frac{dR}{dx} = a - \frac{b}{x^2} = 0 \text{ for turning point} \quad (1 \text{ mark})$$

$$a = \frac{b}{x^2}$$

$$x^2 = \frac{b}{a}$$

$$x = \pm \sqrt{\frac{b}{a}}$$

But $x > 0$

$$\therefore x = \sqrt{\frac{b}{a}} = 674 \quad (1 \text{ mark})$$

When $x < 674$ $\frac{dR}{dx} > 0$

When $x > 674$ $\frac{dR}{dx} < 0$

\therefore Maximum revenue when 674 chips (1 mark)

e.

$$y = a + \frac{b}{x^2}$$

Interchange x and y

$$x = a + \frac{b}{y^2}$$

$$x - a = \frac{b}{y^2}$$

$$y^2 = \frac{b}{x - a}$$

$$y = \pm \sqrt{\frac{b}{x - a}} \quad (1 \text{ mark})$$

But $y > 0$

$$\therefore y = \sqrt{\frac{b}{x - a}}$$

$$C^{-1}(x) = \sqrt{\frac{b}{x - a}} \quad (1 \text{ mark})$$

f.

$$C^{-1}(300) = \sqrt{\frac{62,500,000}{300 - 137.5}} = 1961.16$$

1962 chips must be produced. (1 mark)

<p>Question 4</p> <p>a. $\sin[2\pi(t+0.5)] > 0$ (1 mark) $0 < [2\pi(t+0.5)] < \pi$ (1 mark) $0 < 2t+1 < 1$ $-1 < 2t < 0$ $-\frac{1}{2} < t < 0$ This domain contains $t = -0.25$ So the domain is $(-0.5, 0)$ (1 mark)</p>	<p>b. sin is a cyclic curve that is positive in the domains, $(0, \pi)$, $(2\pi, 3\pi)$, $(4\pi, 5\pi)$.....and negative or zero in the remainder of the domain. You can only get the log of a positive number. (1 mark)</p>
<p>c. $\log_e 1 = 0$ $\therefore \sin 2\pi(t+0.5) = 1$ (1 mark) $\Rightarrow 2\pi(t+0.5) = \frac{\pi}{2}$ $2t+1 = 0.5$ $t = -0.25$ which is in the required domain (1 mark)</p>	<p>d $\int_{-0.25}^{-0.04} f(t) dt$ Find To do this use graphics calculator. $\text{fnInt}(Y_1, t, -0.25, -0.04)$ Required area = 0.0779 (1 mark)</p>
<p>e. Slope = $f^{-1}(t)$ $= \frac{1}{\sin[2\pi(t+0.5)]} \times \cos[2\pi(t+0.5)] \times 2\pi$ (1 mark) $= \frac{2\pi \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}}$ (1 mark) But $\sin \frac{\pi}{4} = \cos \frac{\pi}{4}$ $\therefore f^{-1}(t) = 2\pi$</p>	<p>f. $g(t) = f^{-1}(t)$ Interchange y and t $t = \log_e \sin[2\pi(y+0.5)]$ (1 mark) $e^t = \sin[2\pi(y+0.5)]$ $e^t = \sin(2\pi y + \pi)$ $\sin^{-1}(e^t) = 2\pi y + \pi$ (1 mark) $\sin^{-1}(e^t) - \pi = 2\pi y$ $\therefore y = \frac{1}{2\pi} \sin^{-1}(e^t) - 0.5$ $g(t) = \frac{1}{2\pi} \sin^{-1}(e^t) - 0.5$ (1 mark)</p>

<p>Question 4</p> <p>g. Domain of \sin^{-1} function must contain numbers in the interval $[-1,1]$ (1 mark) $\therefore -1 \leq e^t \leq 1$ But $e^t > 0$ always This reduces to $e^t \leq 1$ (1 mark) $\Rightarrow t \leq 0$ So domain of $g(t)$ is $(-\infty, 0]$</p>	<p>h. $g(t) = \frac{1}{2\pi} \sin^{-1}(e^t) - 0.5$ Since $\sin t$ is a continuous function $\lim_{t \rightarrow \infty} g(t) = \frac{1}{2\pi} \sin^{-1}(\lim_{t \rightarrow \infty} e^t) - 0.5$ (1 mark) $\lim_{t \rightarrow \infty} g(t) = \frac{1}{2\pi} \sin^{-1}(0) - 0.5$ $\lim_{t \rightarrow \infty} g(t) = -0.5$ (1 mark)</p>
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END OF SUGGESTED SOLUTIONS
2003 Mathematical Methods Trial Examination 2

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