Year 2003

VCE

Mathematical Methods Trial Examination 2

Suggested Solutions

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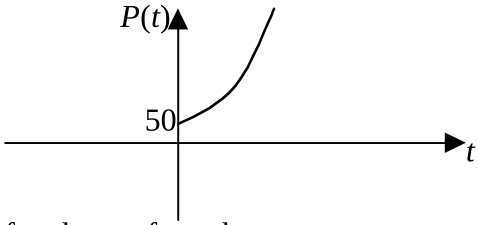
These solutions are suggested solutions only. Teachers and students should carefully read the answers and comments supplied by the Mathematics Examiners.

Question 1

a. i.



1 mark for shape of graph
1 mark for point (0,50)

ii.

$t = 5$

$P(t) = 50 \times e^{0.05\times 5}$

$P(t) = 50 \times e^{0.25}$

$P(t) = 64$ million people to the nearest million (1 mark)

iii.

Average rate of increase $= \dfrac{\Delta P}{\Delta t}$ (1 mark)

In 2003, when $t = 0$, $P = 50 \times e^0 = 50$ (1 mark)

$\therefore \dfrac{\Delta P}{\Delta t} = \dfrac{64-50}{2008-2003} = \dfrac{14}{5} = 2.8$ million / year (1 mark)

iv.

$\dfrac{dP}{dt} = 50 \times 0.05e^{0.05t}$

$\dfrac{dP}{dt} = 2.5e^{0.05t}$ (1mark)

When $t = 20$

$\dfrac{dP}{dt} = 2.5 \times e^{0.05\times 20}$

$\dfrac{dP}{dt} = 2.5e$ million / year (1 mark)

b.i.

$R = \log_{10}\dfrac{3\times 10^6}{3\times 10^{12}} + 12$

$R = \log_{10}10^{-6} + 12$ (1 mark)

$R = -6 + 12$

$R = 6$ (1 mark)

ii

$R = \log_{10}\dfrac{100a}{3\times 10^{12}} + 12$ (1 mark)

$\Rightarrow R = \log_{10}\dfrac{a}{3\times 10^{10}} + 12$

b. iii

$R_1 = \log_{10}\left(\dfrac{a}{T}\right) + B$

$R_2 = \log_{10}\left(\dfrac{100a}{T}\right) + B$

Change in intensity $= R_2 - R_1$

$= \log_{10}\left(\dfrac{100a}{T}\right) + B - \left[\log_{10}\left(\dfrac{a}{T}\right) + B\right]$ (1 mark)

$= \log_{10}100a - \log_{10}T + B - [\log_{10}a - \log_{10}T + B]$

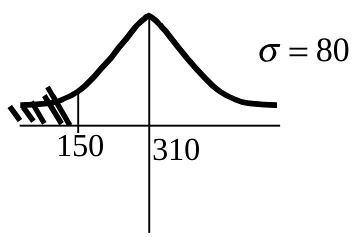
$= \log_{10}100a - \log_{10}T + B - \log_{10}a + \log_{10}T - B$ (1 mark)

$= \log_{10}100a - \log_{10}a = \log_{10}\dfrac{100a}{a}$ (1 mark)

$= \log_{10}100 = 2$ (1 mark)

Question 2

a.



$\Pr(X < 150) = \Pr(Z < -2)$ $\quad Z = \dfrac{x-\mu}{\sigma}$

$\Pr(X < 150) = \Pr(Z > 2)$ $\quad Z = \dfrac{150-310}{80} = -2$

$\Pr(X < 150) = 1 - \Pr(Z < 2)$ (1 mark)

$\Pr(X < 150) = 1 - 0.9772$

$\Pr(X < 150) = 0.0228$ (1 mark)

Question 2

b.

Binomial with

$p = 0.0228$

$q = 0.9772$

$n = 10$

$x = 4$ (1 mark)

$\Pr(X = 4) = \binom{10}{4}(0.0228)^4(0.9772)^6$

$\Rightarrow \Pr(X = 4) = 5 \times 10^{-5}$ (1 mark)

c.

Conditional probability

$\Pr x < 150 / x < 310$

310 is the mean

$\therefore \Pr x < 310 = 0.5$ (1 mark)

$\Pr x < 150 / x < 310 = \frac{\Pr x < 150}{\Pr x < 310}$ (1 mark)

$= \frac{0.0228}{0.5}$

$= 0.0456$ (1 mark)

d.

In 2110, $t = 7$ (1 mark)

$r(7) = 30 \times 7 + 220 = 430$ (1 mark)

$Z = \frac{x - \mu}{\sigma}$

$Z = \frac{430 - 310}{80} = 1.5$ (1 mark)

$\Pr(X > 430) = \Pr(Z > 1.5)$

$\Pr(X > 430) = 1 - \Pr(Z < 1.5)$

$\Pr(X > 430) = 1 - 0.9332$

$\Pr(X > 430) = 0.0668$ (1 mark)

e.

$\Pr(X < -a) = 0.346$

$\Pr(X > a) = 0.346$ by symmetry

$\Pr(X < a) = 1 - 0.346 = 0.654$ (1 mark)

$a = 0.396$ (from tables)

$\therefore -a = -0.396$ (1 mark)

$Z = \frac{x - \mu}{\sigma}$

$-0.396 = \frac{x - 310}{80}$

$x = -0.396 \times 80 + 310 = 278.32$ cm (1 mark)

Question 3

a.

$C = a + \frac{b}{x^2}$

$200 = a + \frac{b}{1{,}000{,}000}$

$140 = a + \frac{b}{25{,}000{,}000}$

$\Rightarrow 200 - \frac{b}{1{,}000{,}000} = 140 - \frac{b}{25{,}000{,}000}$

$\Rightarrow 60 = \frac{b}{1{,}000{,}000} - \frac{b}{25{,}000{,}000}$

$\Rightarrow 60 = \frac{25b}{25{,}000{,}000} - \frac{b}{25{,}000{,}000}$

$60 \times 25{,}000{,}000 = 24b$

$\Rightarrow b = \frac{60 \times 25{,}000{,}000}{24}$ (1 mark)

$\Rightarrow b = 62{,}500{,}000$

b.

$200 = a + \frac{62{,}500{,}000}{1{,}000{,}000}$

$200 = a + 62.5$

$a = 200 - 62.5$

$a = 137.5$ (1 mark)

Question 3

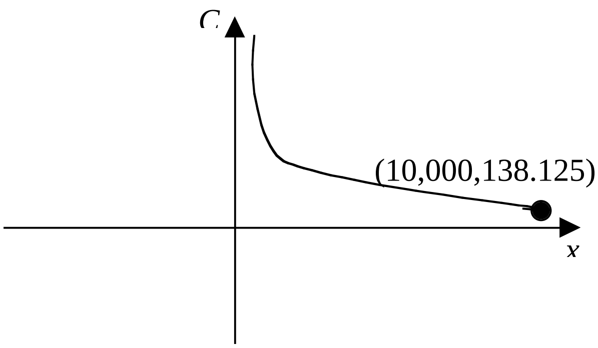
c.

$$C = 137.5 + \frac{62{,}500{,}000}{x^2}$$

When $x = 1000$, $C = 200$

When $x = 5000$, $C = 140$

When $x = 10{,}000$, $C = 137.5 + 0.625 = 138.125$



1 mark for shape, 1 mark for end point

d.

$$R = Cx = \left(a + \frac{b}{x^2}\right)x$$

$$R = ax + \frac{b}{x}$$

$$\frac{dR}{dx} = a - \frac{b}{x^2} = 0 \text{ for turning point} \quad \text{(1 mark)}$$

$$a = \frac{b}{x^2}$$

$$x^2 = \frac{b}{a}$$

$$x = \pm\sqrt{\frac{b}{a}}$$

But $x > 0$

$$\therefore x = \sqrt{\frac{b}{a}} = 674 \quad \text{(1 mark)}$$

When $x < 674$ $\quad \frac{dR}{dx} > 0$

When $x > 674$ $\quad \frac{dR}{dx} < 0$

$\therefore$ Maximum revenue when 674 chips (1 mark)

e.

$$y = a + \frac{b}{x^2}$$

Interchange x and y

$$x = a + \frac{b}{y^2}$$

$$x - a = \frac{b}{y^2}$$

$$y^2 = \frac{b}{x-a}$$

$$y = \pm\sqrt{\frac{b}{x-a}} \quad \text{(1 mark)}$$

But $y > 0$

$$\therefore y = \sqrt{\frac{b}{x-a}}$$

$$C^{-1}(x) = \sqrt{\frac{b}{x-a}} \quad \text{(1 mark)}$$

f.

$$C^{-1}(300) = \sqrt{\frac{62{,}500{,}000}{300 - 137.5}} = 1961.16$$

1962 chips must be produced. (1 mark)

Question 4

a.

$\sin[2\pi(t+0.5)]>0$ (1 mark)

$0<[2\pi(t+0.5)]<\pi$ (1 mark)

$0<2t+1<1$

$-1<2t<0$

$-\frac{1}{2}<t<0$

This domain contains $t=-0.25$

So the domain is (–0.5,0) (1 mark)

b.

sin is a cyclic curve that is positive in the domains, (0, π),

(2π,3π), (4π,5π)…….and negative or zero in the remainder of the domain. You can only get the log of a positive number.

(1 mark)

c.

$\log_e 1=0$

$\therefore \sin 2\pi(t+0.5)=1$ (1 mark)

$\Rightarrow 2\pi(t+0.5)=\frac{\pi}{2}$

$2t+1=0.5$

$t=-0.25$ which is in the required domain

(1 mark)

d

Find $\int_{-0.25}^{-0.04} f(t)dt$

To do this use graphics calculator.

fnInt(Y_1, t,–0.25,–0.04)

Required area = 0.0779

(1 mark)

e.

Slope $= f^1(t)$

$=\frac{1}{\sin[2\pi(t+0.5)[}\times\cos[2\pi(t+0.5)]\times 2\pi$(1 mark)

$=\frac{2\pi\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}$ (1 mark)

But $\sin\frac{\pi}{4}=\cos\frac{\pi}{4}$

$\therefore f^1(t)=2\pi$

f.

$g(t)=f^{-1}(t)$

Interchange y and t

$t=\log_e \sin[2\pi(y+0.5)]$ (1 mark)

$e^t=\sin[2\pi(y+0.5)]$

$e^t=\sin(2\pi y+\pi)]$

$\sin^{-1}(e^t)=2\pi y+\pi$ (1 mark)

$\sin^{-1}(e^t)-\pi=2\pi y$

$\therefore y=\frac{1}{2\pi}\sin^{-1}(e^t)-0.5$

$g(t)=\frac{1}{2\pi}\sin^{-1}(e^t)-0.5$ (1 mark)

Question 4	**h.**
g. Domain of $\sin^{-1}$ function must contain numbers in the interval [-1,1] (1 mark) $\therefore -1 \le e^t \le 1$ But $e^t > 0$ always This reduces to $e^t \le 1$ (1 mark) $\Rightarrow t \le 0$ So domain of *g(t)* is $(-\infty, 0]$	$g(t) = \frac{1}{2\pi}\sin^{-1}(e^t) - 0.5$ Since $\sin t$ is a continuous function $\lim_{t\to\infty} g(t) = \frac{1}{2\pi}\sin^{-1}(\lim_{t\to\infty} e^t) - 0.5$ (1 mark) $\lim_{t\to\infty} g(t) = \frac{1}{2\pi}\sin^{-1}(0) - 0.5$ $\lim_{t\to\infty} g(t) = -0.5$ (1 mark)

END OF SUGGESTED SOLUTIONS
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